Photonic Crystals:
A Crash Course in Designer Electromagnetism

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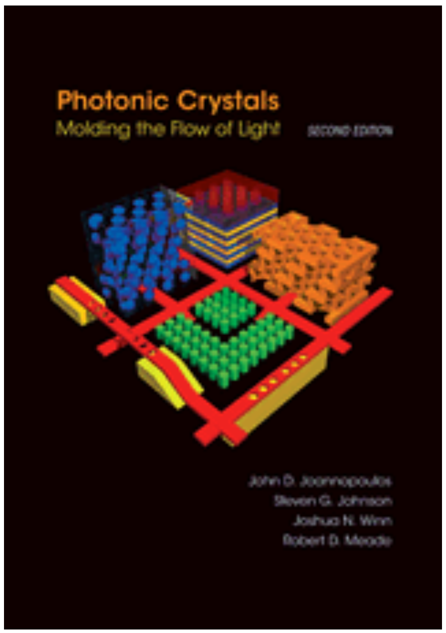
MIT Applied Mathematics

Free Materials Online



Photonic Crystals book: jdj.mit.edu/book

Tutorial slides: jdj.mit.edu/photons/tutorial

Free electromagnetic simulation software (FDTD, mode solver, etc.)
jdj.mit.edu/wiki

Outline

- Preliminaries: waves in periodic media
- Photonic crystals in theory and practice
- Bulk crystal properties
- Intentional defects and devices
- Index-guiding and incomplete gaps
- Photonic-crystal fibers
- Perturbations, tuning, and disorder

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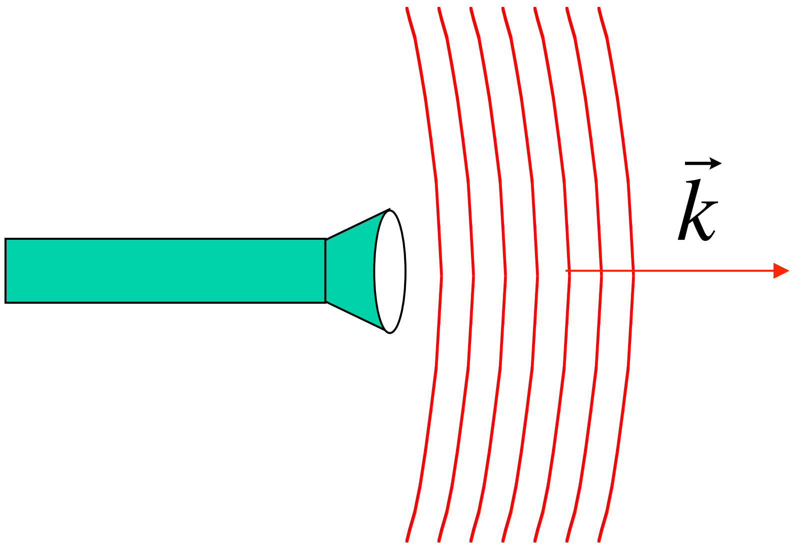
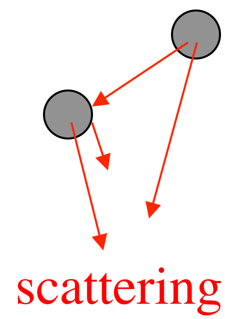
To Begin: A Cartoon in 2d

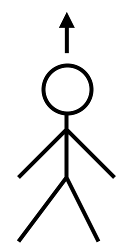


planewave

$$\vec{E},\vec{H} \sim e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\left|\vec{k}\right| = \omega / c = \frac{2\pi}{\lambda}$$

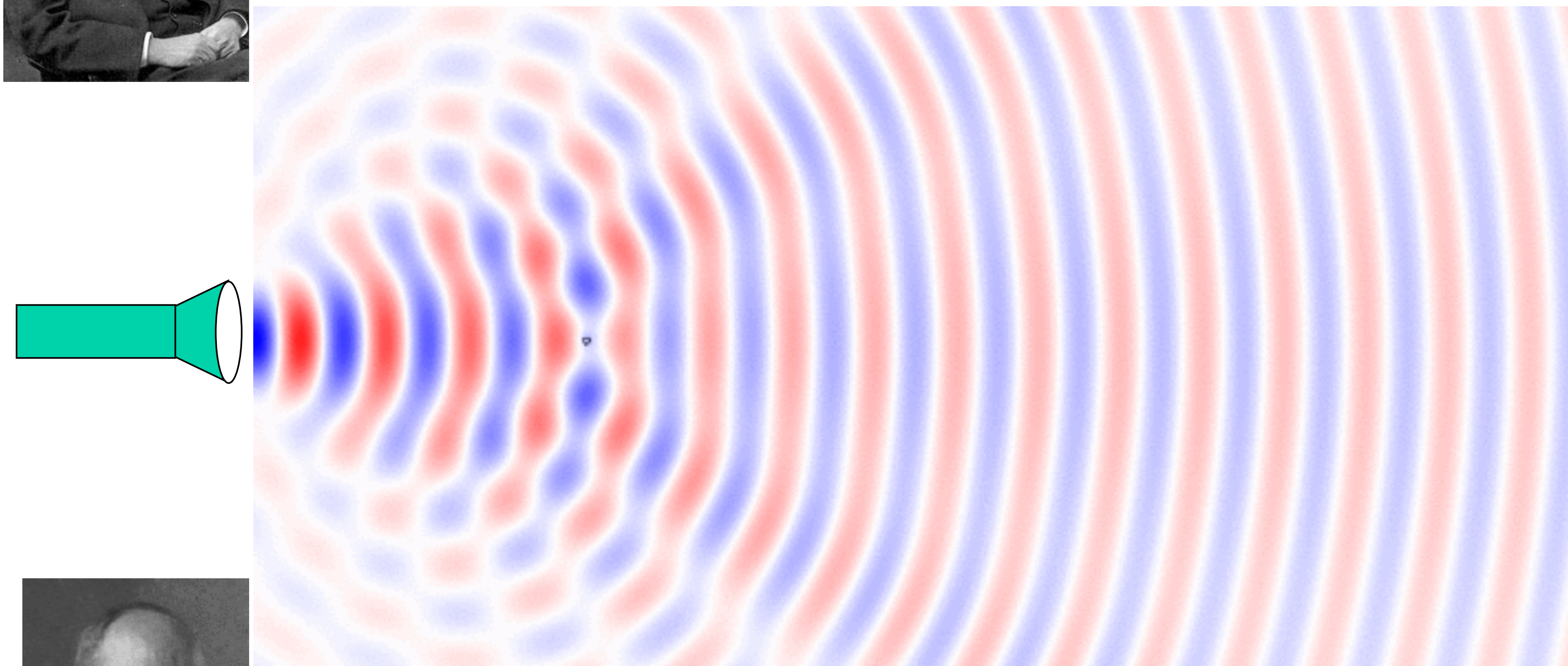




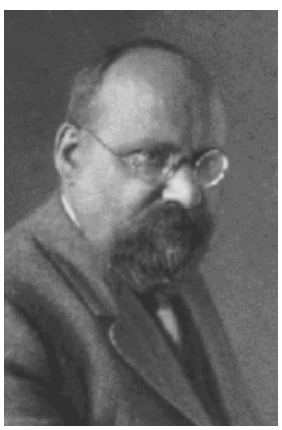
…Waves Can Scatter

small particles:
Lord Rayleigh (1871)
why the sky is blue

here: a little circular speck of silicon



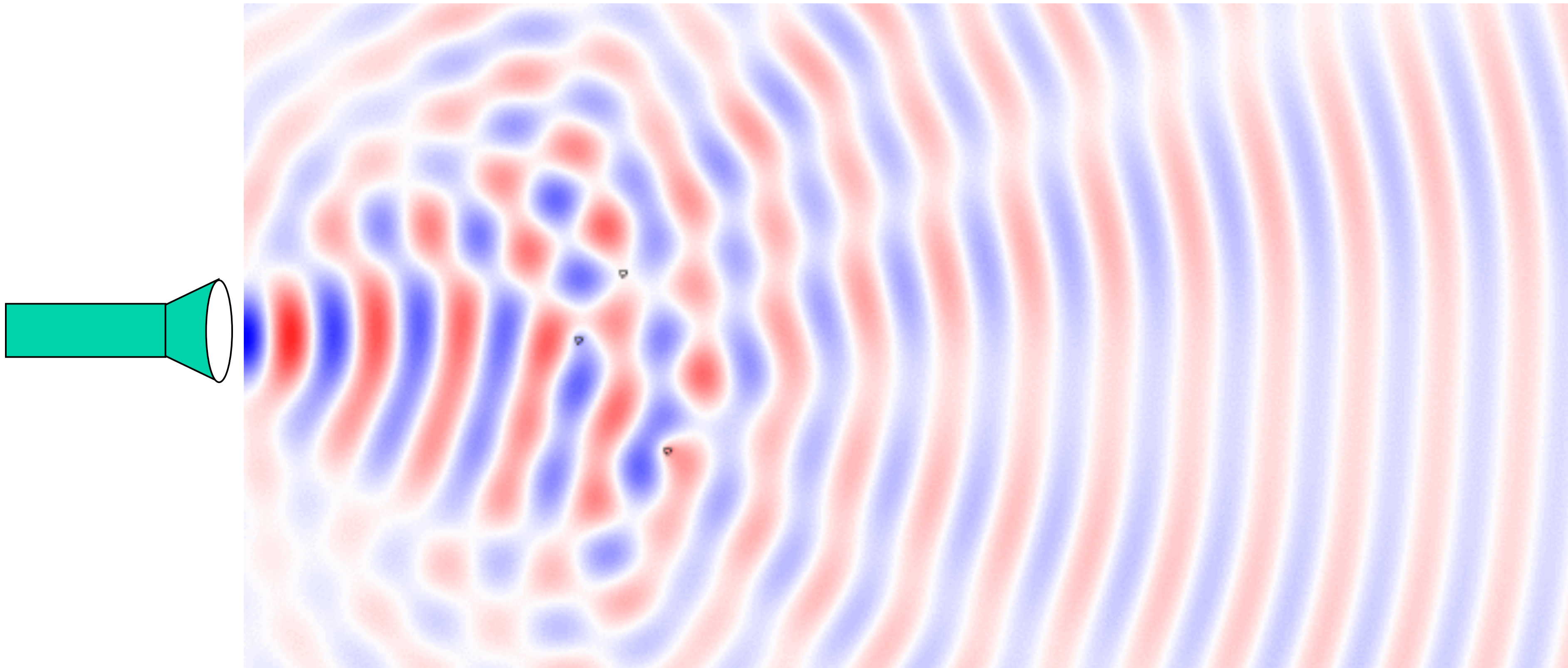
checkerboard pattern: interference of waves traveling in different directions



scattering by spheres:
solved by Gustave Mie (1908)

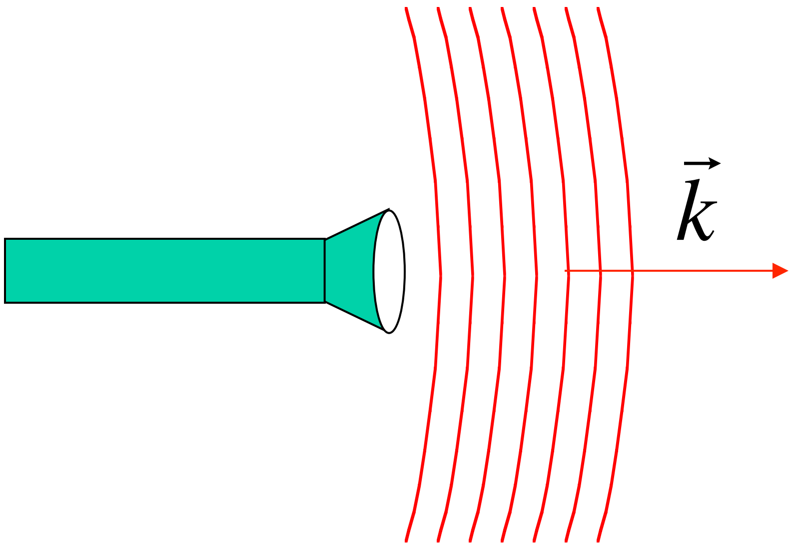
Multiple Scattering is Just Messier?

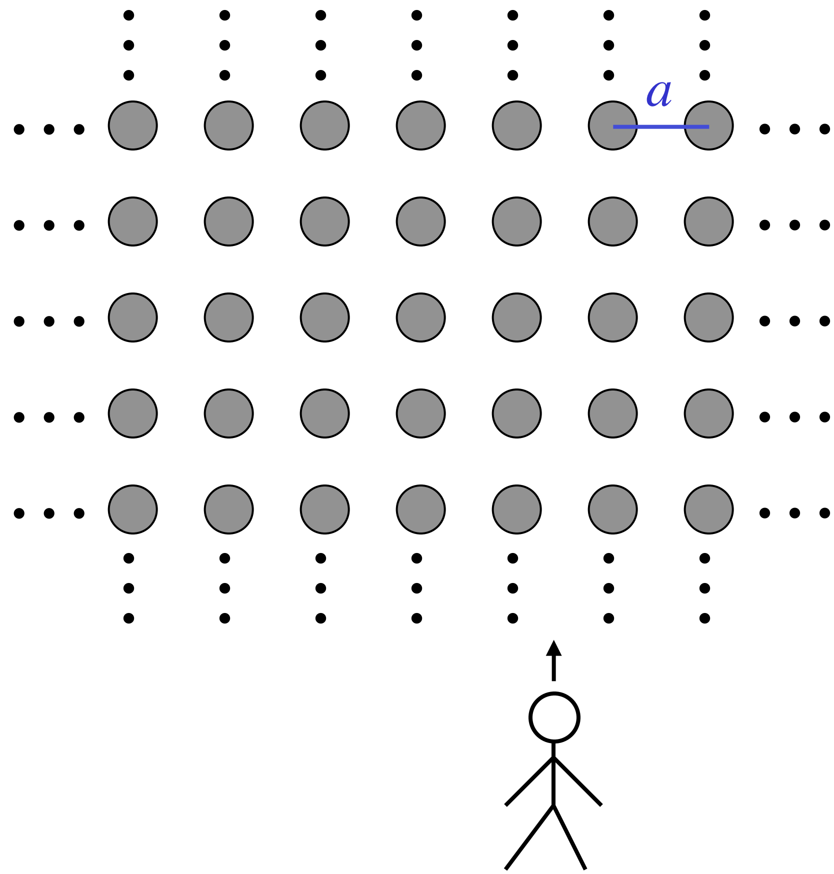
here: scattering off three specks of silicon



can be solved on a computer, but not terribly interesting…

To Begin: A Cartoon in 2d





planewave

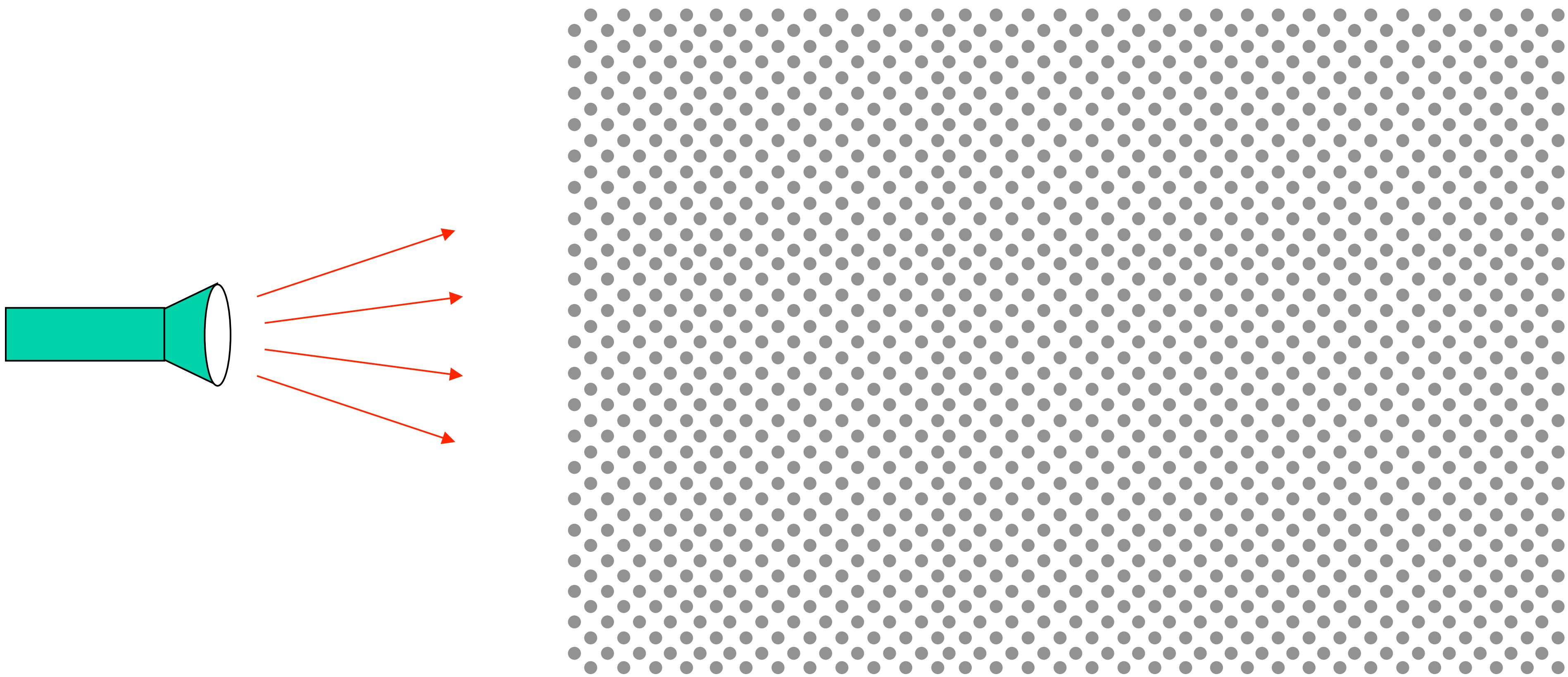
$$\vec{E}, \vec{H} \sim e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\left|\vec{k}\right| = \omega / c = \frac{2\pi}{\lambda}$$

for most λ, beam(s) propagate through crystal without scattering (scattering cancels coherently)

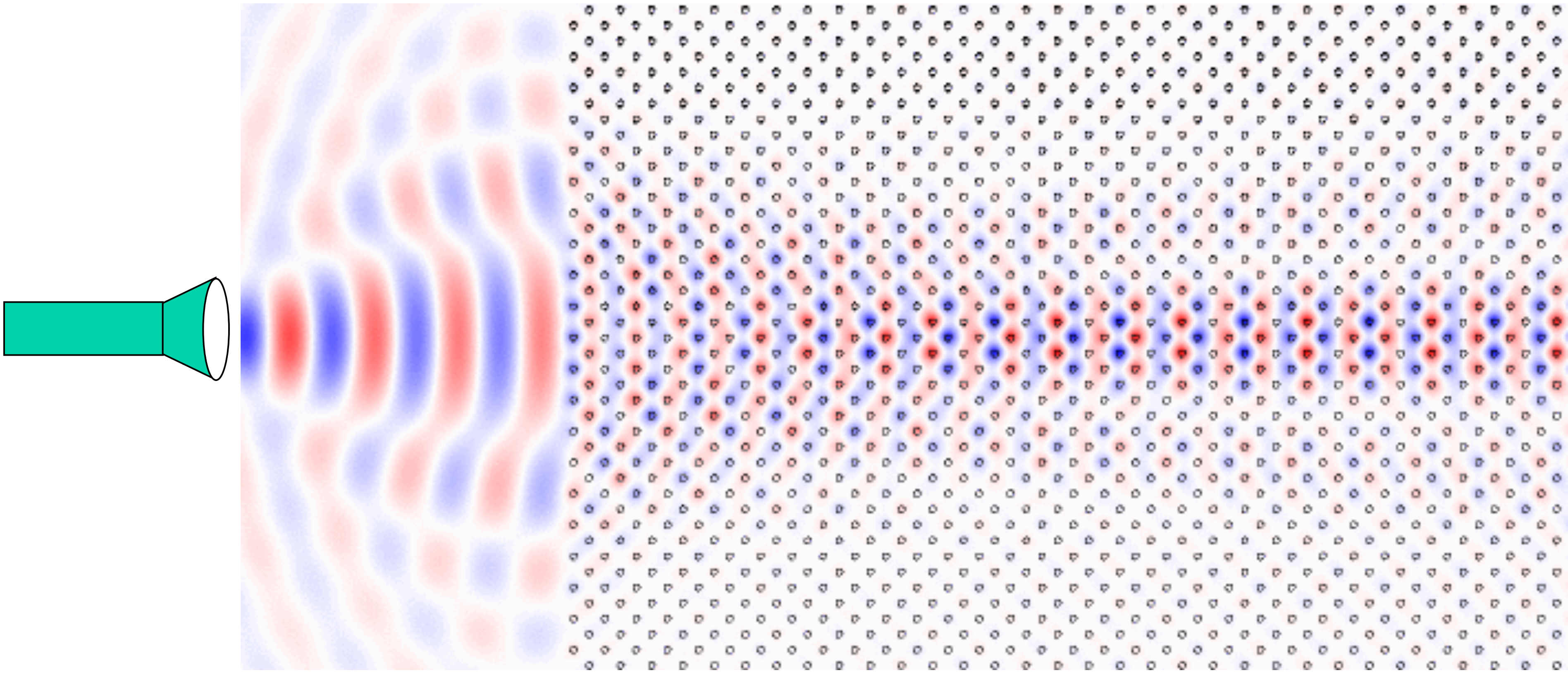
...but for some λ (~ 2a), no light can propagate: a photonic band gap

An even bigger mess?
zillons of scatterers



Blech, light will just scatter like crazy
and go all over the place … how boring!

Not so messy, not so boring…



the light seems to form several *coherent beams*
that propagate *without scattering*
… and almost *without diffraction* (*supercollimation*)

…the magic of symmetry…

[Emmy Noether, 1915]

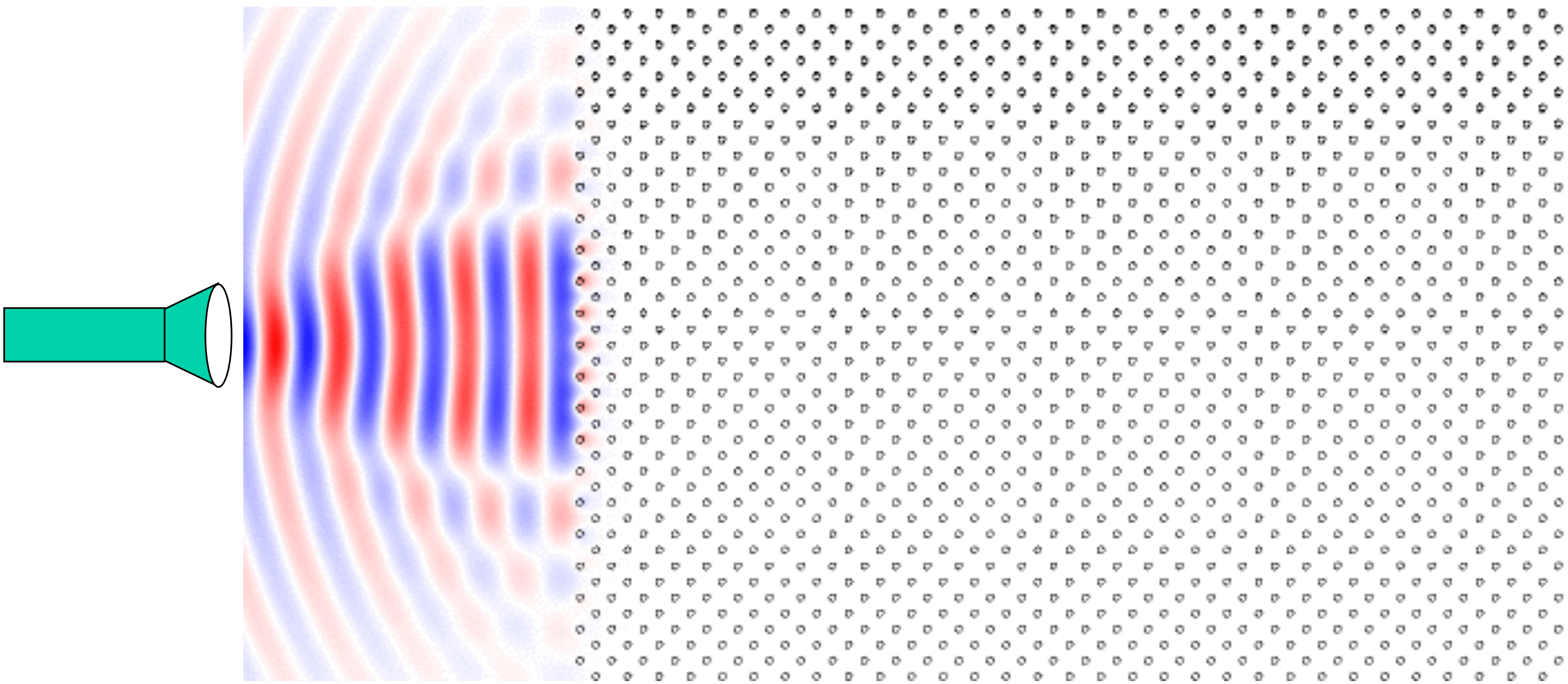
Noether's theorem:
symmetry = conservation laws

In this case, periodicity
= conserved "momentum"
= wave solutions without scattering
[Bloch waves]

Felix Bloch
(1928)

A slight change? Shrink λ by 20%

an "optical insulator" (photonic bandgap)

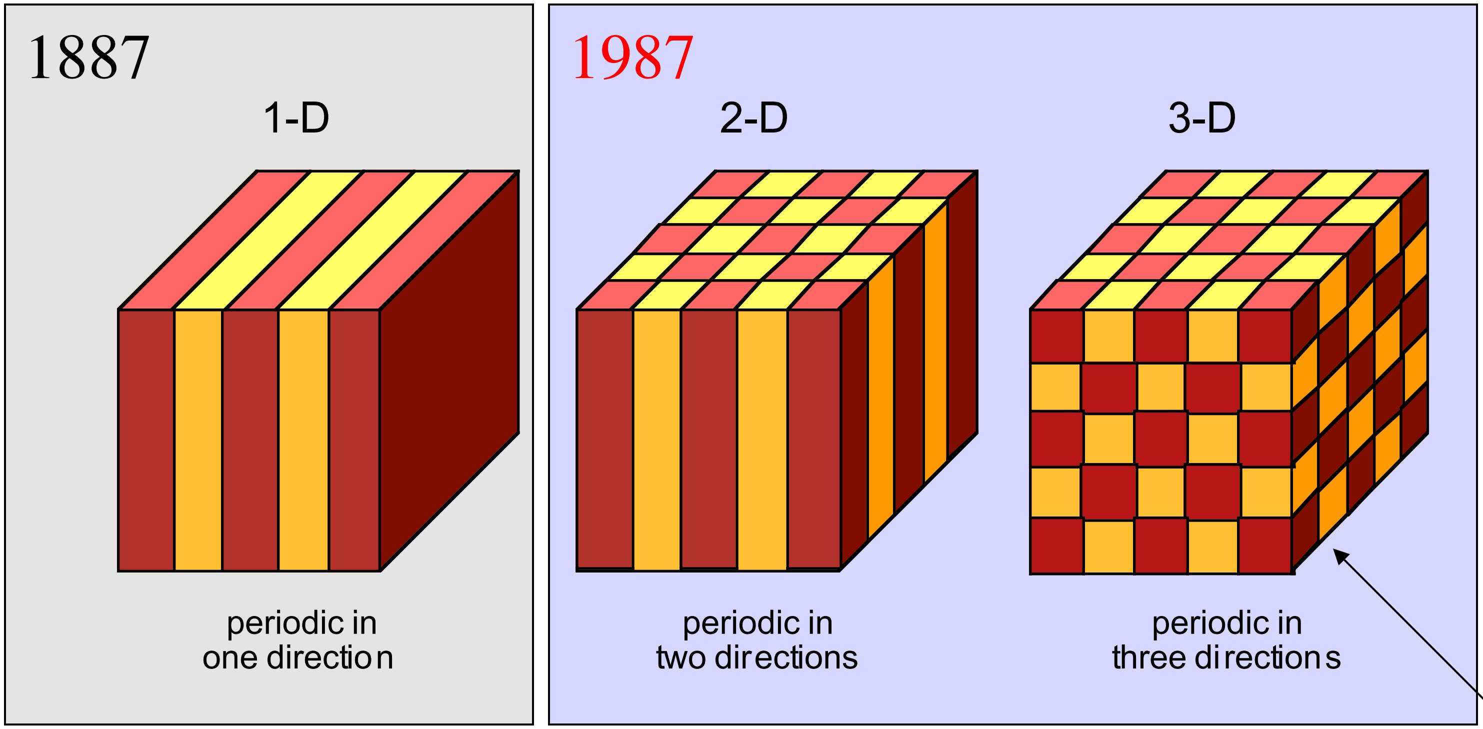


light cannot penetrate the structure at this wavelength!

all of the scattering destructively interferes

Photonic Crystals

periodic electromagnetic media



(need a more complex topology)

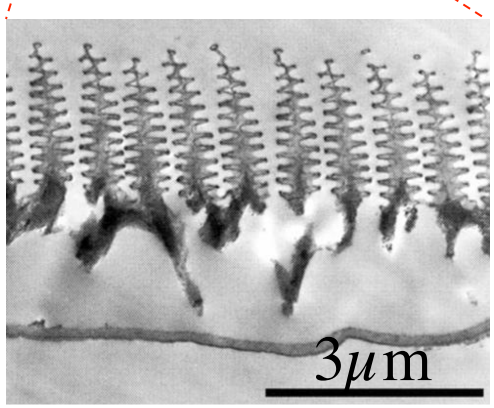
with photonic band gaps: "optical insulators"

Photonic Crystals in Nature

Morpho rhetenor butterfly

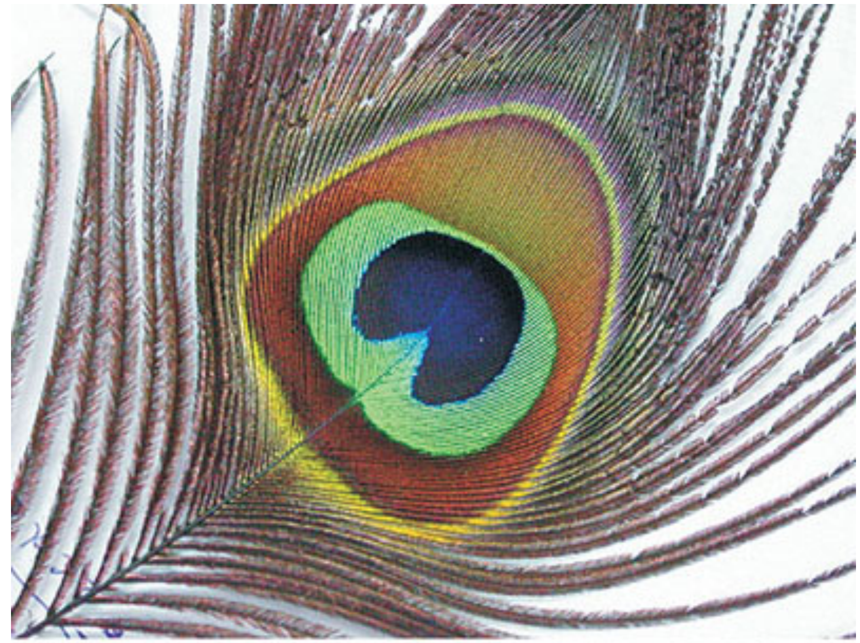
wing scale:

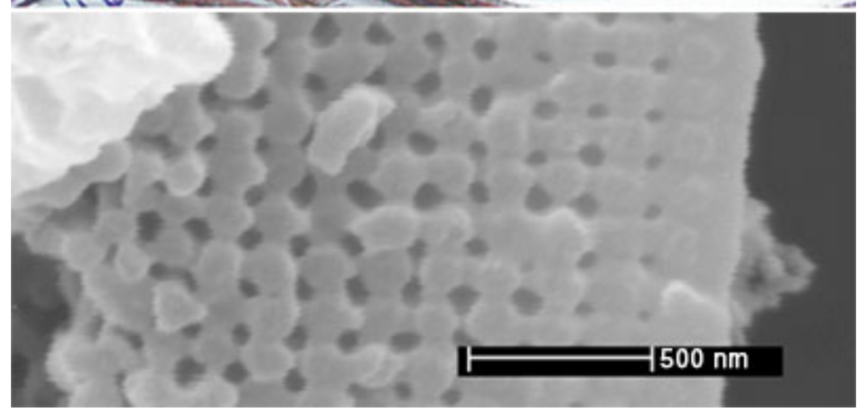
[P. Vukosic *et al.*, *Proc. Roy. Soc: Bio. Sci.* **266**, 1403 (1999)]



[also: B. Gralak *et al.*, *Opt. Express* **9**, 567 (2001)]

Peacock feather

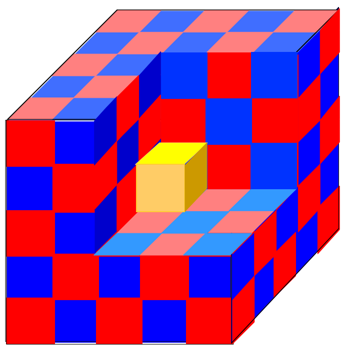


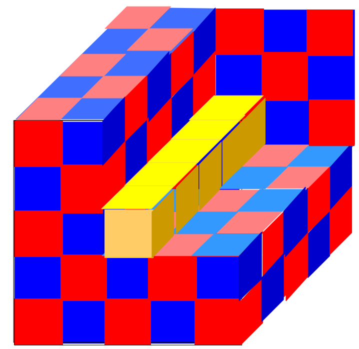


[J. Zi *et al*, *Proc. Nat. Acad. Sci. USA*, **100**, 12576 (2003)]
[figs: Blau, *Physics Today* **57**, 18 (2004)]

Photonic Crystals

periodic electromagnetic media



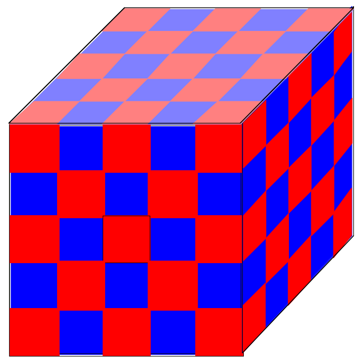


can trap light in cavities and waveguides ("wires")

with photonic band gaps:
"optical insulators"
for holding and controlling light

Photonic Crystals

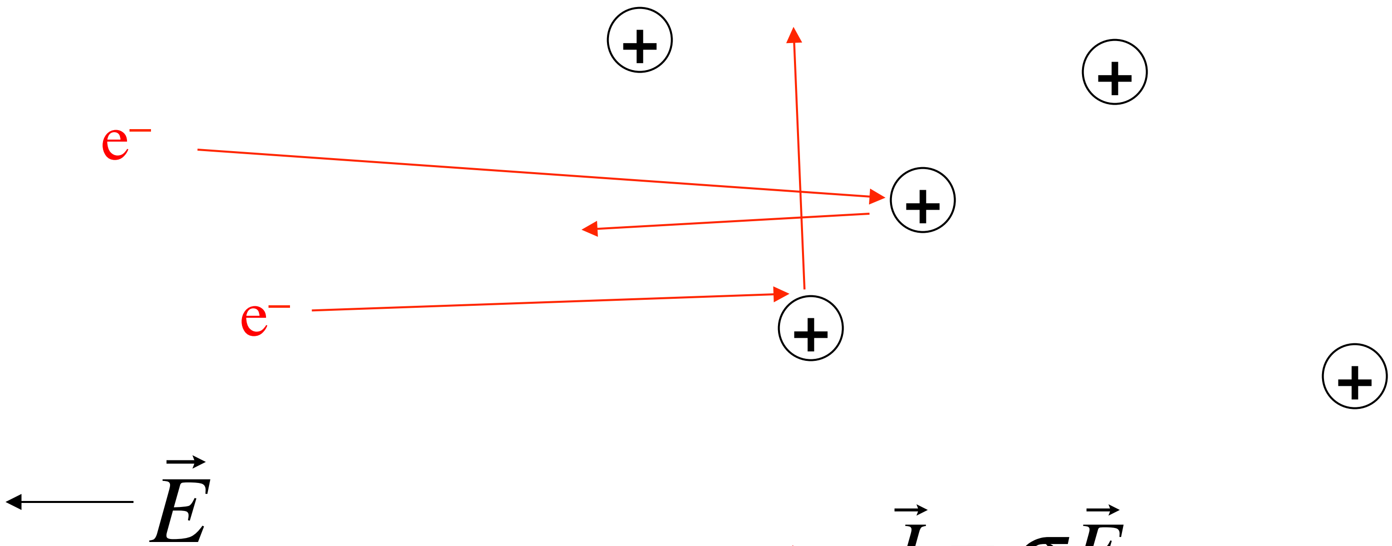
periodic electromagnetic media



But how can we understand such complex systems?
Add up the infinite sum of scattering? Ugh!

A mystery from the 19th century

conductive material

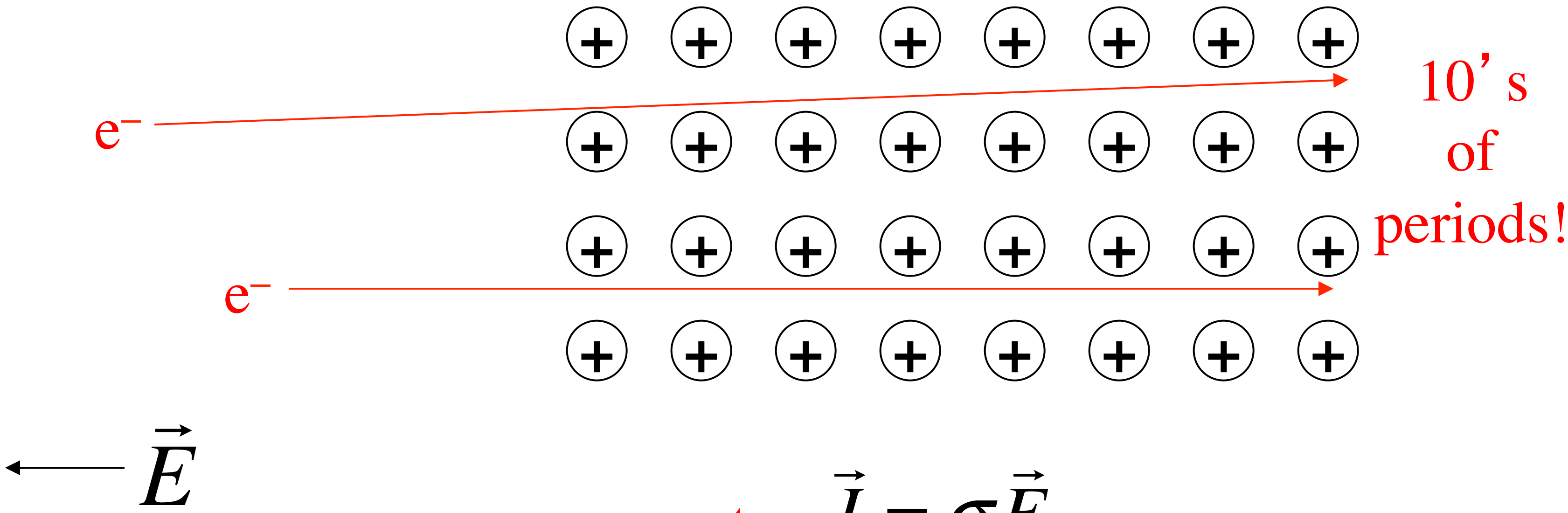


current: $\vec{J} = \sigma \vec{E}$

conductivity (measured)

mean free path (distance) of electrons

A mystery from the 19th century



current: $\vec{J} = \sigma \vec{E}$

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mean free path (distance) of electrons

A mystery solved…

1 electrons are waves (quantum mechanics)

2 waves in a periodic medium can propagate without scattering:

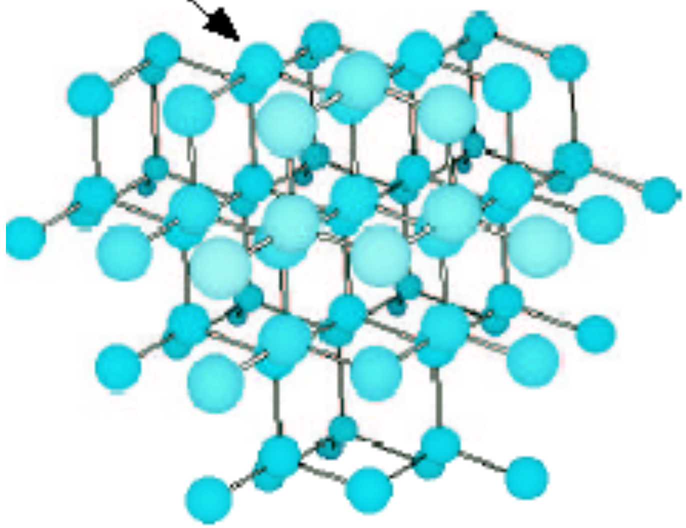
Bloch's Theorem (1d: Floquet's)

The foundations do not depend on the specific wave equation.

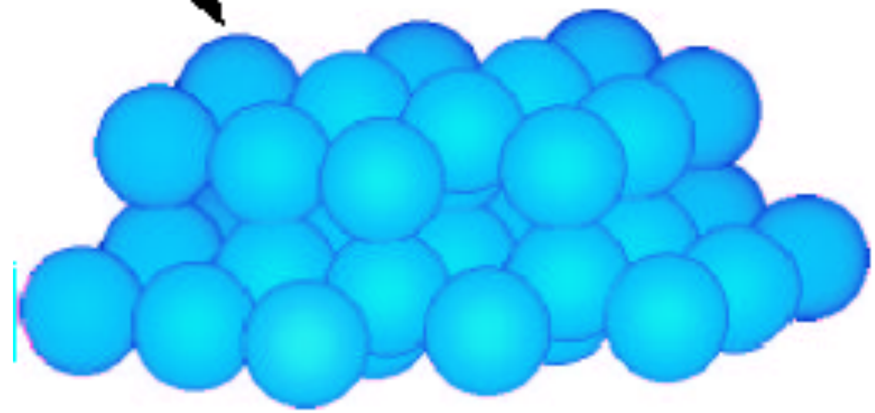
Electronic and Photonic Crystals

Periodic Medium

atoms in diamond structure

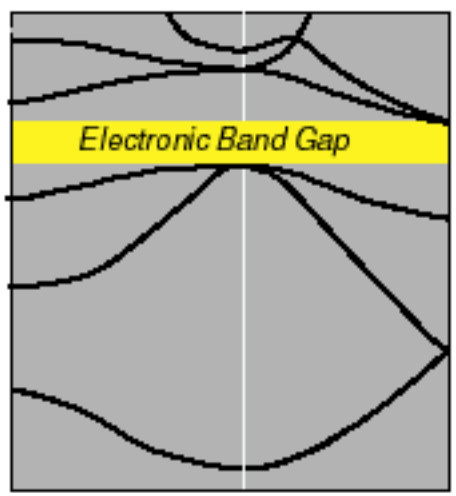


dielectric spheres, diamond lattice



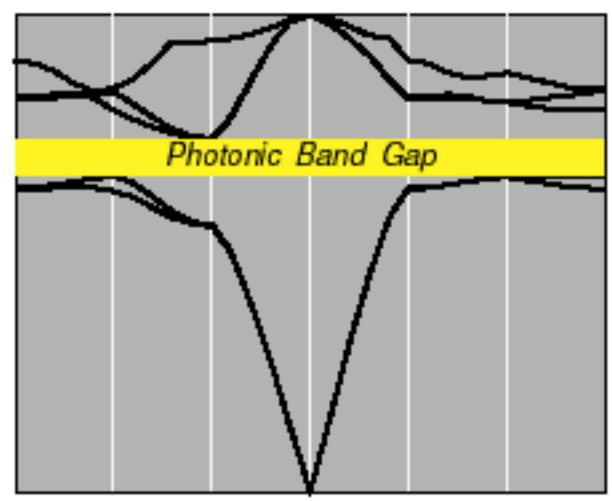
Bloch waves: **Band Diagram**

electron energy



wavevector

photon frequency

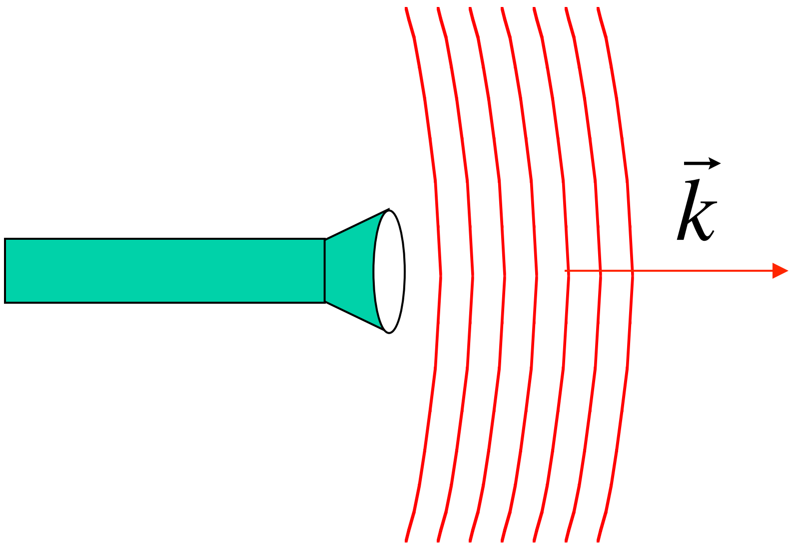


wavevector

strongly interacting fermions

weakly-interacting bosons

Time to Analyze the Cartoon



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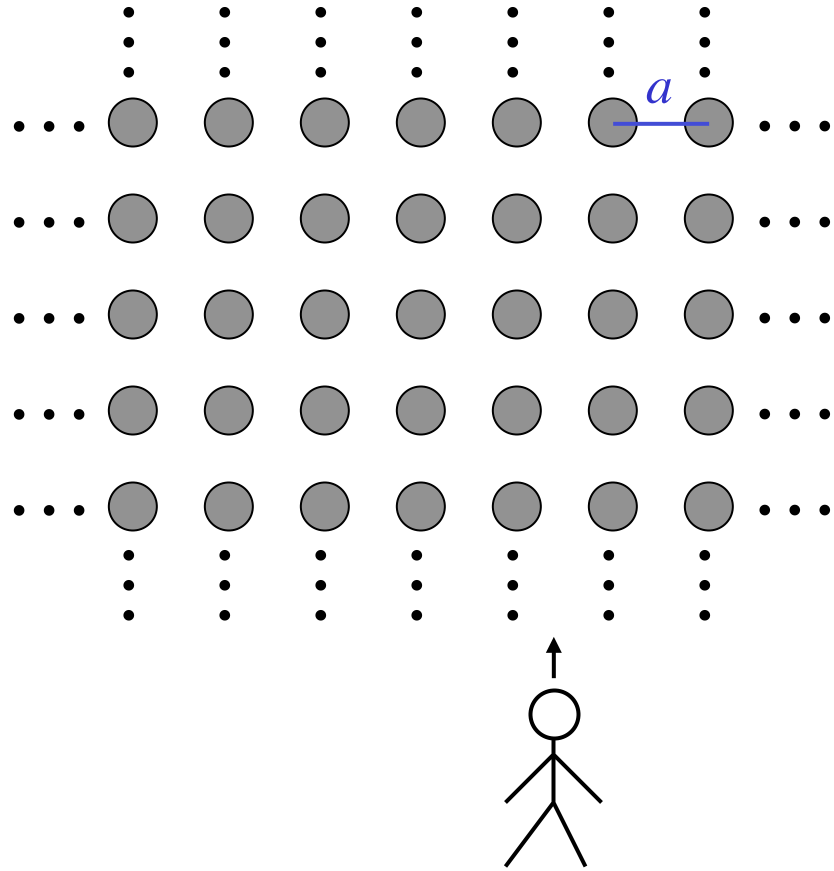

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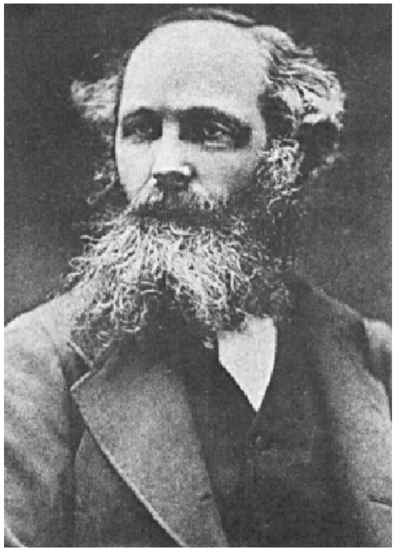


James Clerk Maxwell
1864

Maxwell's Equations

Gauss:

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{D} = \rho$$

Ampere:

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$

Faraday:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

constitutive relations:

$$\varepsilon_0 \mathbf{E} = \mathbf{D} - \mathbf{P}$$

$$\mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}$$

electromagnetic fields:

E = electric field
D = displacement field
H = magnetic field / induction
B = magnetic field / flux density

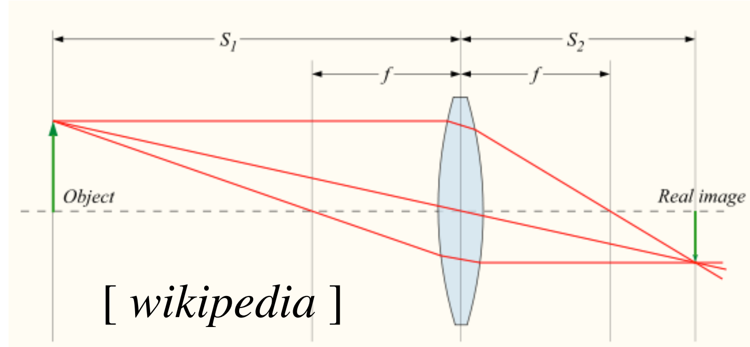
sources: **J** = current density
ρ = charge density

material response to fields:
P = polarization density
M = magnetization density

constants: ε_0, μ_0 = vacuum permittivity/permeability
c = vacuum speed of light = $(\varepsilon_0\ \mu_0)^{-1/2}$

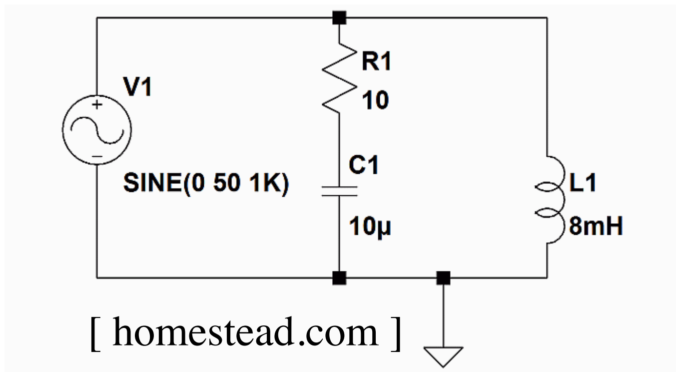
When can we solve this mess?

- Very small wavelengths: ray optics



[*wikipedia*]

- Very large wavelengths:
  quasistatics (freshman E&M)
  & lumped circuit models



[homestead.com]

- Wavelengths comparable to geometry?
  — handful of cases can be ~solved analytically:
  planes, spheres, cylinders, empty space
  — everything else just a mess for computer…?

Mathematically, use *structure* of the equations, not explicit solution:

linear algebra, group theory, functional analysis,
perturbative methods, …

This lecture: omit proofs & derivations,
jump from starting points to results

Fun with Math

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}\vec{H} = i\frac{\omega}{c}\vec{H}$$

First task:
get rid of this mess

$$\vec{\nabla} \times \vec{H} = \varepsilon\frac{1}{c}\frac{\partial}{\partial t}\vec{E} + \cancelto{0}{\vec{J}} = -i\frac{\omega}{c}\varepsilon\vec{E}$$

dielectric function $\varepsilon(\mathbf{x}) = n^2(\mathbf{x})$

$$\underbrace{\nabla \times \frac{1}{\varepsilon}\nabla \times}_{\text{eigen-operator}} \vec{H} = \underbrace{\left(\frac{\omega}{c}\right)^2}_{\text{eigen-value}} \vec{H}$$

eigen-state

+ constraint
$$\nabla \cdot \vec{H} = 0$$

Hermitian Eigenproblems

$$\underbrace{\nabla \times \frac{1}{\varepsilon} \nabla \times}_{\text{eigen-operator}} \underbrace{\vec{H}}_{\text{eigen-state}} = \underbrace{\left(\frac{\omega}{c}\right)^2}_{\text{eigen-value}} \vec{H}$$

+ constraint

$$\nabla \cdot \vec{H} = 0$$

eigen-operator

eigen-value

eigen-state

Hermitian for real (lossless) ε

➡ well-known properties from linear algebra:

ω are real (lossless)

eigen-states are orthogonal

eigen-states are complete (give all solutions)*

* Technically, completeness requires slightly more than just Hermitian-ness.

Periodic Hermitian Eigenproblems

[G. Floquet, "Sur les équations différentielles linéaries à coefficients périodiques," *Ann. École Norm. Sup.* **12**, 47–88 (1883).]
[F. Bloch, "Über die quantenmechanik der electronen in kristallgittern," *Z. Physik* **52**, 555–600 (1928).]

if eigen-operator is periodic, then Bloch-Floquet theorem applies:

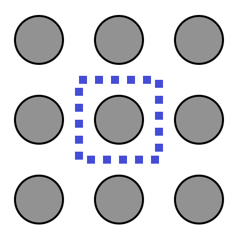
can choose: $$\vec{H}(\vec{x},t) = e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}\vec{H}_{\vec{k}}(\vec{x})$$

planewave

periodic "envelope"

Corollary 1: **k** is conserved, *i.e.* no scattering of Bloch wave

Corollary 2: $\vec{H}_{\vec{k}}$ given by finite unit cell,
so ω are discrete $\omega_n(\mathbf{k})$



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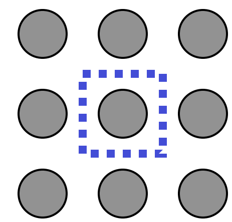
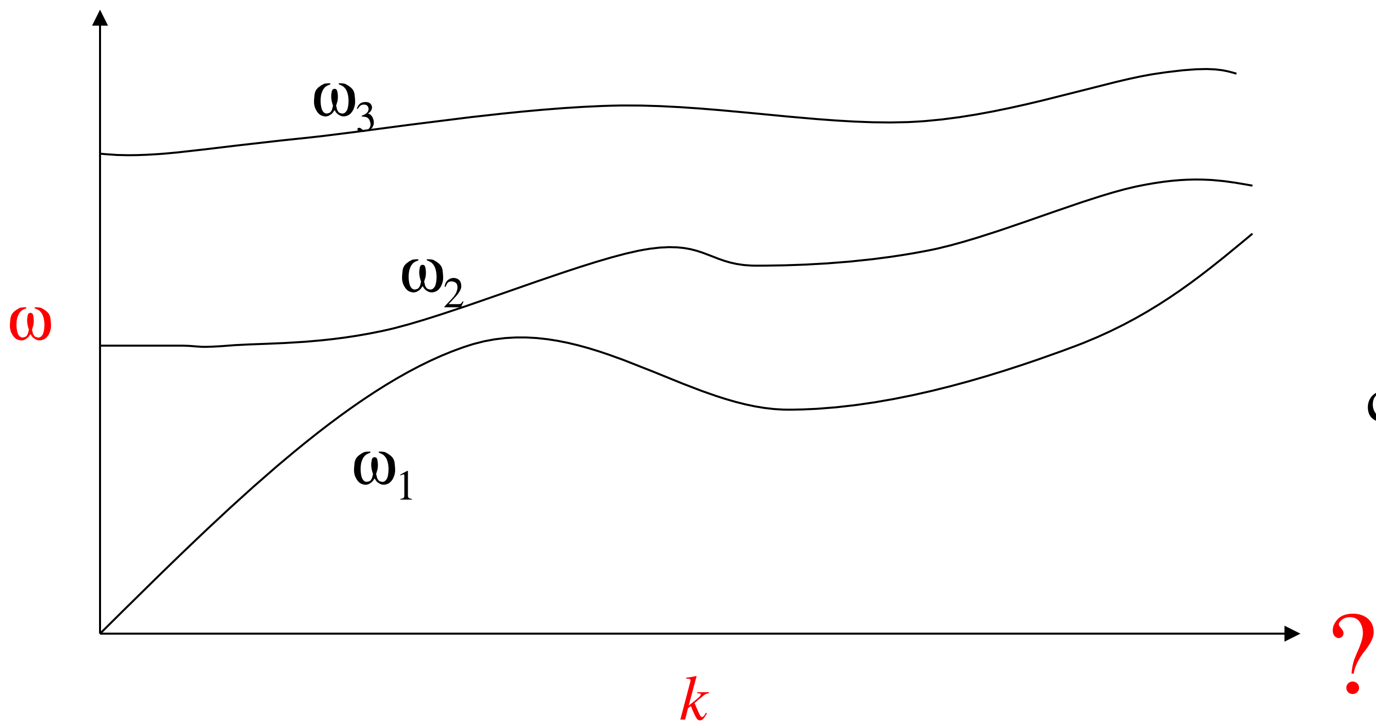


band diagram (dispersion relation)

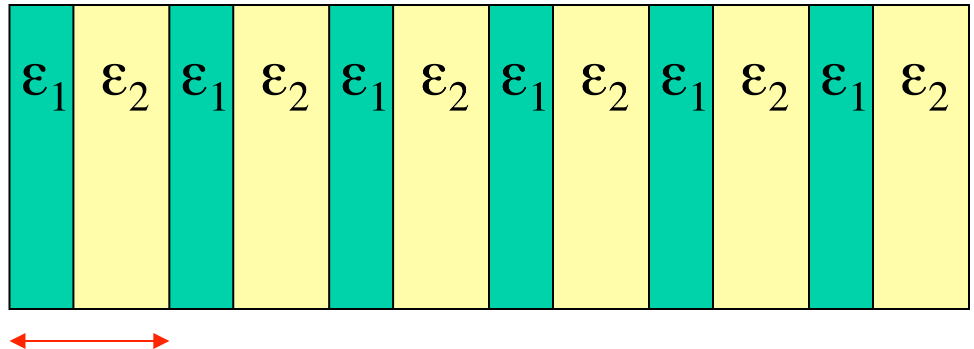


map of
what states
exist &
can interact

range of *k*?

Periodic Hermitian Eigenproblems in 1d

$$H(x) = e^{ikx} H_k(x)$$



a

$$\varepsilon(x) = \varepsilon(x+a)$$

Consider $k+2\pi/a$: $e^{i\left(k+\frac{2\pi}{a}\right)x} H_{k+\frac{2\pi}{a}}(x) = e^{ikx}\left[\underbrace{e^{i\frac{2\pi}{a}x} H_{k+\frac{2\pi}{a}}(x)}_{} \right]$

periodic!
satisfies same equation as H_k
$= H_k$

k is periodic:
$k + 2\pi/a$ equivalent to k
"quasi-phase-matching"

Periodic Hermitian Eigenproblems in 1d

k is periodic:

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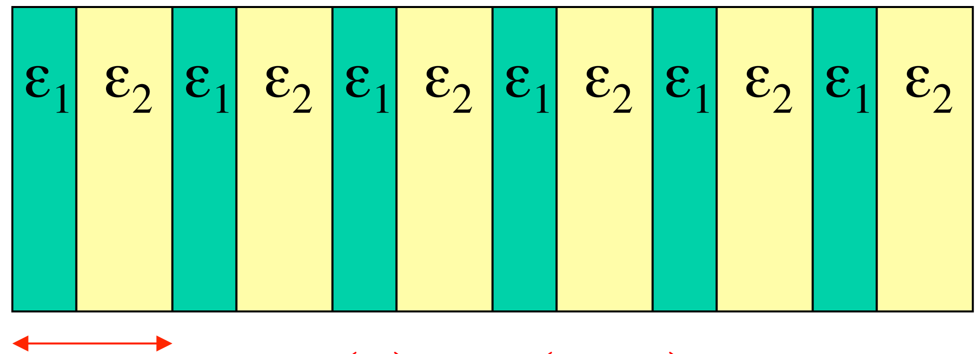
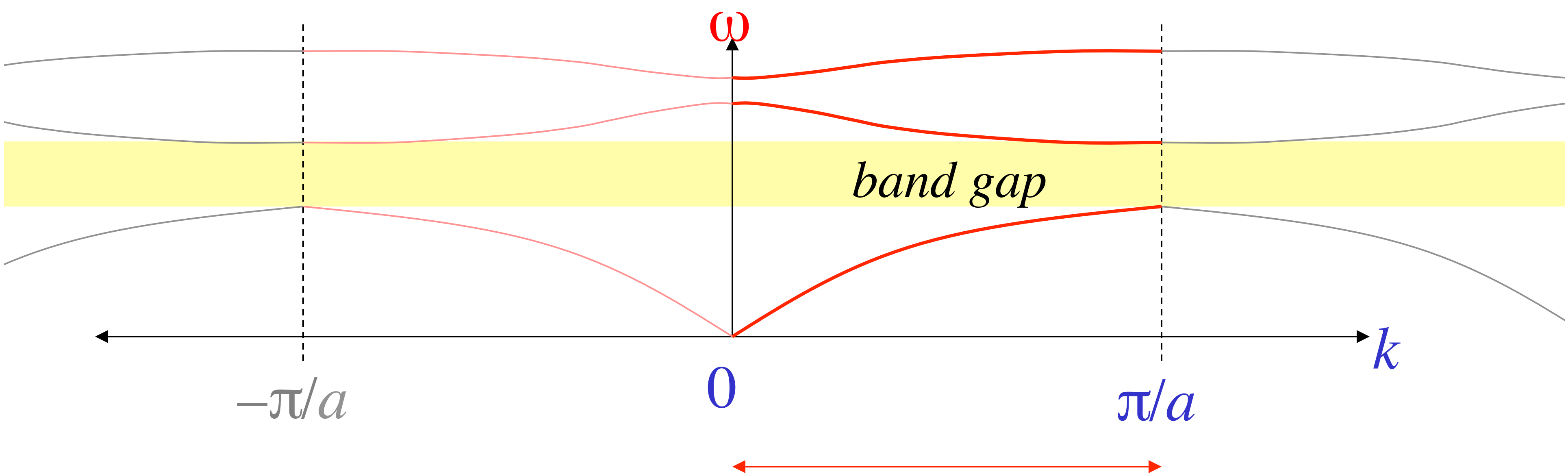


$\varepsilon(x) = \varepsilon(x+a)$

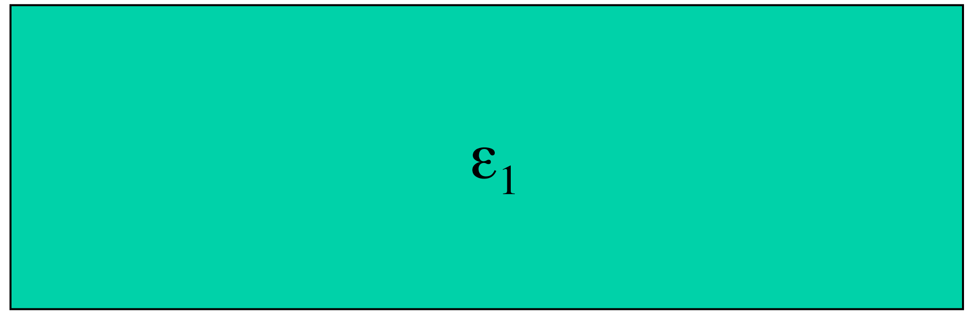


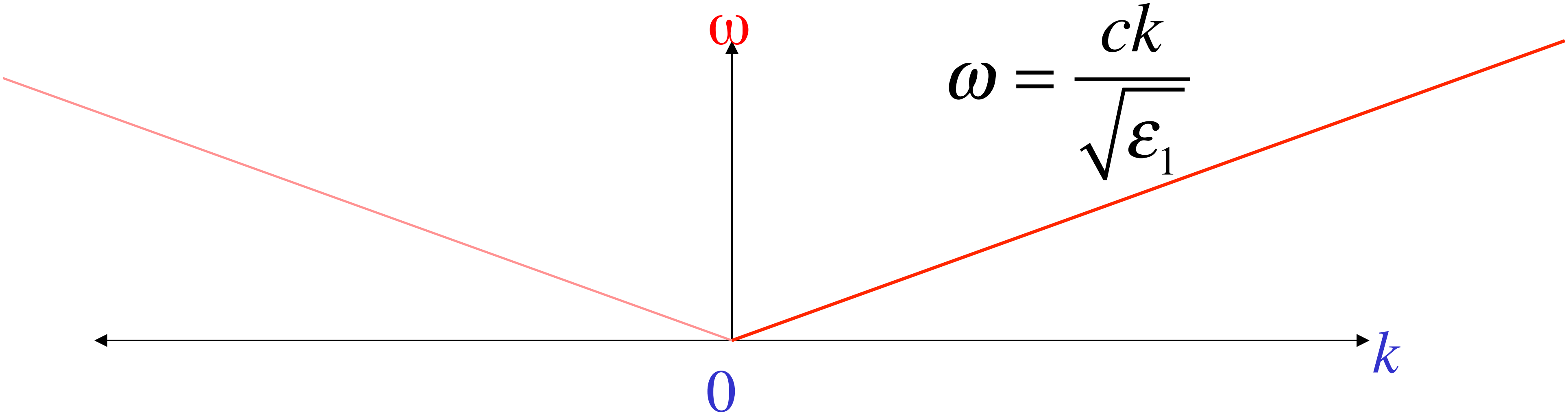
irreducible Brillouin zone

Any 1d Periodic System has a Gap

[Lord Rayleigh, "On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure," *Philosophical Magazine* **24**, 145–159 (1887).]

Start with
a uniform (1d) medium:



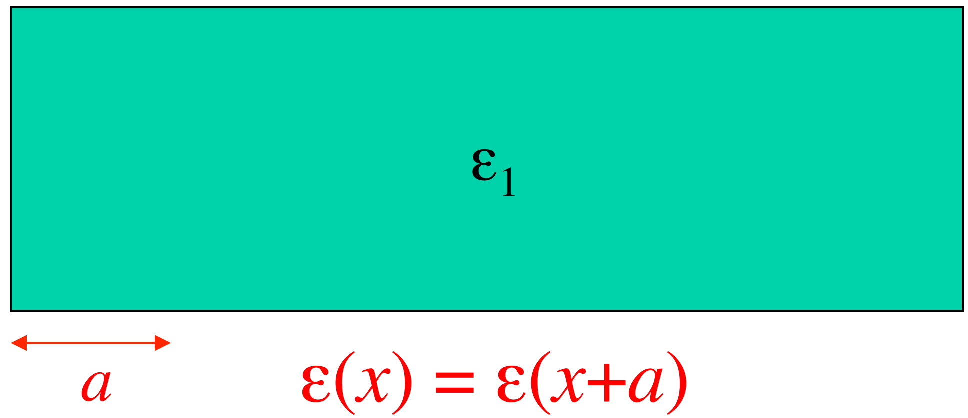


$$\omega = \frac{ck}{\sqrt{\varepsilon_1}}$$

Any 1d Periodic System has a Gap

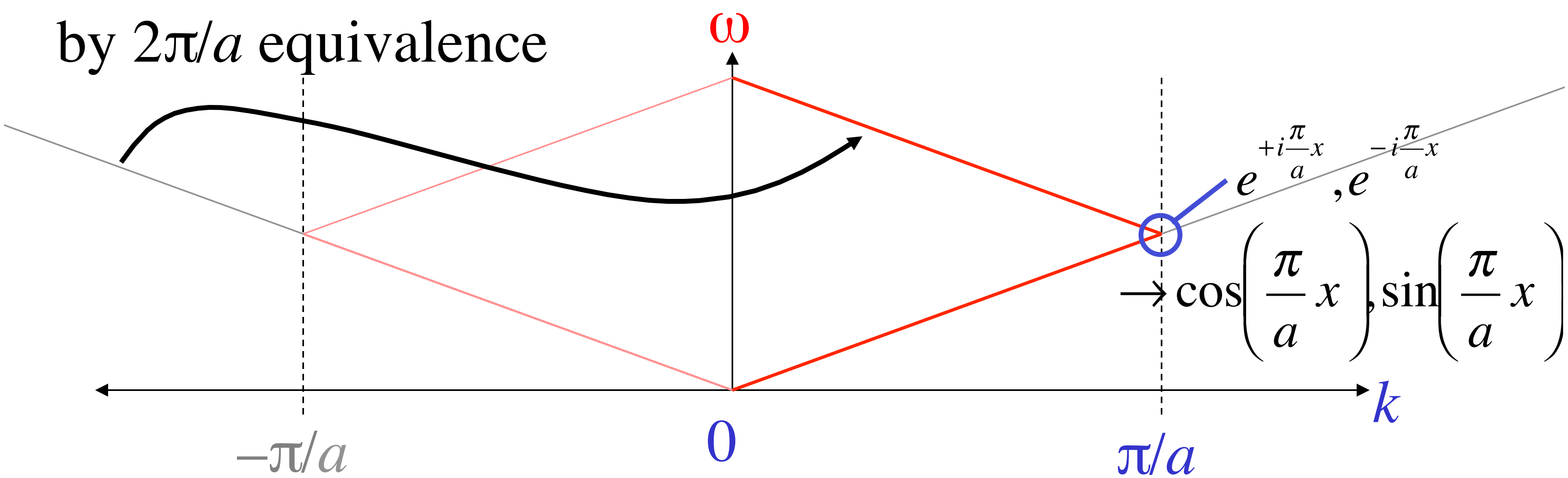
[Lord Rayleigh, "On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure," *Philosophical Magazine* **24**, 145–159 (1887).]

Treat it as
"artificially" periodic



$\varepsilon(x) = \varepsilon(x+a)$

bands are "folded"
by $2\pi/a$ equivalence



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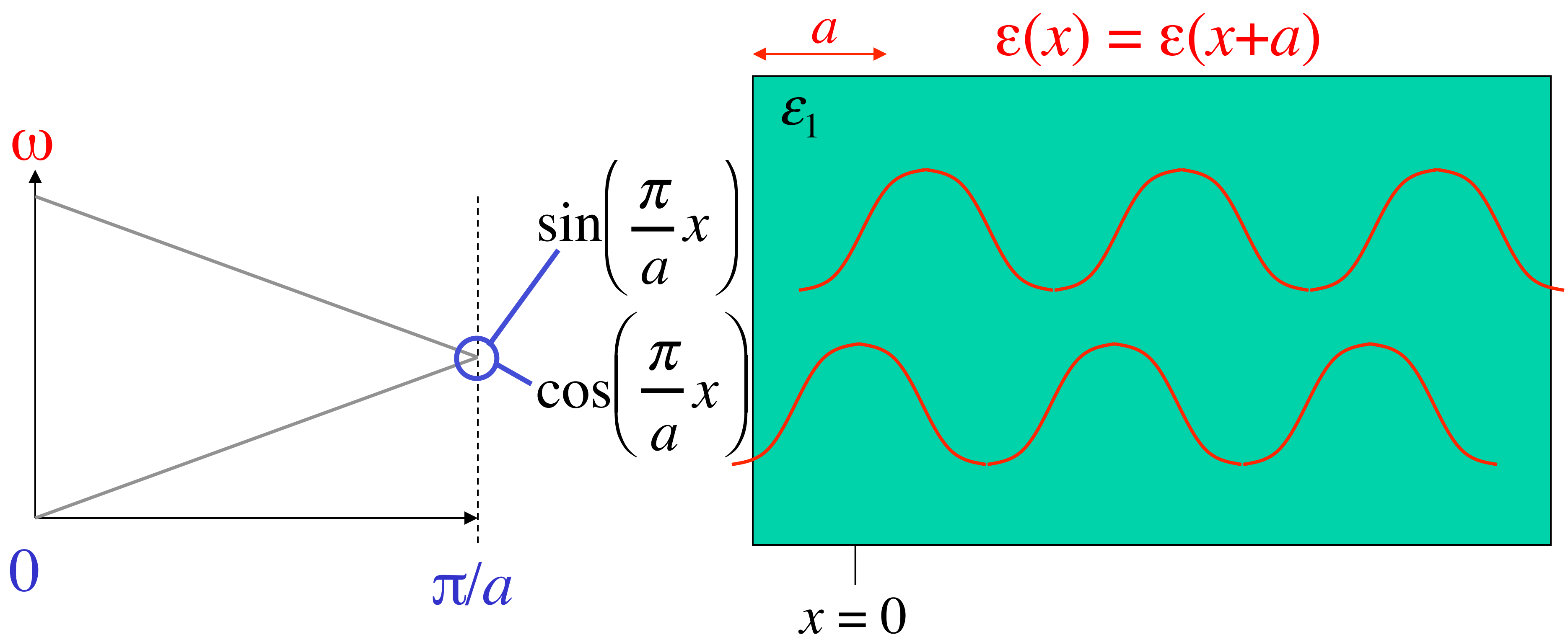

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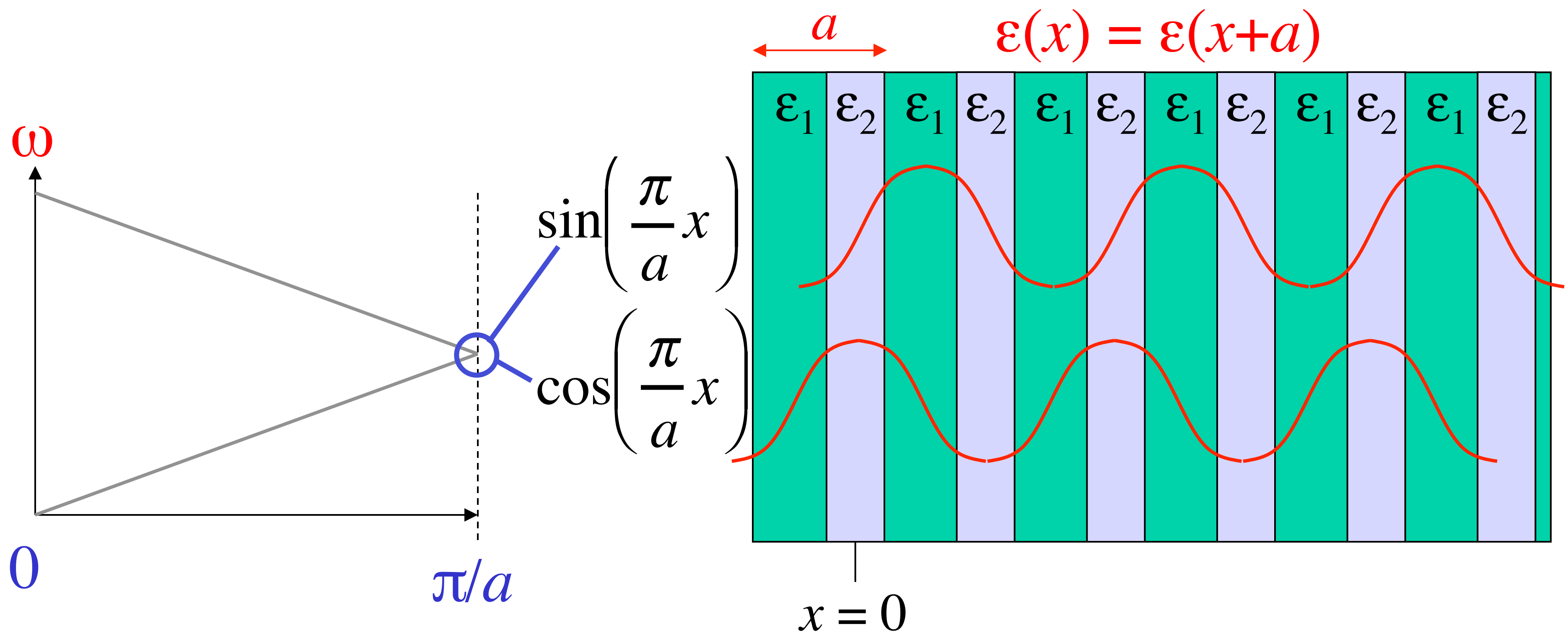
Add a small
"real" periodicity
$\varepsilon_2 = \varepsilon_1 + \Delta\varepsilon$



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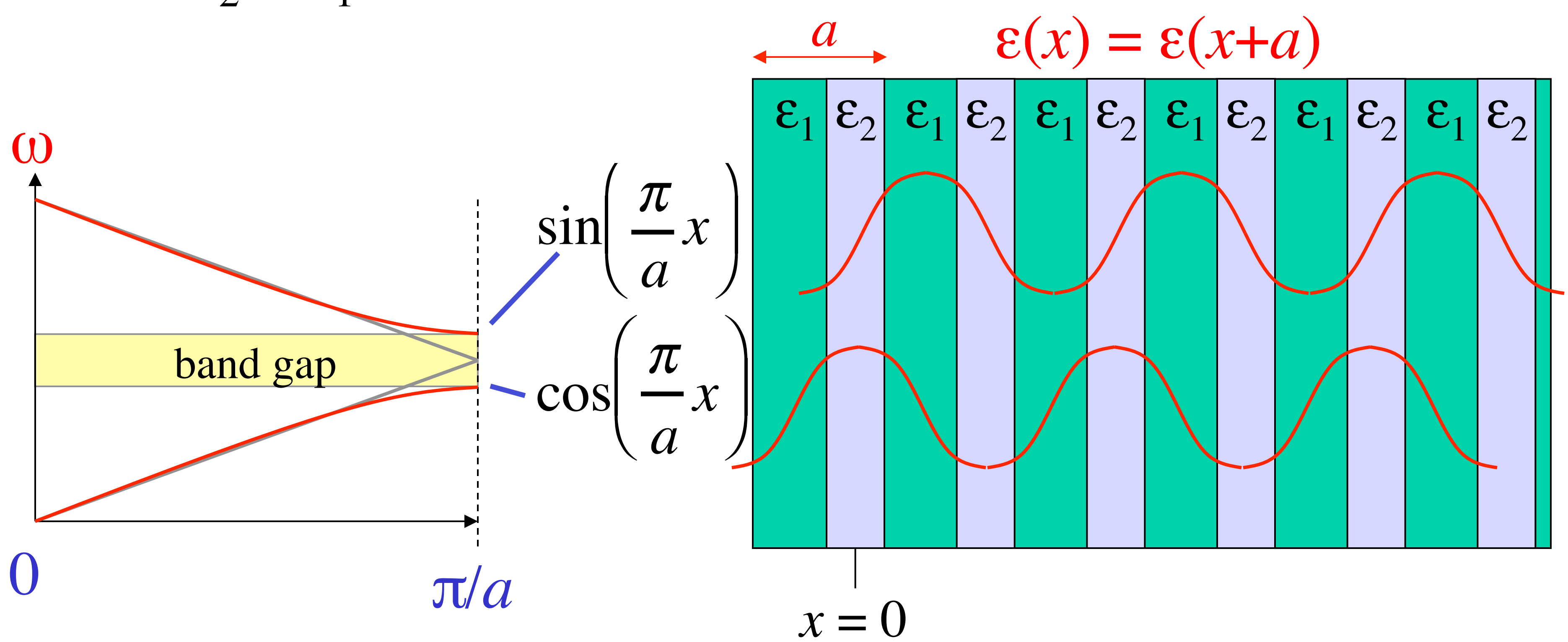
$\varepsilon_2 = \varepsilon_1 + \Delta\varepsilon$

Splitting of degeneracy:
state concentrated in higher index (ε_2) has lower frequency



Some 2d and 3d systems have gaps

- In general, eigen-frequencies satisfy Variational Theorem:

$$\omega_1(\vec{k})^2 = \min_{\substack{\vec{E}_1 \\ \nabla\cdot\varepsilon\vec{E}_1=0}} \frac{\int \left|\left(\nabla + i\vec{k}\right)\times \vec{E}_1\right|^2}{\int \varepsilon\left|\vec{E}_1\right|^2} c^2$$

"kinetic"

inverse "potential"

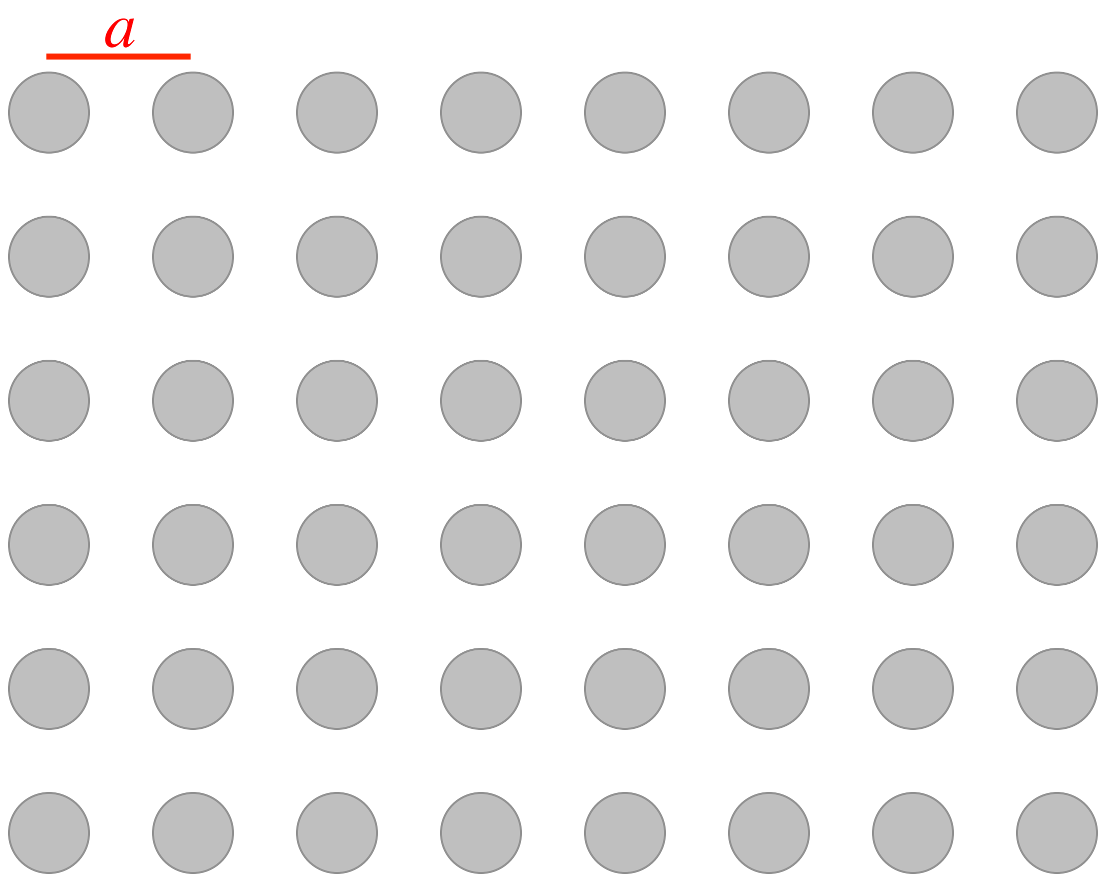
$$\omega_2(\vec{k})^2 = \min_{\substack{\vec{E}_2 \\ \nabla\cdot\varepsilon\vec{E}_2=0 \\ \int \varepsilon E_1^* \cdot E_2 = 0}} \text{"}\cdots\text{"}$$

bands "want" to be in high-ε

…but are forced out by orthogonality

➔ band gap (maybe)

A 2d Model System

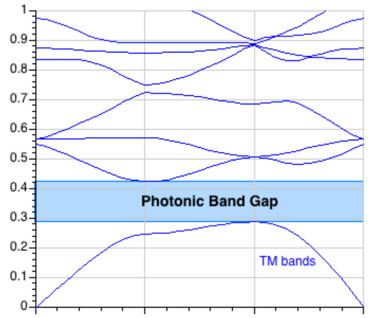


Square lattice of dielectric rods (ε = 12 ~ Si) in air (ε = 1)

Solving the Maxwell Eigenproblem

Finite cell ➔ *discrete* eigenvalues ω_n

Want to solve for $\omega_n(\mathbf{k})$,
& plot vs. "all" **k** for "all" n,



$$(\nabla + i\mathbf{k}) \times \frac{1}{\varepsilon}(\nabla + i\mathbf{k}) \times \mathbf{H}_n = \frac{\omega_n^{\,2}}{c^2}\mathbf{H}_n$$

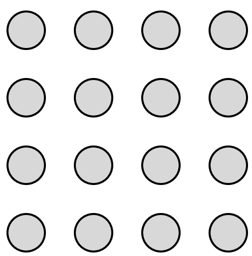
constraint: $(\nabla + i\mathbf{k}) \cdot \mathbf{H} = 0$

where magnetic field = $\mathbf{H}(\mathbf{x})\, e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}$

1. Limit range of **k**: irreducible Brillouin zone
2. Limit degrees of freedom: expand **H** in finite basis
3. Efficiently solve eigenproblem: iterative methods

Solving the Maxwell Eigenproblem: 1

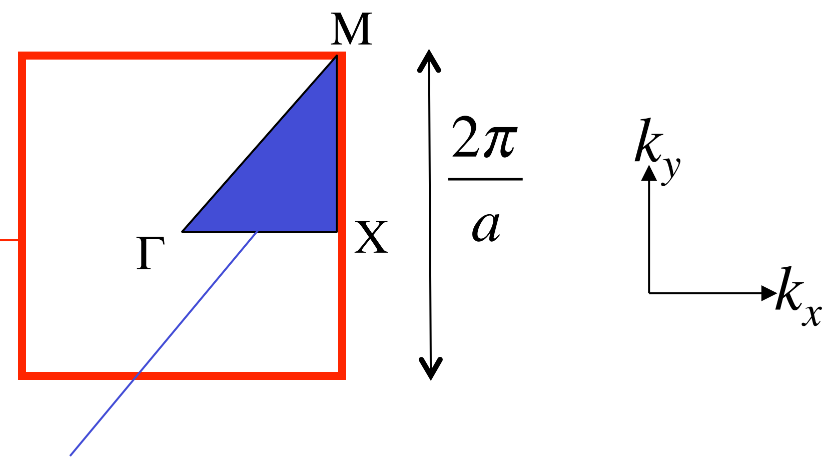
1 Limit range of **k**: irreducible Brillouin zone



—Bloch's theorem: solutions are periodic in **k**

first Brillouin zone

= minimum |**k**| "primitive cell"



irreducible Brillouin zone: reduced by symmetry

2 Limit degrees of freedom: expand **H** in finite basis

3 Efficiently solve eigenproblem: iterative methods

Solving the Maxwell Eigenproblem: 2a

1. Limit range of $\mathbf{k}$: irreducible Brillouin zone

2. Limit degrees of freedom: expand $\mathbf{H}$ in finite basis (N)

$$|\mathbf{H}\rangle = \mathbf{H}(\mathbf{x}_t) = \sum_{m=1}^{N} h_m \mathbf{b}_m(\mathbf{x}_t)$$

solve: $\hat{A}|\mathbf{H}\rangle = \omega^2 |\mathbf{H}\rangle$

finite matrix problem: $Ah = \omega^2 Bh$

$$\langle \mathbf{f} | \mathbf{g} \rangle = \int \mathbf{f}^* \cdot \mathbf{g}$$

$$A_{m\ell} = \langle \mathbf{b}_m | \hat{A} | \mathbf{b}_\ell \rangle \qquad B_{m\ell} = \langle \mathbf{b}_m | \mathbf{b}_\ell \rangle$$

3. Efficiently solve eigenproblem: iterative methods

Solving the Maxwell Eigenproblem: 2b

1. Limit range of **k**: irreducible Brillouin zone

2. Limit degrees of freedom: expand **H** in finite basis

— must satisfy constraint: $(\nabla + i\mathbf{k}) \cdot \mathbf{H} = 0$

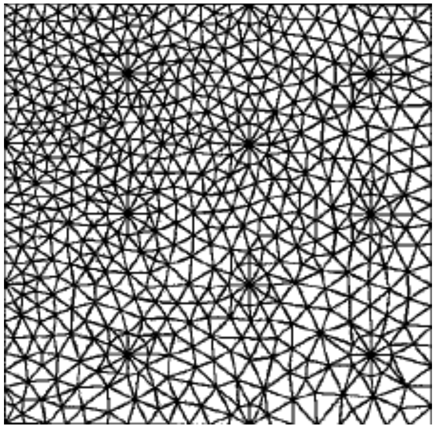
Planewave (FFT) basis

$$\mathbf{H}(\mathbf{x}_t) = \sum_{\mathbf{G}} \mathbf{H}_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{x}_t}$$

constraint: $\mathbf{H}_{\mathbf{G}} \cdot (\mathbf{G} + \mathbf{k}) = 0$

uniform "grid," periodic boundaries,
simple code, O(N log N)

Finite-element basis



[figure: Peyrilloux *et al.*,
J. Lightwave Tech.
21, 536 (2003)]

constraint, boundary conditions:

Nédélec elements

[Nédélec, *Numerische Math.*
35, 315 (1980)]

nonuniform mesh,
more arbitrary boundaries,
complex code & mesh, O(N)

3. Efficiently solve eigenproblem: iterative methods

Solving the Maxwell Eigenproblem: 3a

1. Limit range of **k**: irreducible Brillouin zone
2. Limit degrees of freedom: expand **H** in finite basis
3. Efficiently solve eigenproblem: iterative methods

$$Ah = \omega^2 Bh$$

Slow way: compute A & B, ask LAPACK for eigenvalues
— requires $O(N^2)$ storage, $O(N^3)$ time

Faster way:
— start with *initial guess* eigenvector h_0
— *iteratively* improve
— $O(Np)$ storage, ~ $O(Np^2)$ time for p eigenvectors
(p smallest eigenvalues)

Solving the Maxwell Eigenproblem: 3b

1. Limit range of **k**: irreducible Brillouin zone
2. Limit degrees of freedom: expand **H** in finite basis
3. Efficiently solve eigenproblem: iterative methods

$$Ah = \omega^2 Bh$$

Many iterative methods:

— Arnoldi, Lanczos, Davidson, Jacobi-Davidson, …, Rayleigh-quotient minimization

Solving the Maxwell Eigenproblem: 3c

1. Limit range of **k**: irreducible Brillouin zone
2. Limit degrees of freedom: expand **H** in finite basis
3. Efficiently solve eigenproblem: iterative methods

$$Ah = \omega^2 Bh$$

Many iterative methods:

— Arnoldi, Lanczos, Davidson, Jacobi-Davidson, …, Rayleigh-quotient minimization

for Hermitian matrices, smallest eigenvalue ω_0 minimizes:

"variational theorem"

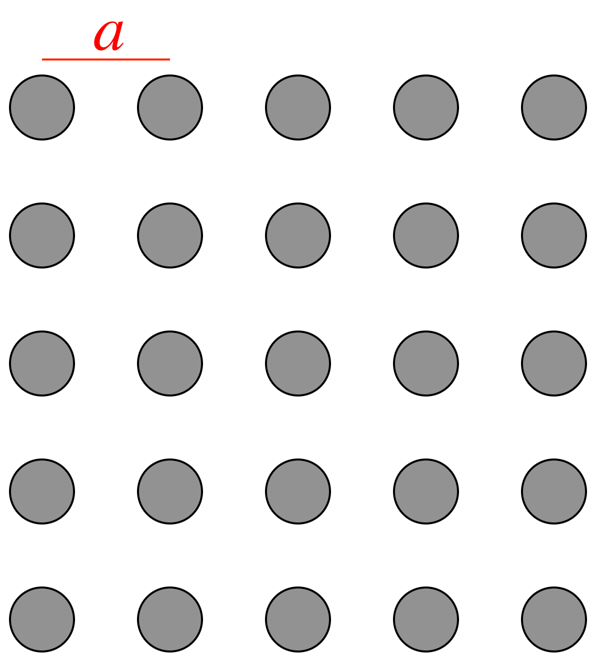
$$\omega_0^2 = \min_h \frac{h' Ah}{h' Bh}$$

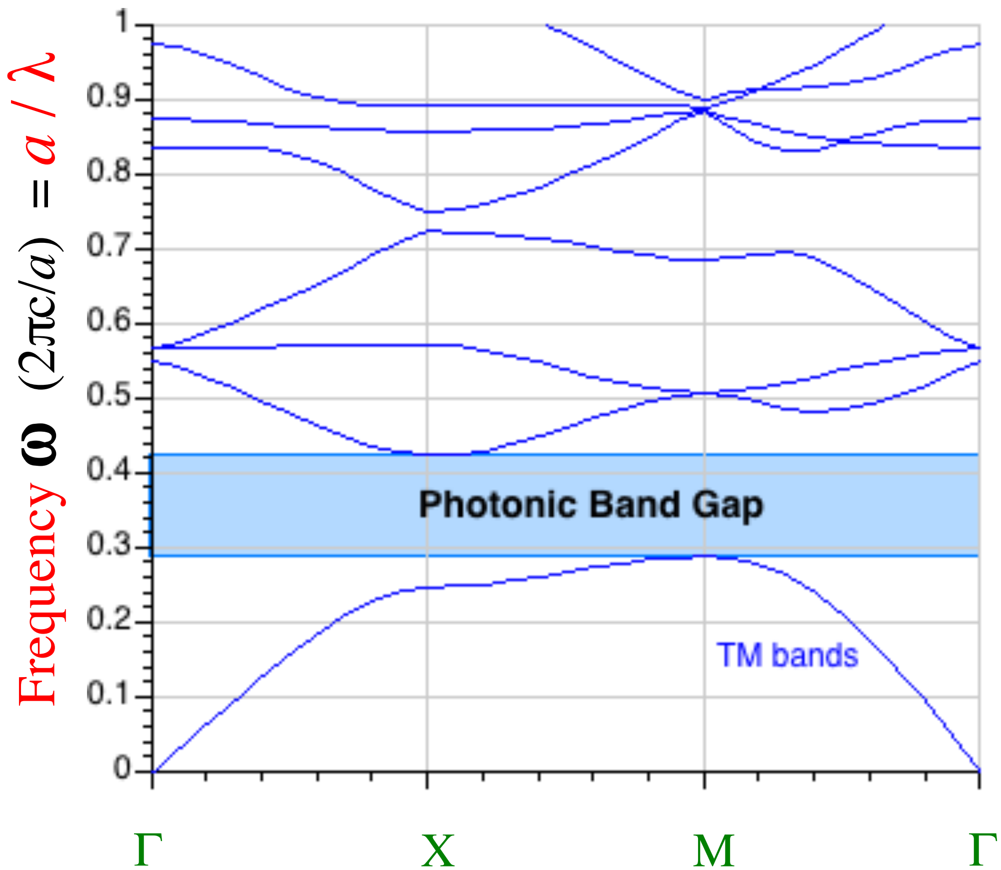
minimize by preconditioned conjugate-gradient (or…)

Outline

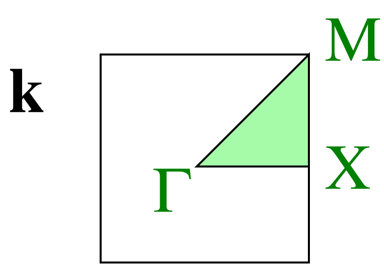
- Preliminaries: waves in periodic media
- Photonic crystals in theory and practice
- Bulk crystal properties
- Intentional defects and devices
- Index-guiding and incomplete gaps
- Photonic-crystal fibers
- Perturbations, tuning, and disorder

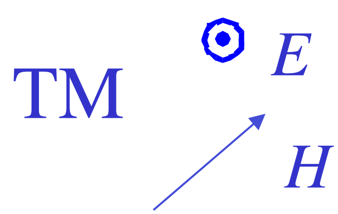
2d periodicity, ε=12:1





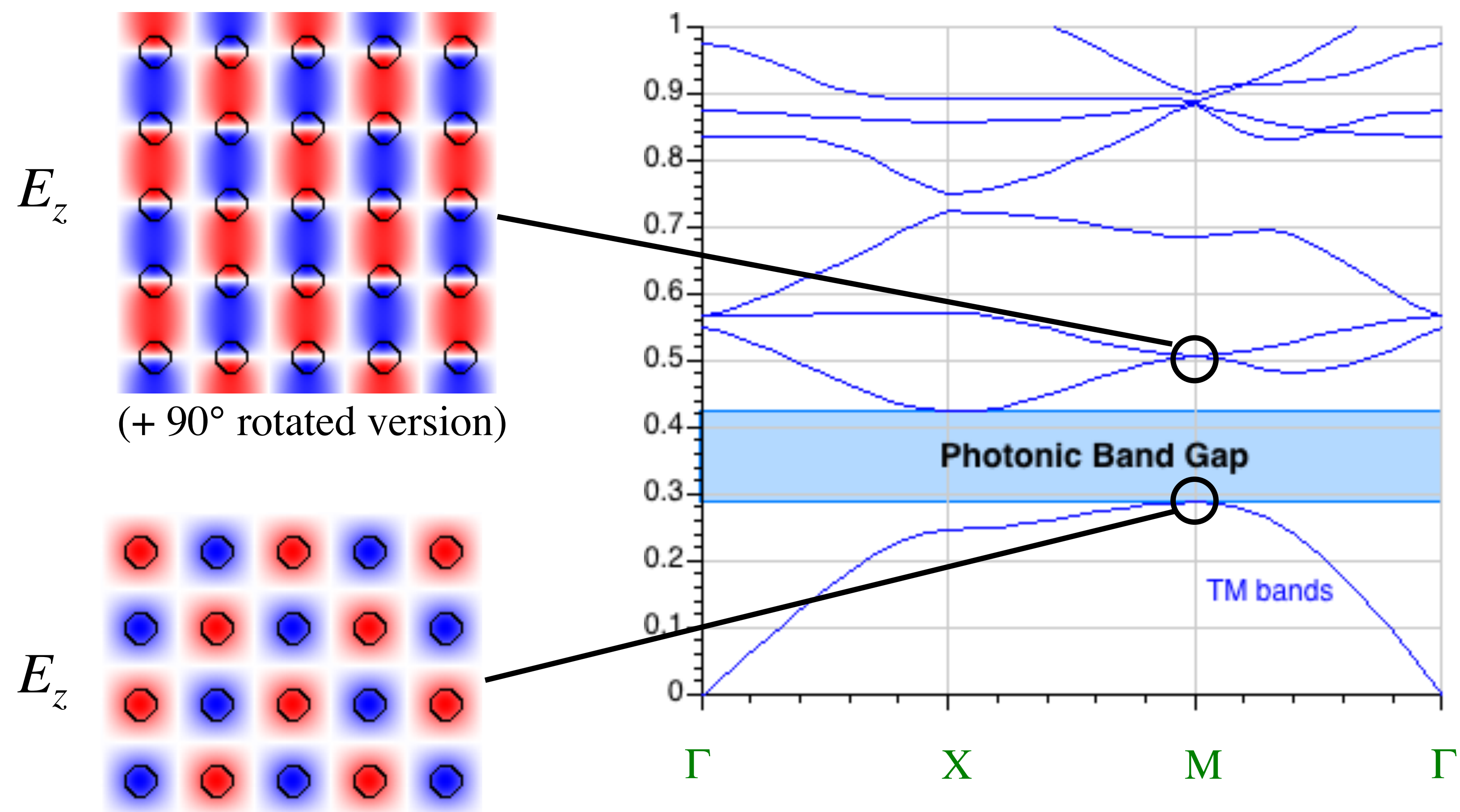
irreducible Brillouin zone





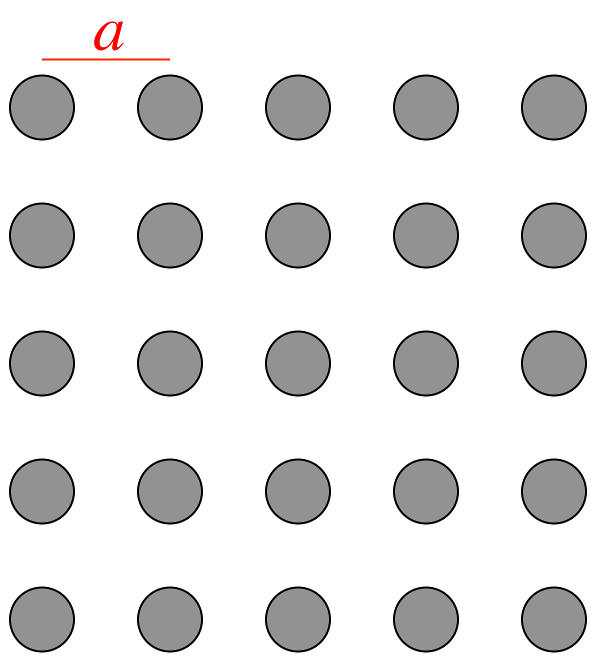
gap for
$n >$ ~1.75:1

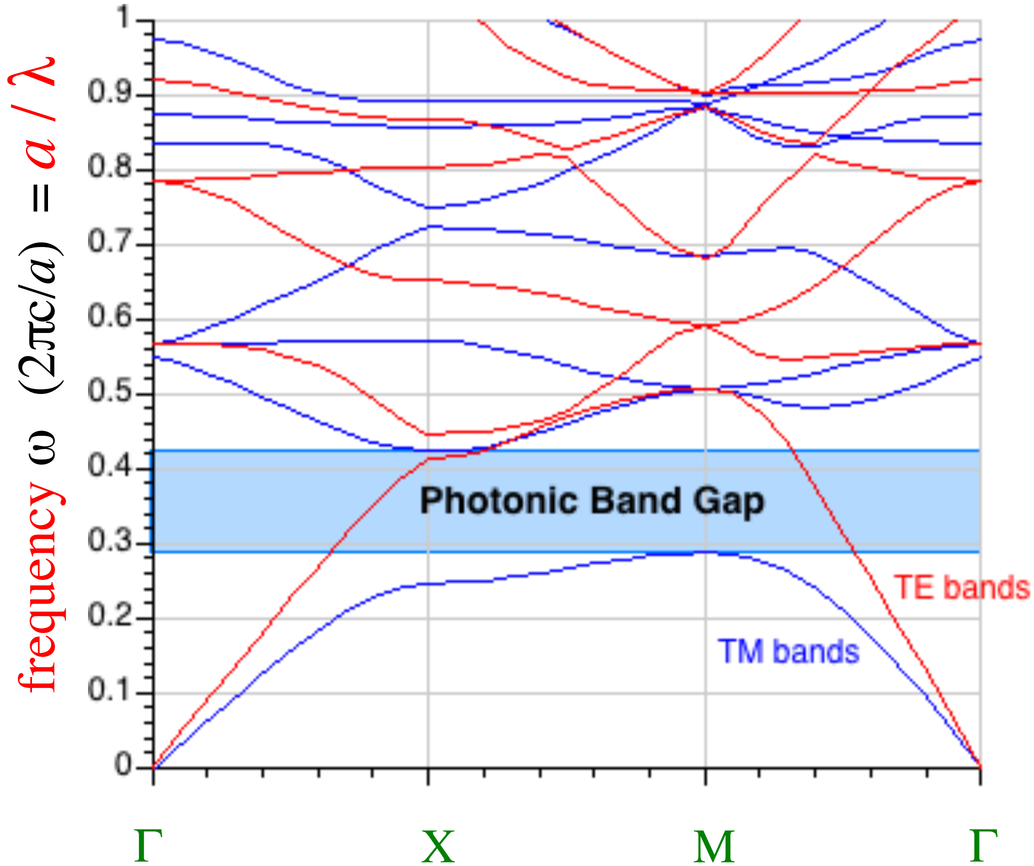
2d periodicity, ε=12:1



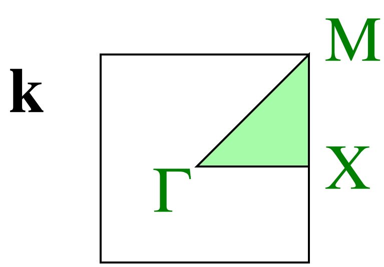
gap for
$n > \sim 1.75:1$

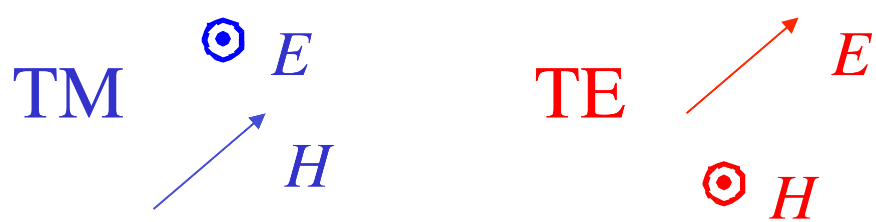
2d periodicity, ε=12:1



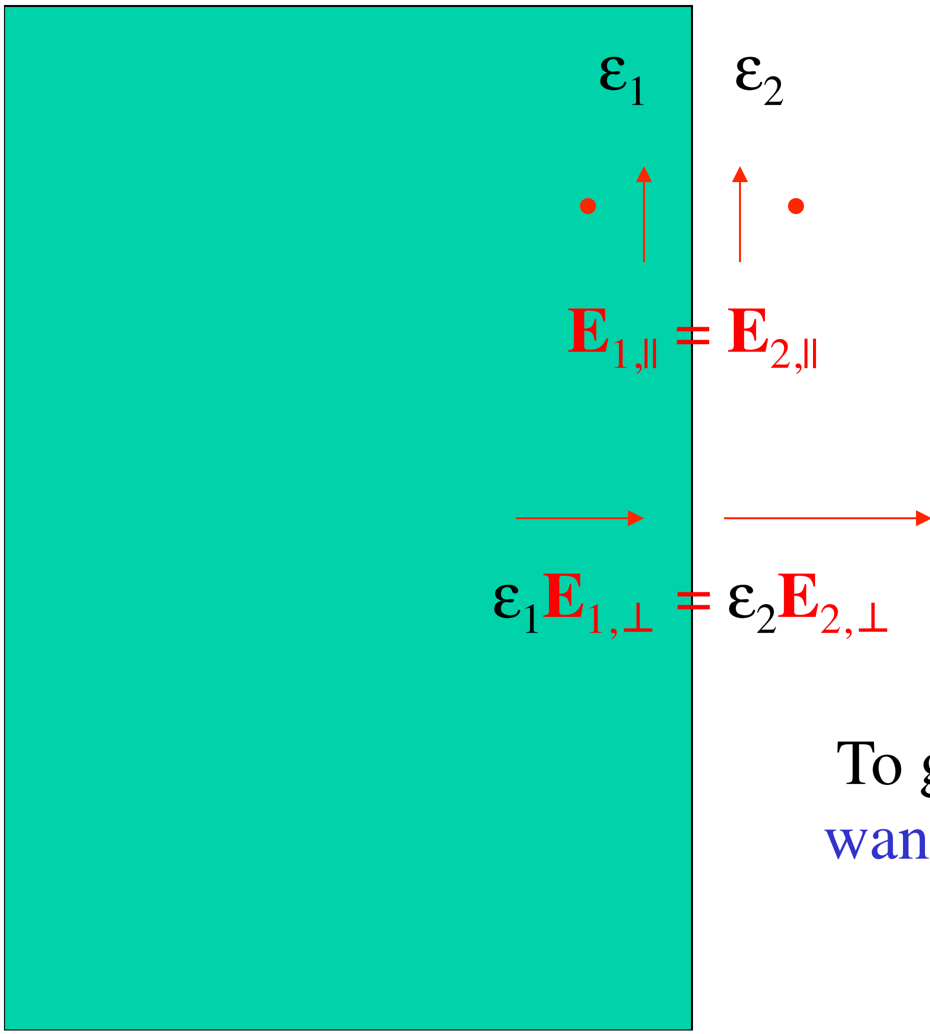


irreducible Brillouin zone





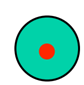
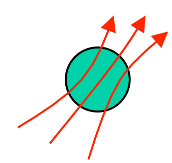
What a difference a boundary condition makes…



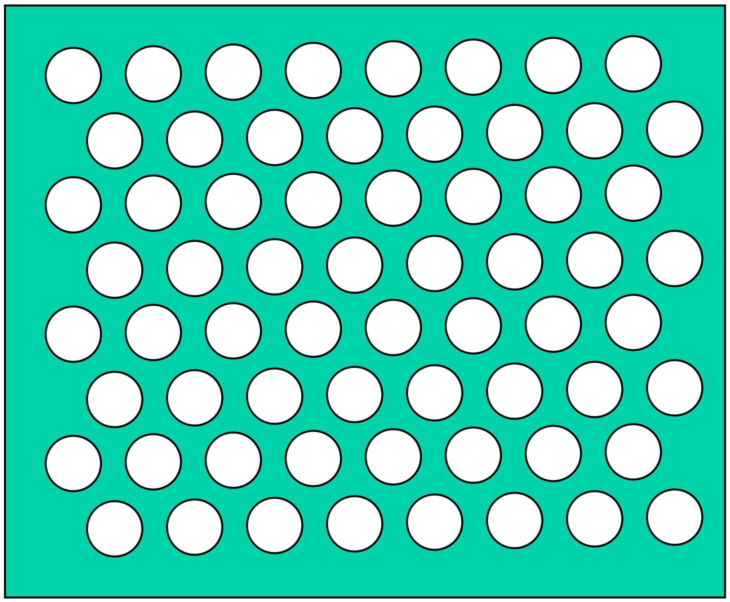
$\mathbf{E}_{\parallel}$ is continuous:
energy density $\varepsilon|\mathbf{E}|^2$
more in larger ε

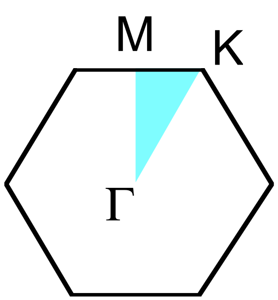
$\varepsilon\mathbf{E}_{\perp}$ is continuous:
energy density $|\varepsilon\mathbf{E}|^2/\varepsilon$
more in smaller ε

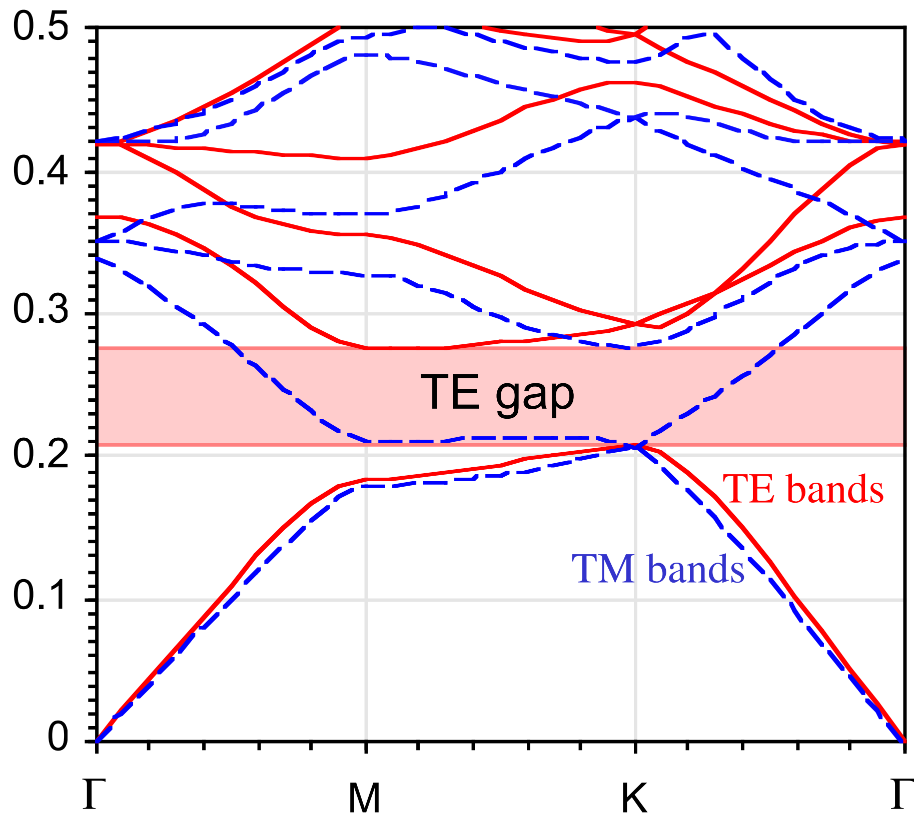
To get strong confinement & gaps,
want **E** mostly parallel to interfaces

TM: $\parallel$  TE: $\perp$ 

2d photonic crystal: TE gap, ε=12:1





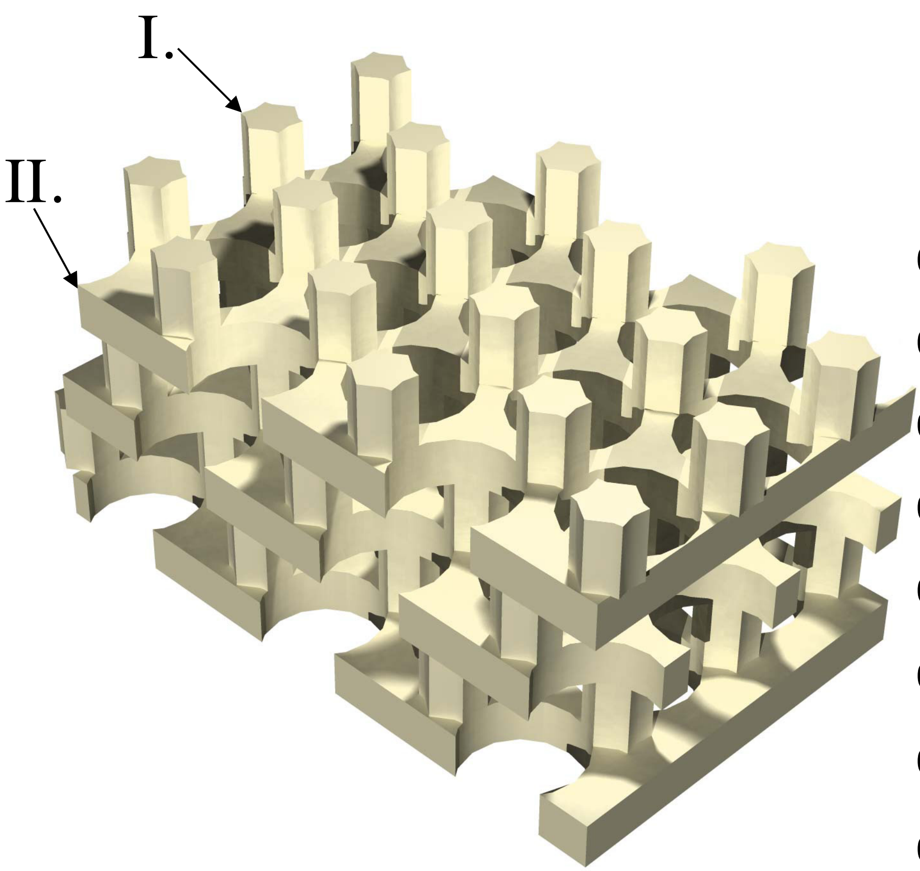


TE *E*

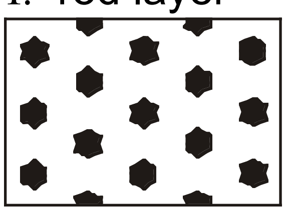
H

gap for n > ~1.4:1

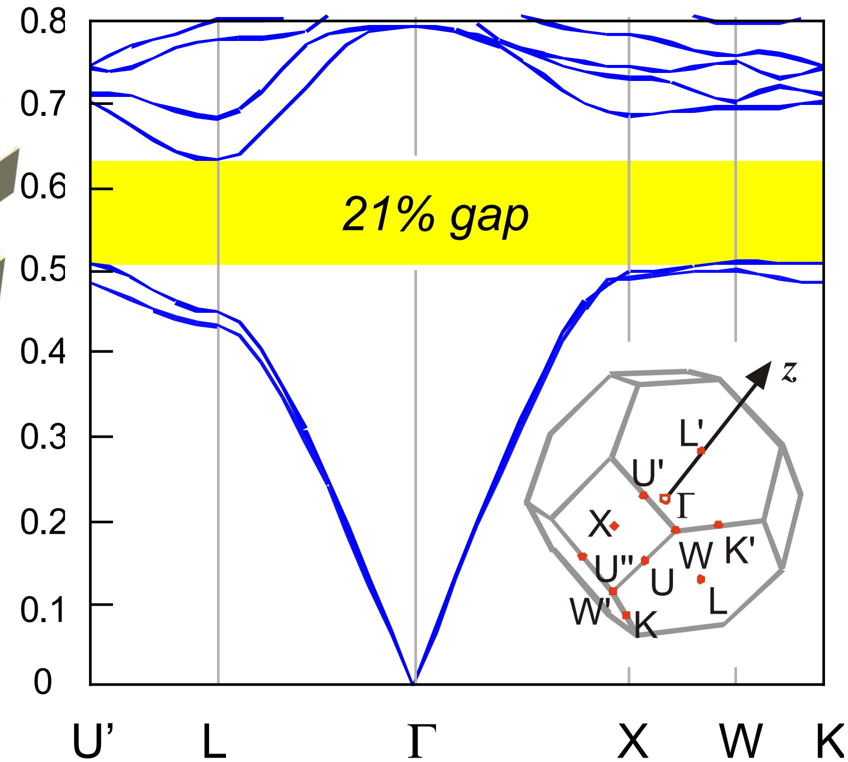
3d photonic crystal: complete gap , ε=12:1



I: rod layer



II: hole layer



gap for n > ~2:1

[S. G. Johnson *et al*., *Appl. Phys. Lett.* **77**, 3490 (2000)]

You, too, can compute photonic eigenmodes!

MIT Photonic-Bands (MPB) package:

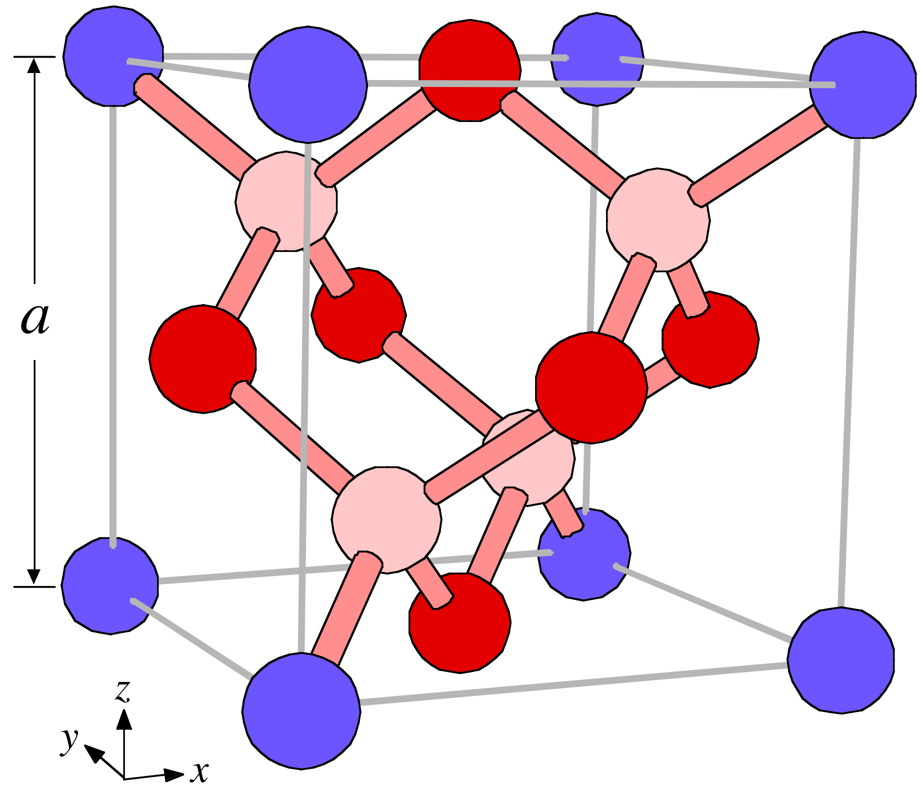
`http://ab-initio.mit.edu/mpb`

The Mother of (almost) All Bandgaps

The diamond lattice:

fcc (face-centered-cubic)
with two "atoms" per unit cell

(primitive)



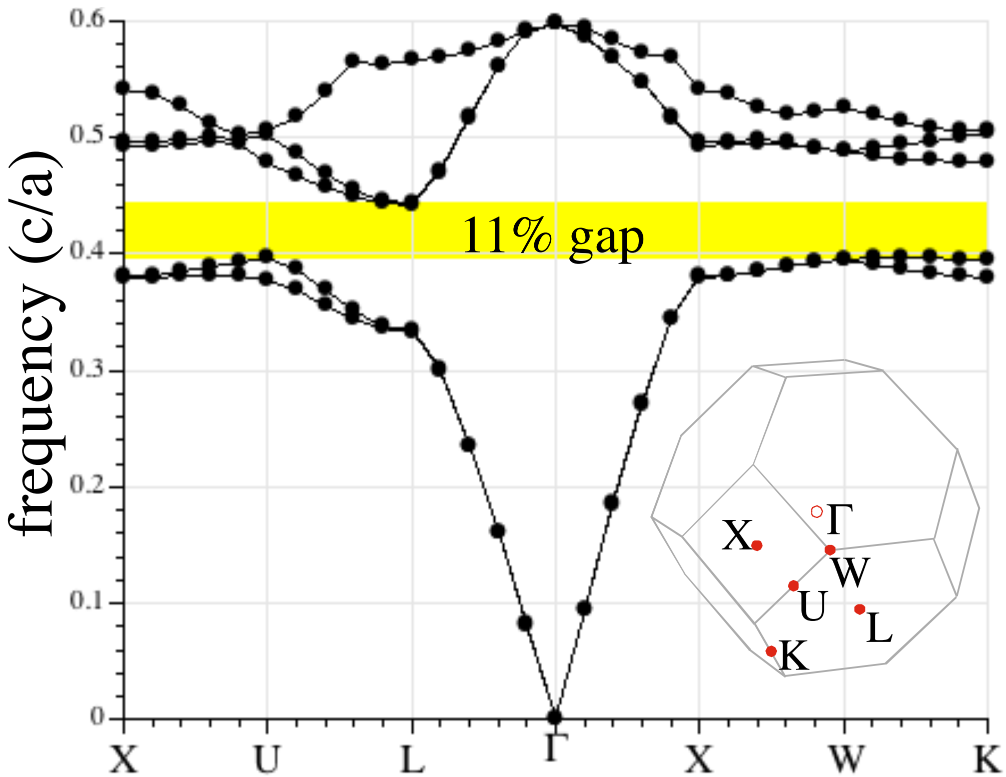
Recipe for a complete gap:

fcc = most-spherical Brillouin zone

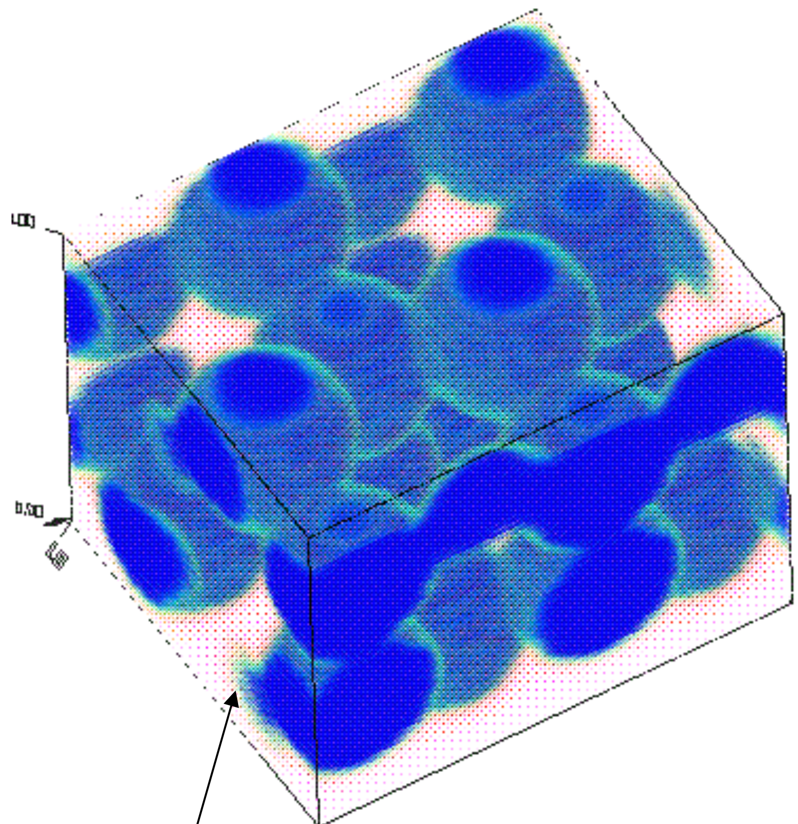
+ diamond "bonds" = lowest (two) bands can concentrate in lines

The First 3d Bandgap Structure

K. M. Ho, C. T. Chan, and C. M. Soukoulis, *Phys. Rev. Lett.* **65**, 3152 (1990).



for gap at $\lambda = 1.55\mu m$,
sphere diameter ~ 330nm



overlapping Si spheres

MPB tutorial, `http://ab-initio.mit.edu/mpb`

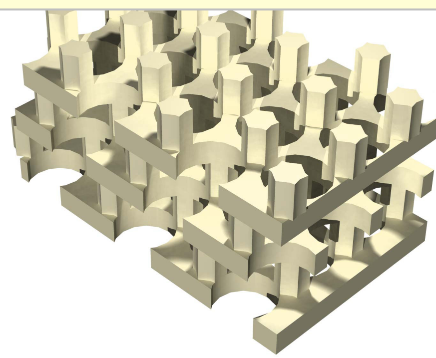
Layer-by-Layer Lithography

• Fabrication of 2d patterns in Si or GaAs is very advanced (think: Pentium IV, 50 million transistors)

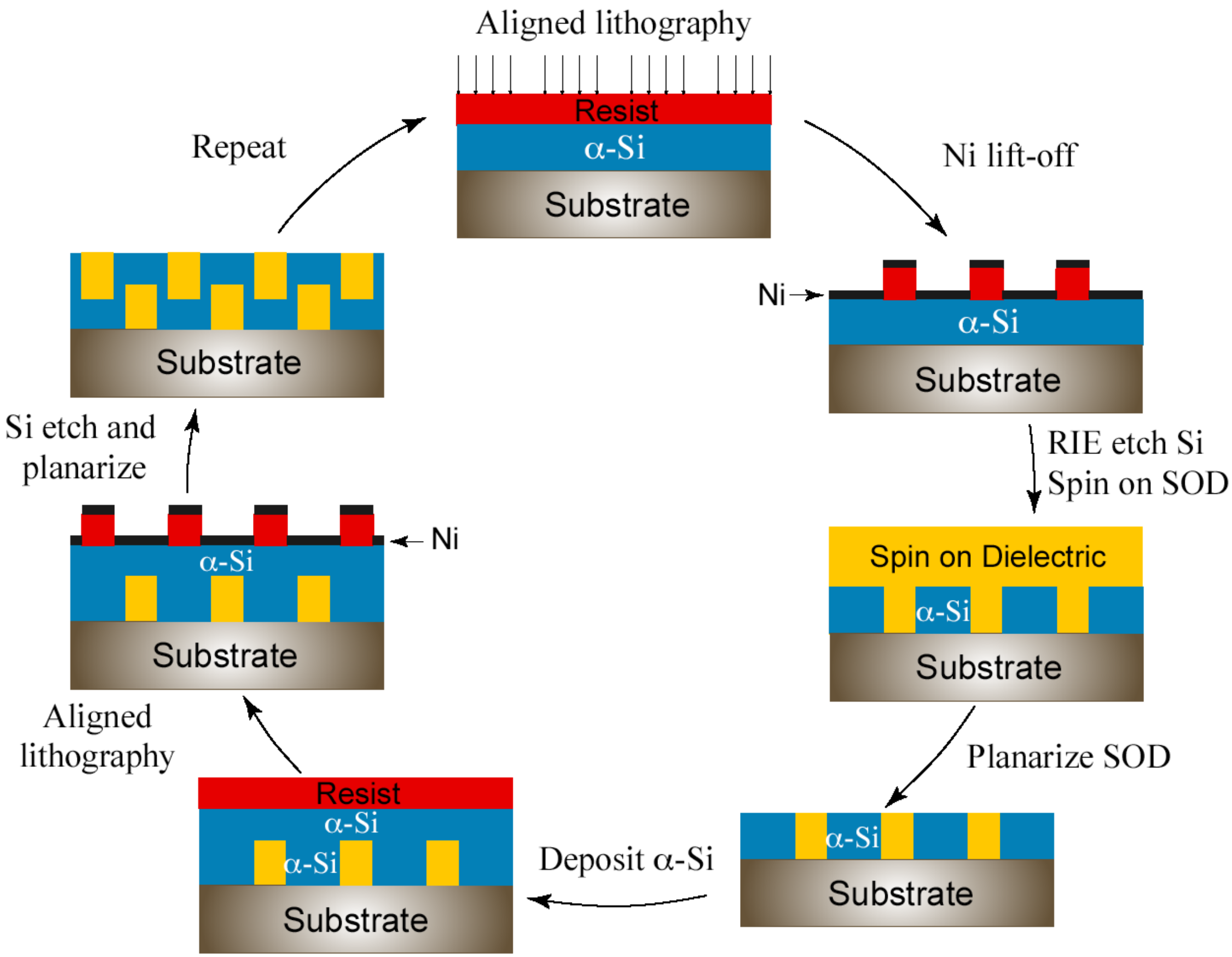
…inter-layer alignment techniques are only slightly more exotic

So, make 3d structure one layer at a time

Need a 3d crystal with constant cross-section layers



A Schematic



[M. Qi, H. Smith, MIT]

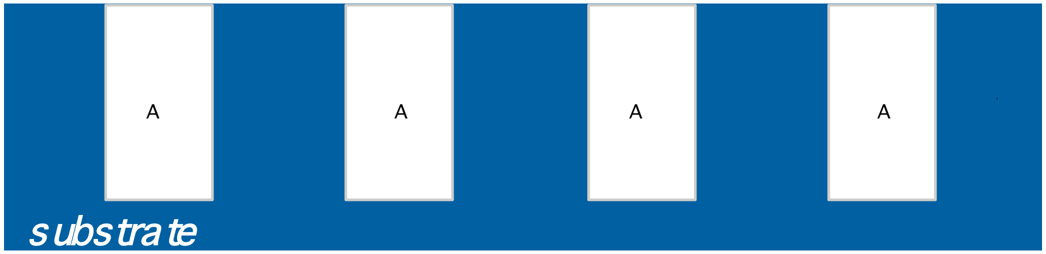
Making Rods & Holes Simultaneously

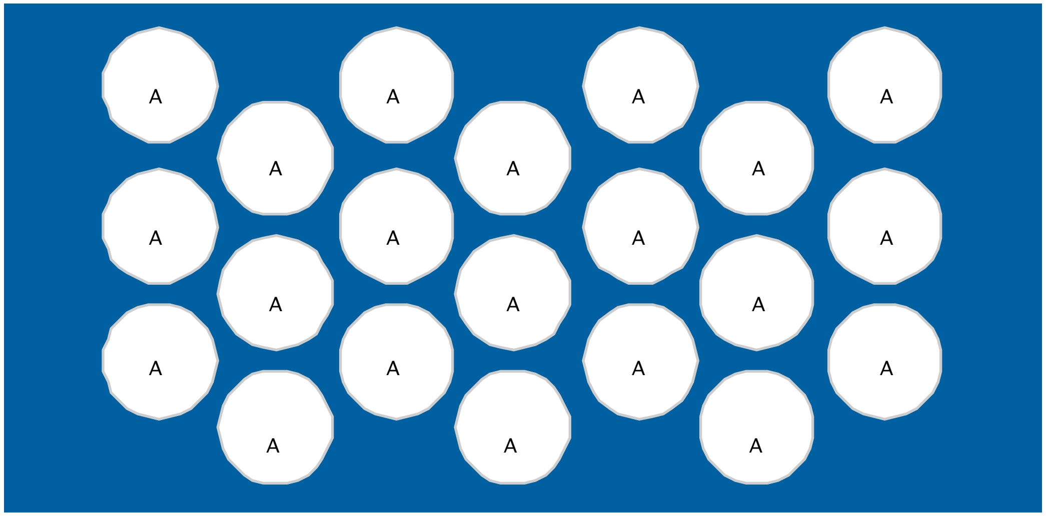
side view

top view

Making Rods & Holes Simultaneously

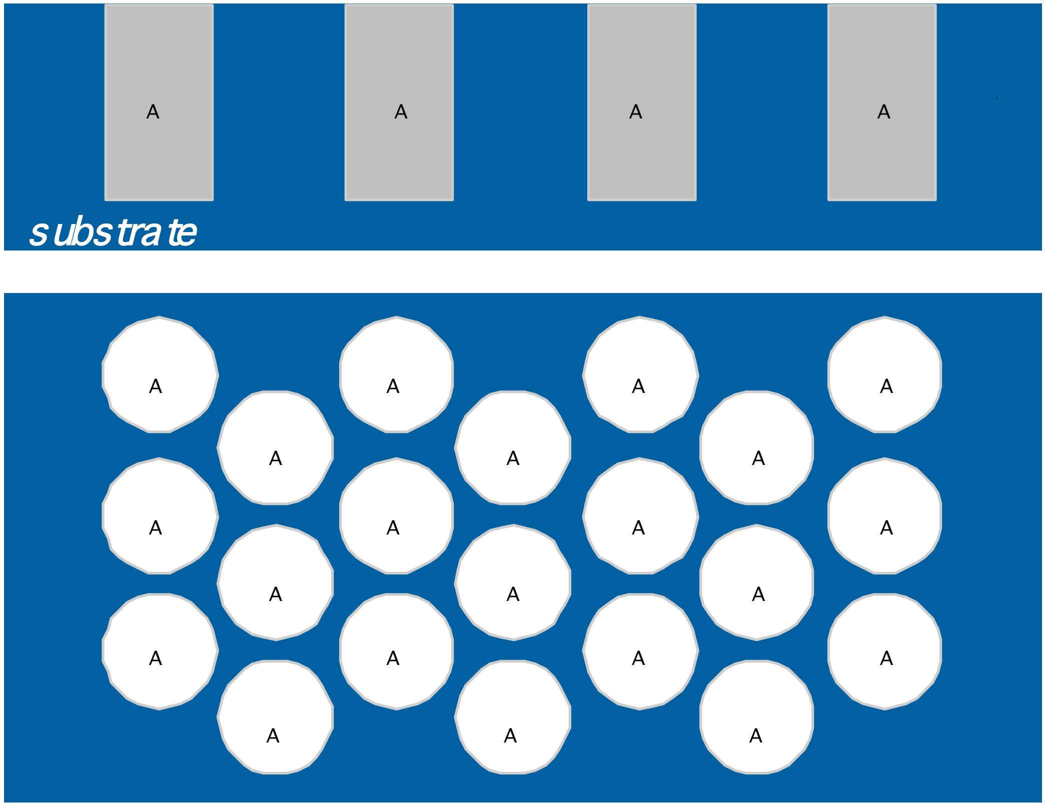
expose/etch holes





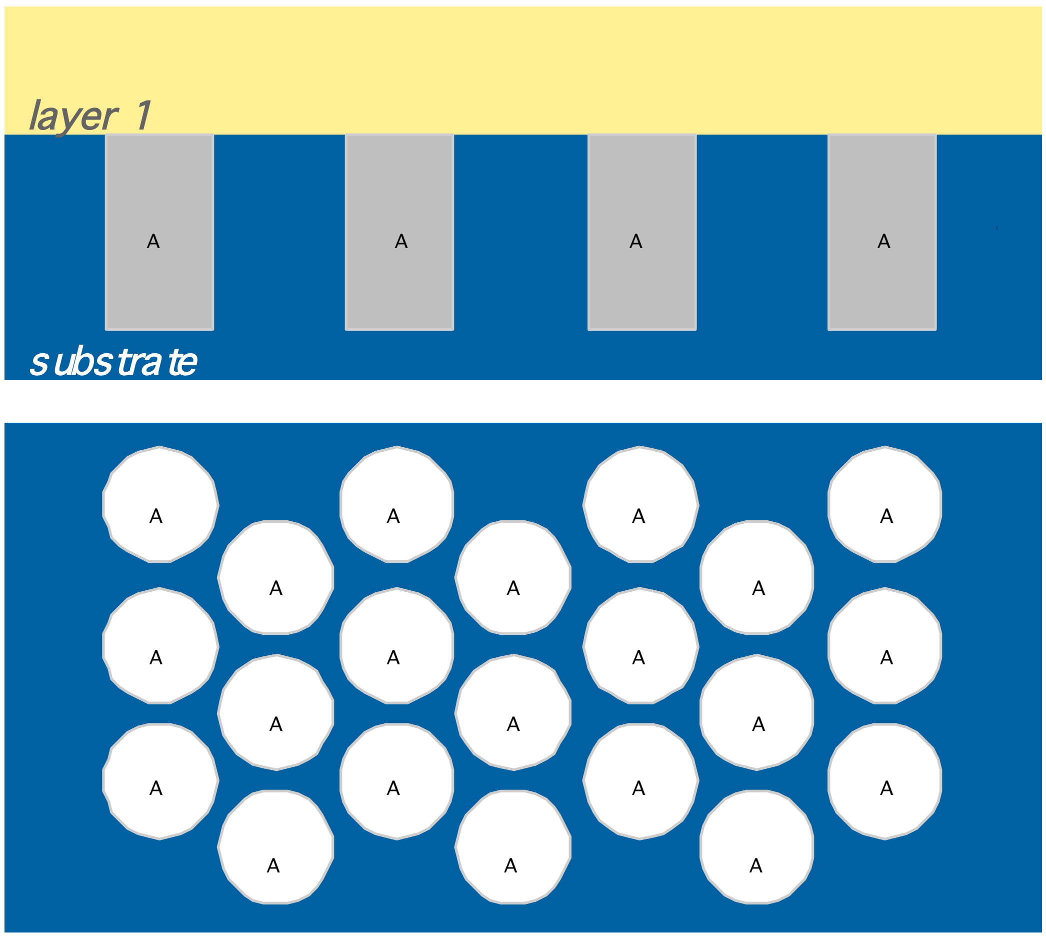
Making Rods & Holes Simultaneously

backfill with silica (SiO_2) & polish



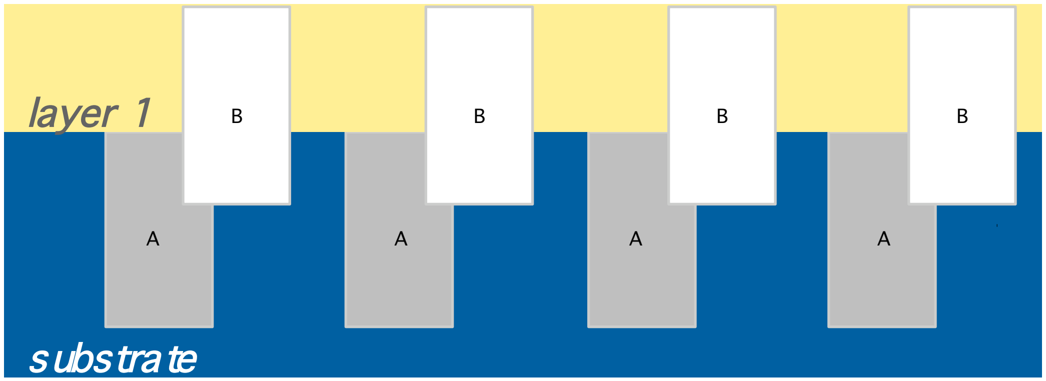
Making Rods & Holes Simultaneously

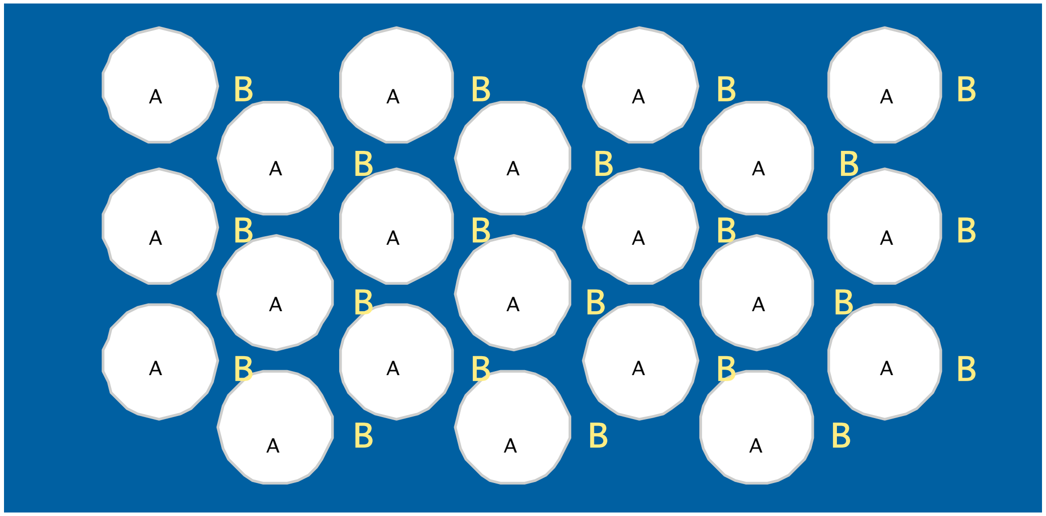
deposit another
Si layer



Making Rods & Holes Simultaneously

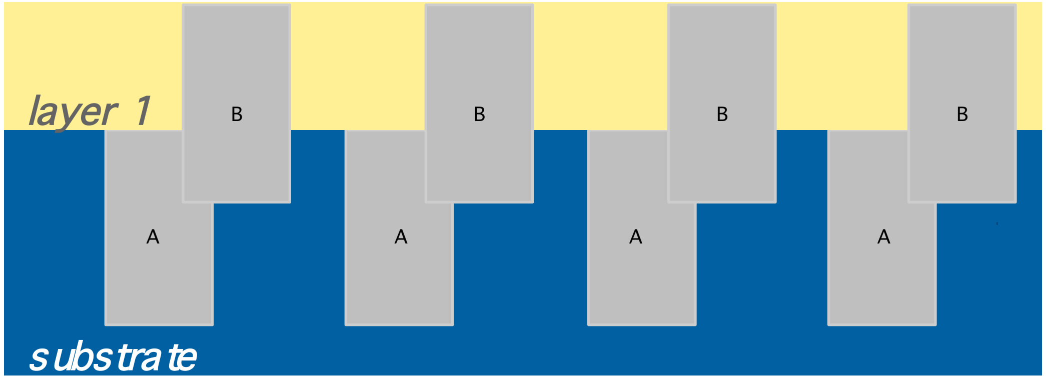
dig more holes
offset
& overlapping

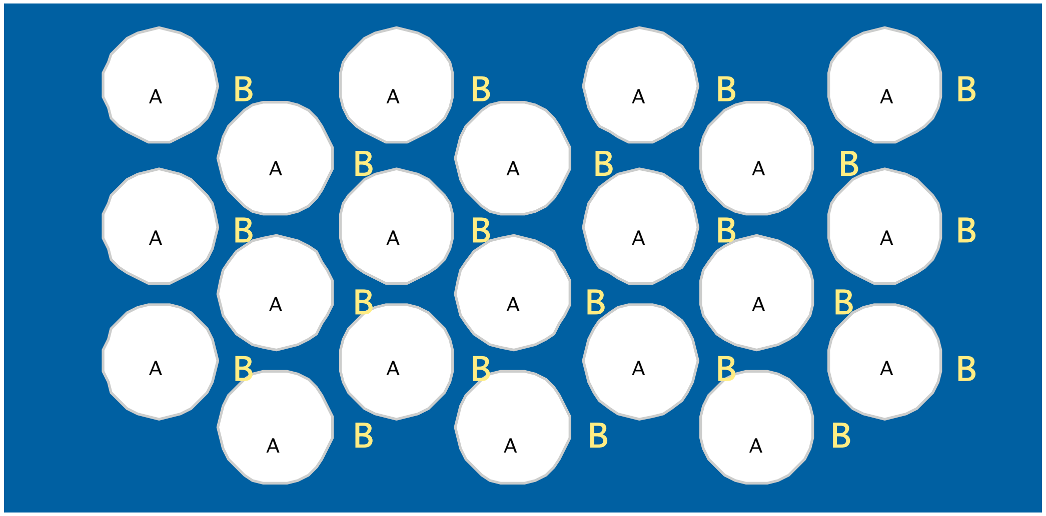




Making Rods & Holes Simultaneously

backfill

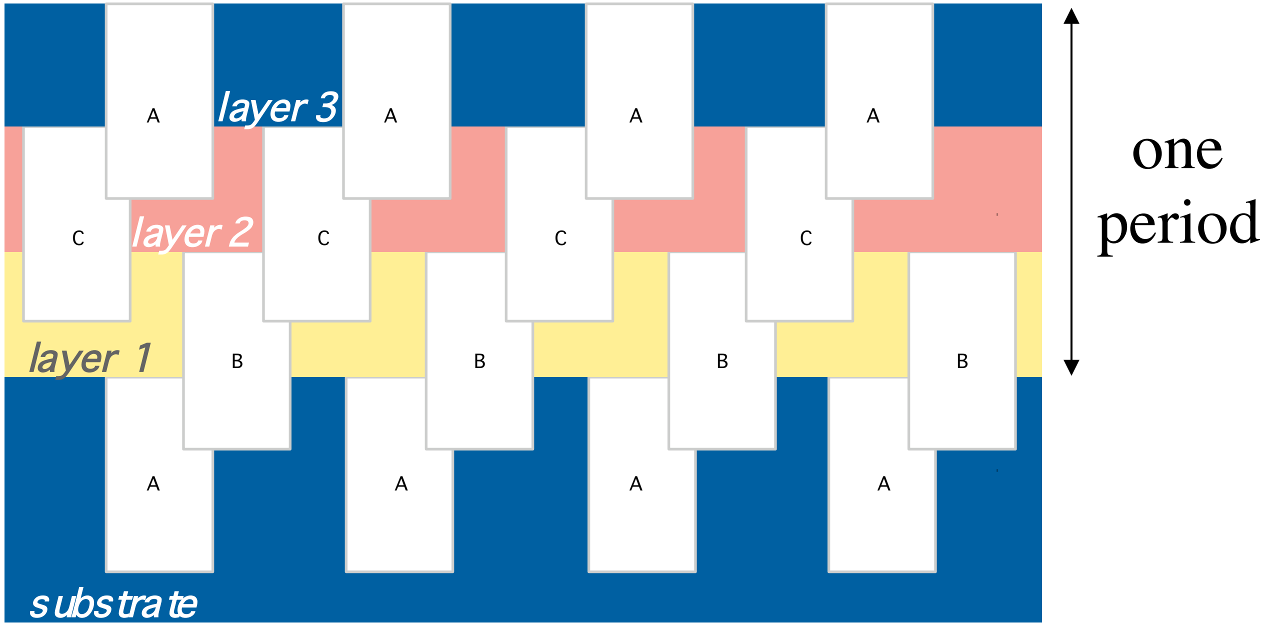


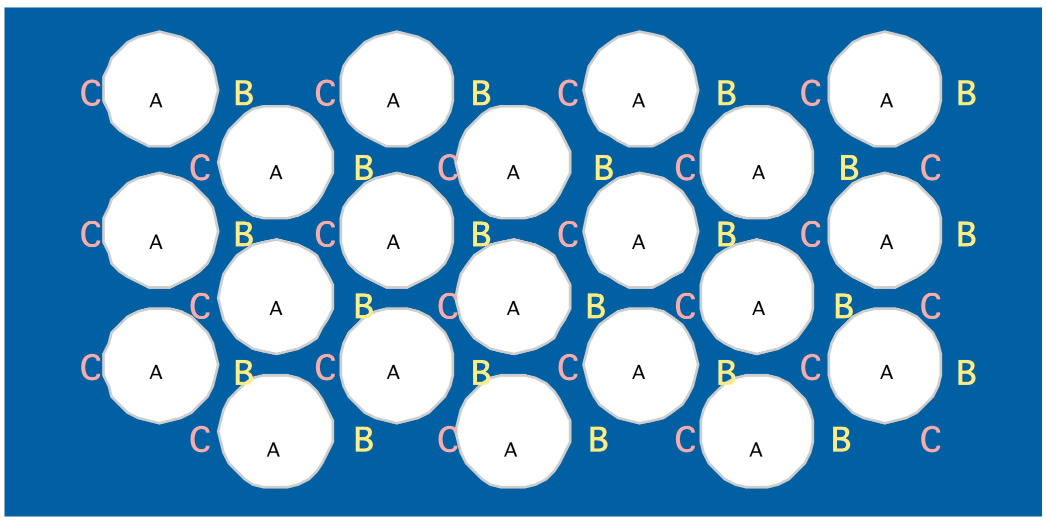


Making Rods & Holes Simultaneously

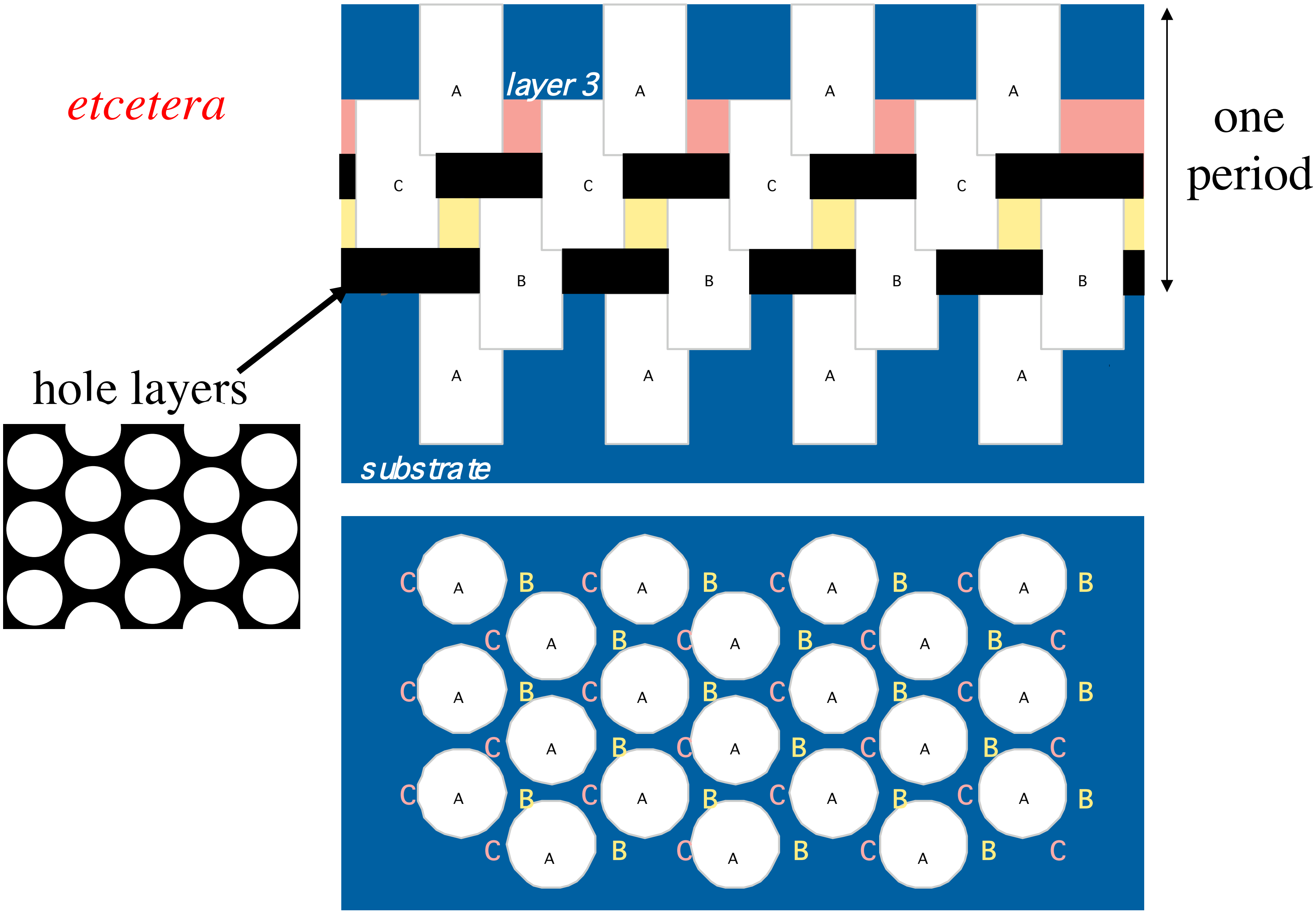
etcetera

(dissolve silica when done)



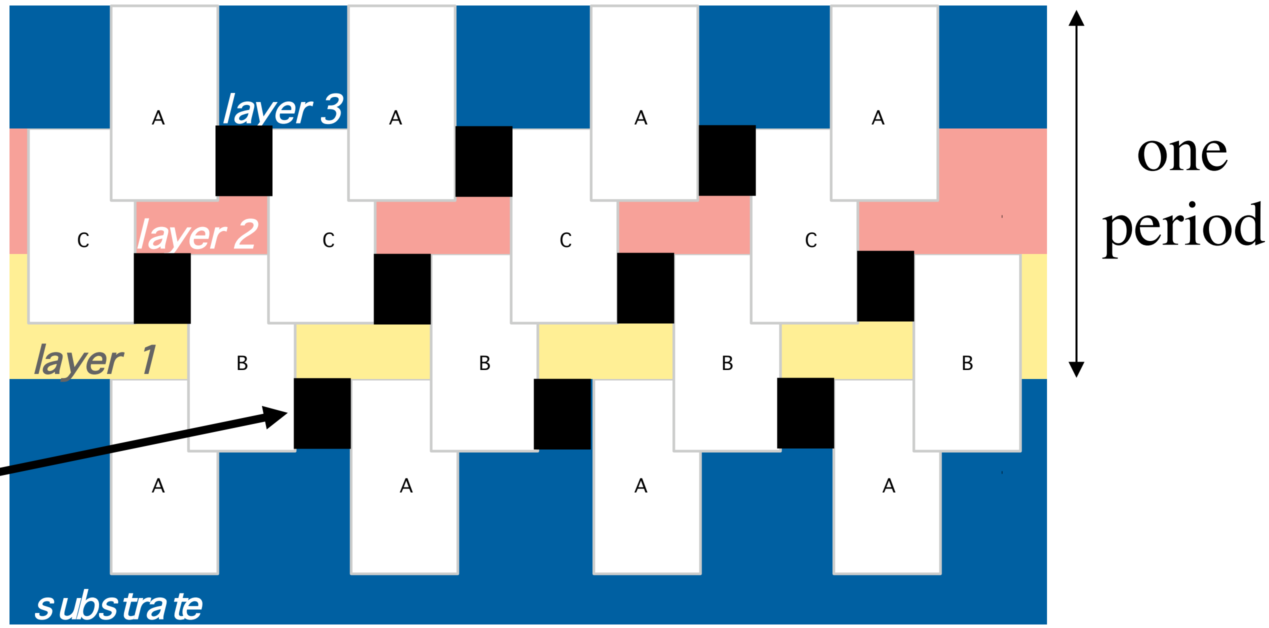


Making Rods & Holes Simultaneously

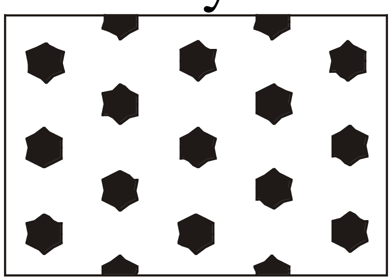


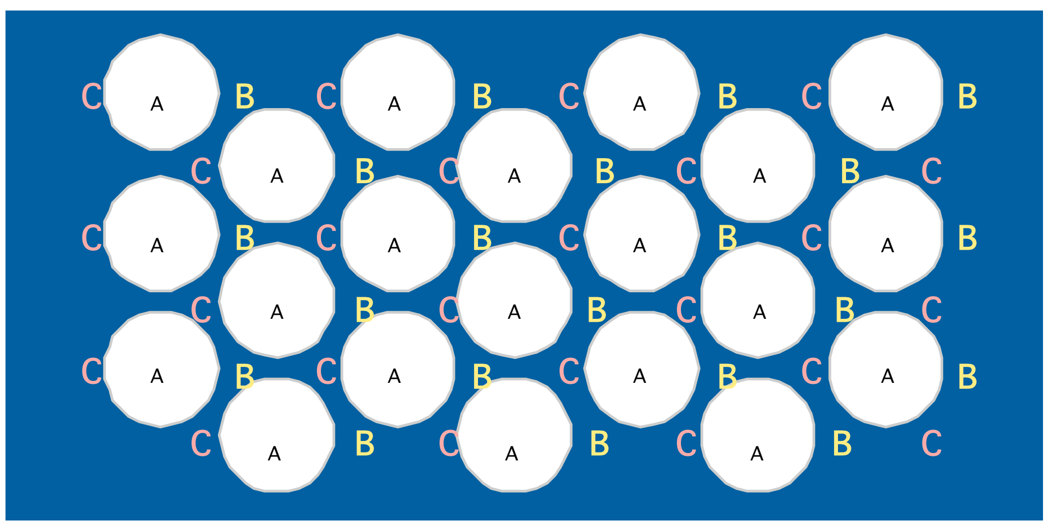
Making Rods & Holes Simultaneously

etcetera

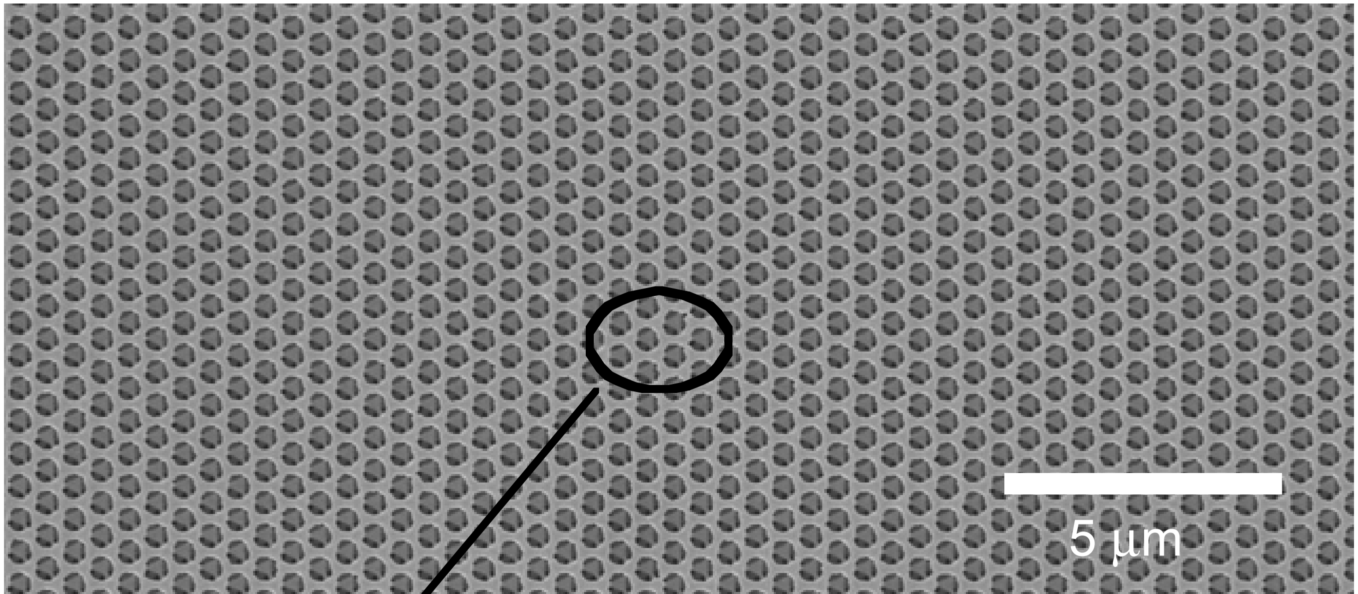


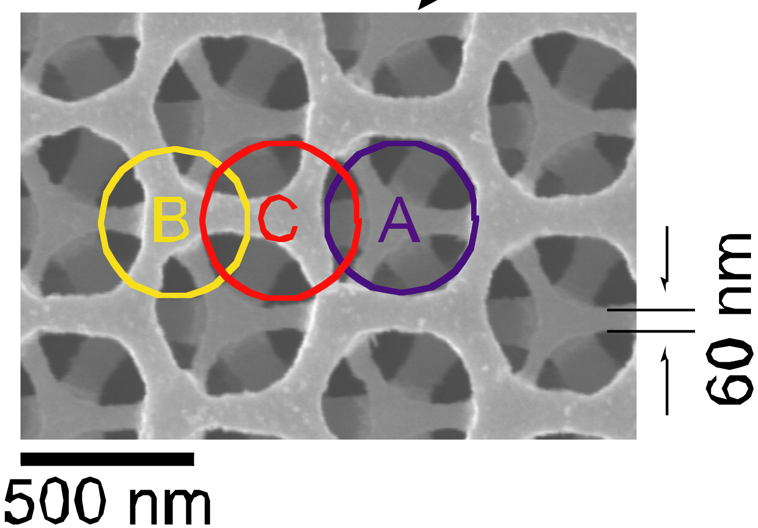
rod layers

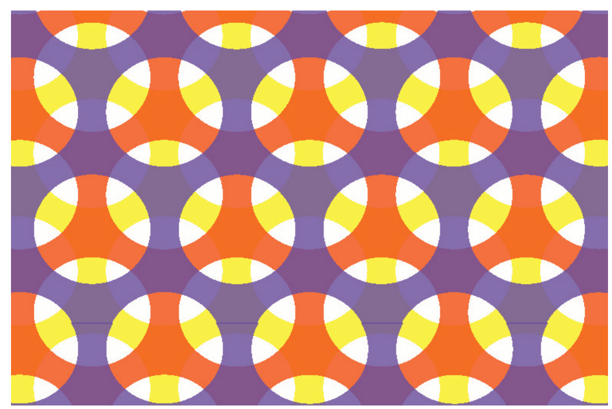




7-layer E-Beam Fabrication







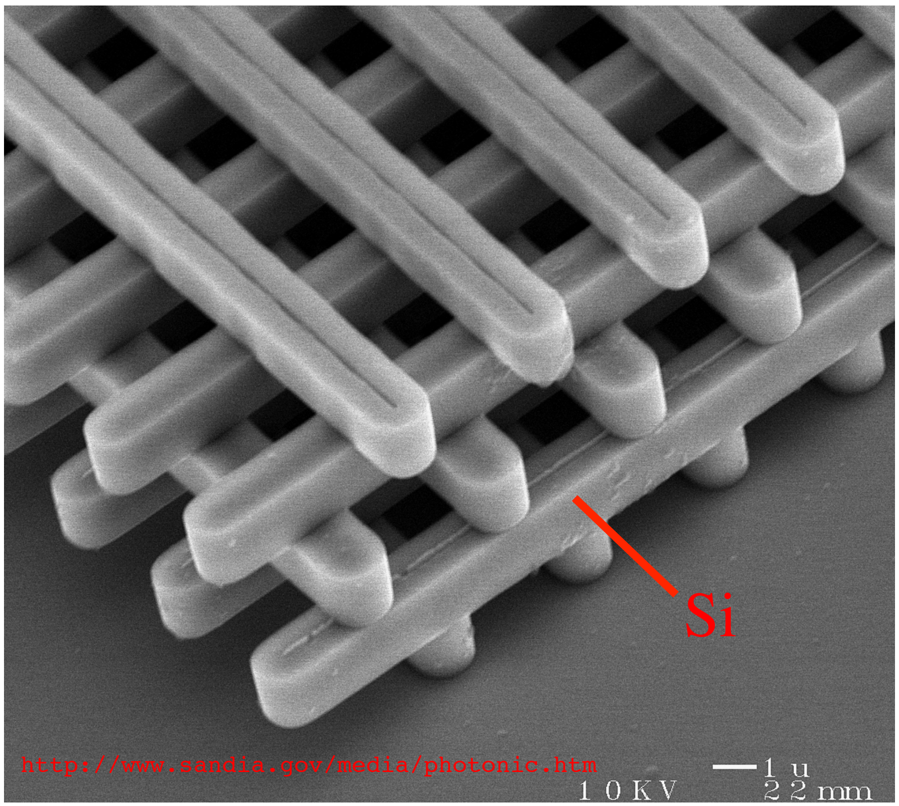
[M. Qi, *et al*., *Nature* **429**, 538 (2004)]

an earlier design:
(& currently more popular)

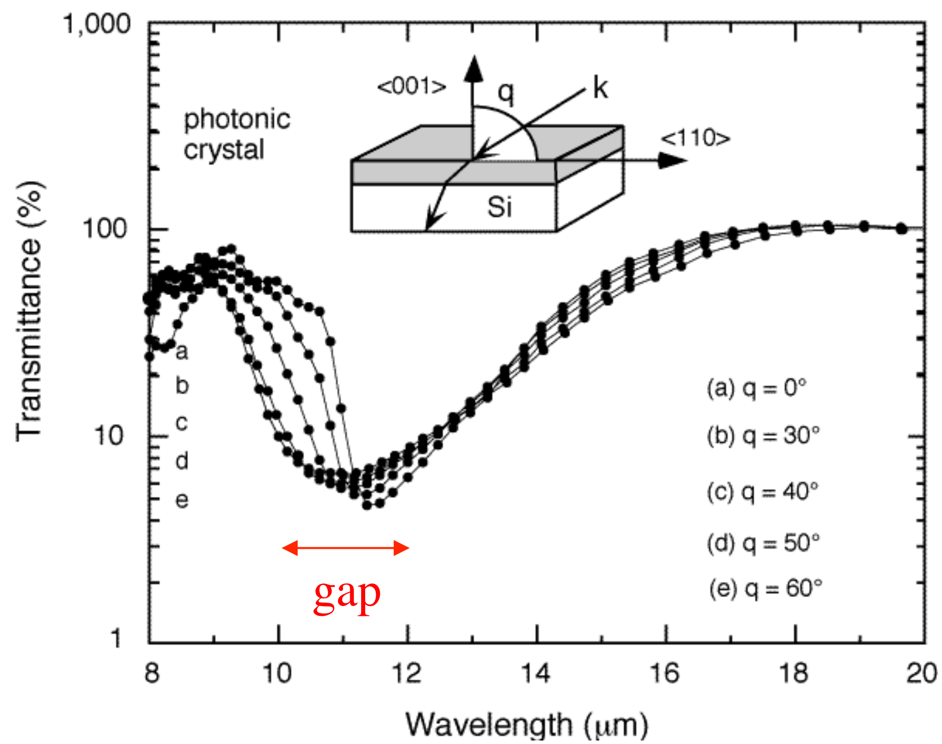
The Woodpile Crystal

[K. Ho *et al.*, *Solid State Comm.* **89**, 413 (1994)] [H. S. Sözüer *et al.*, *J. Mod. Opt.* **41**, 231 (1994)]

(4 “log” layers = 1 period)



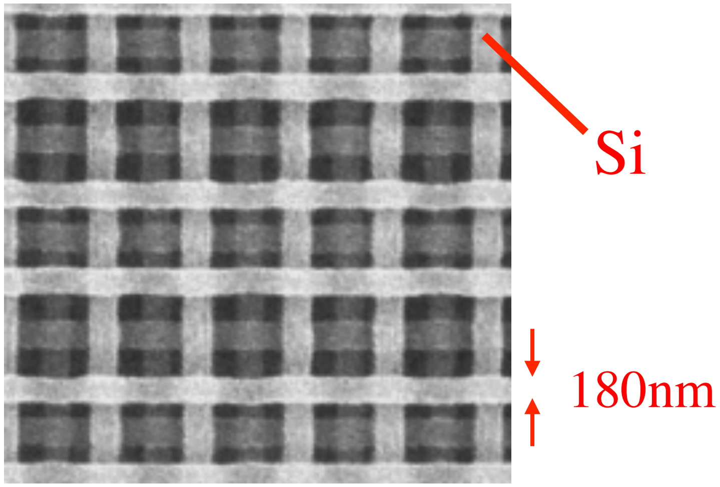
[S. Y. Lin *et al.*, *Nature* **394**, 251 (1998)]

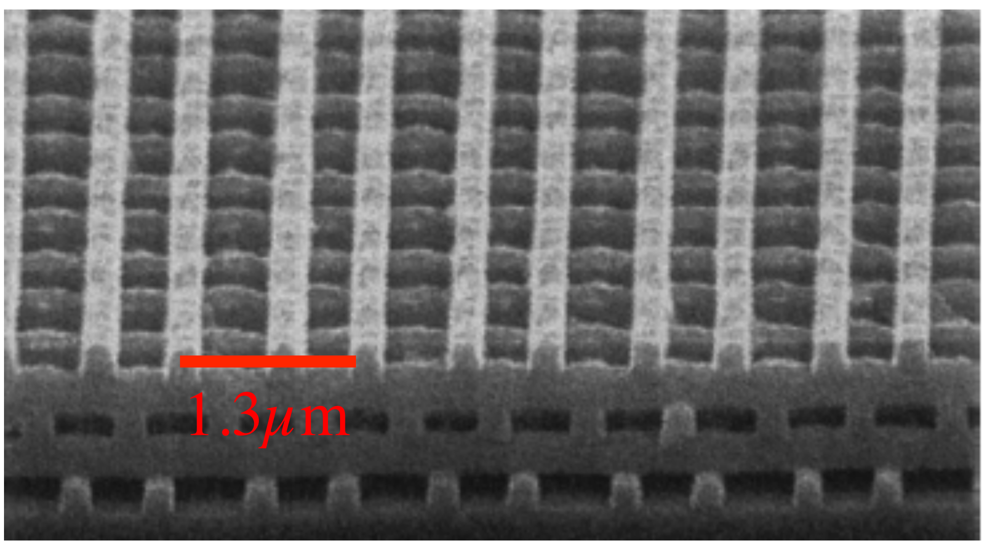


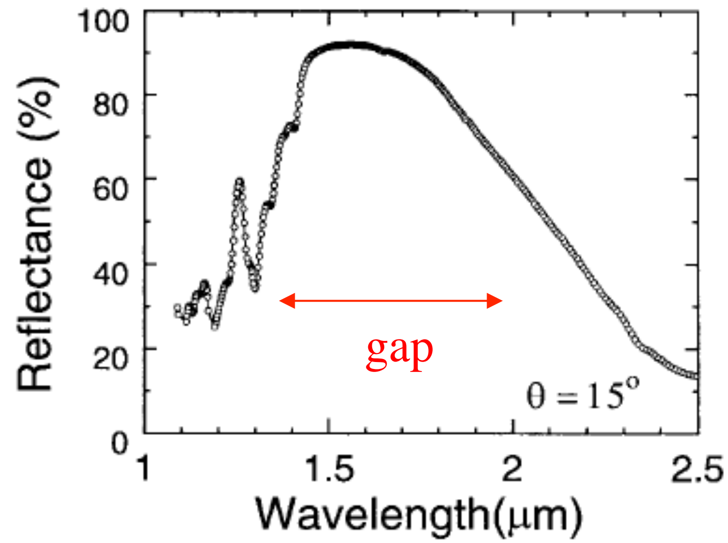
1.25 Periods of Woodpile @ 1.55μm

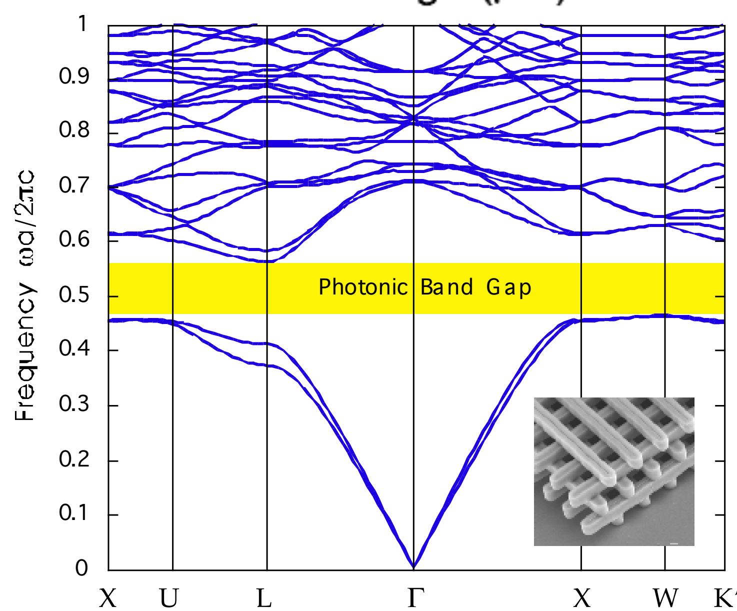
(4 “log” layers = 1 period)

[Lin & Fleming, *JLT* **17**, 1944 (1999)]

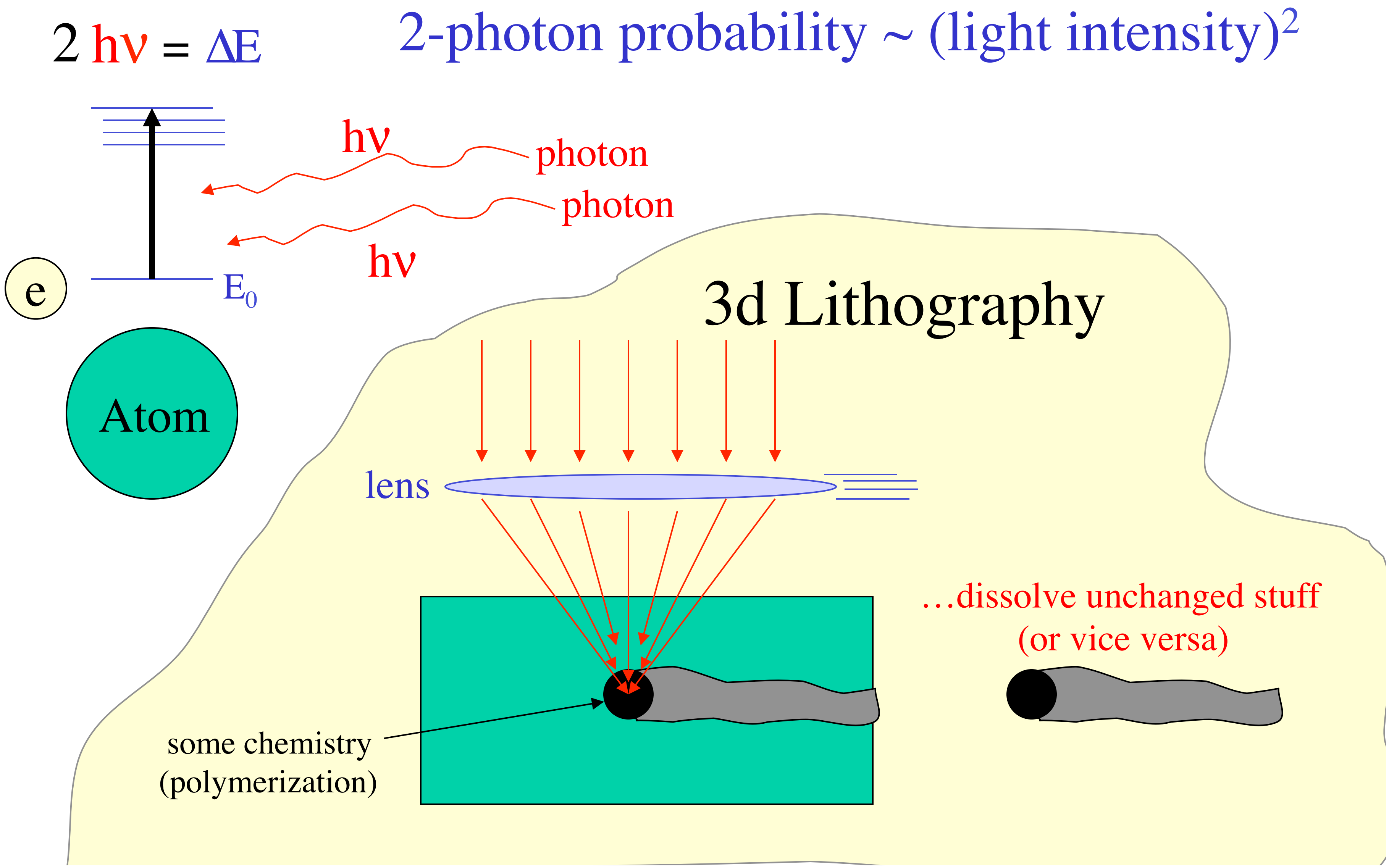






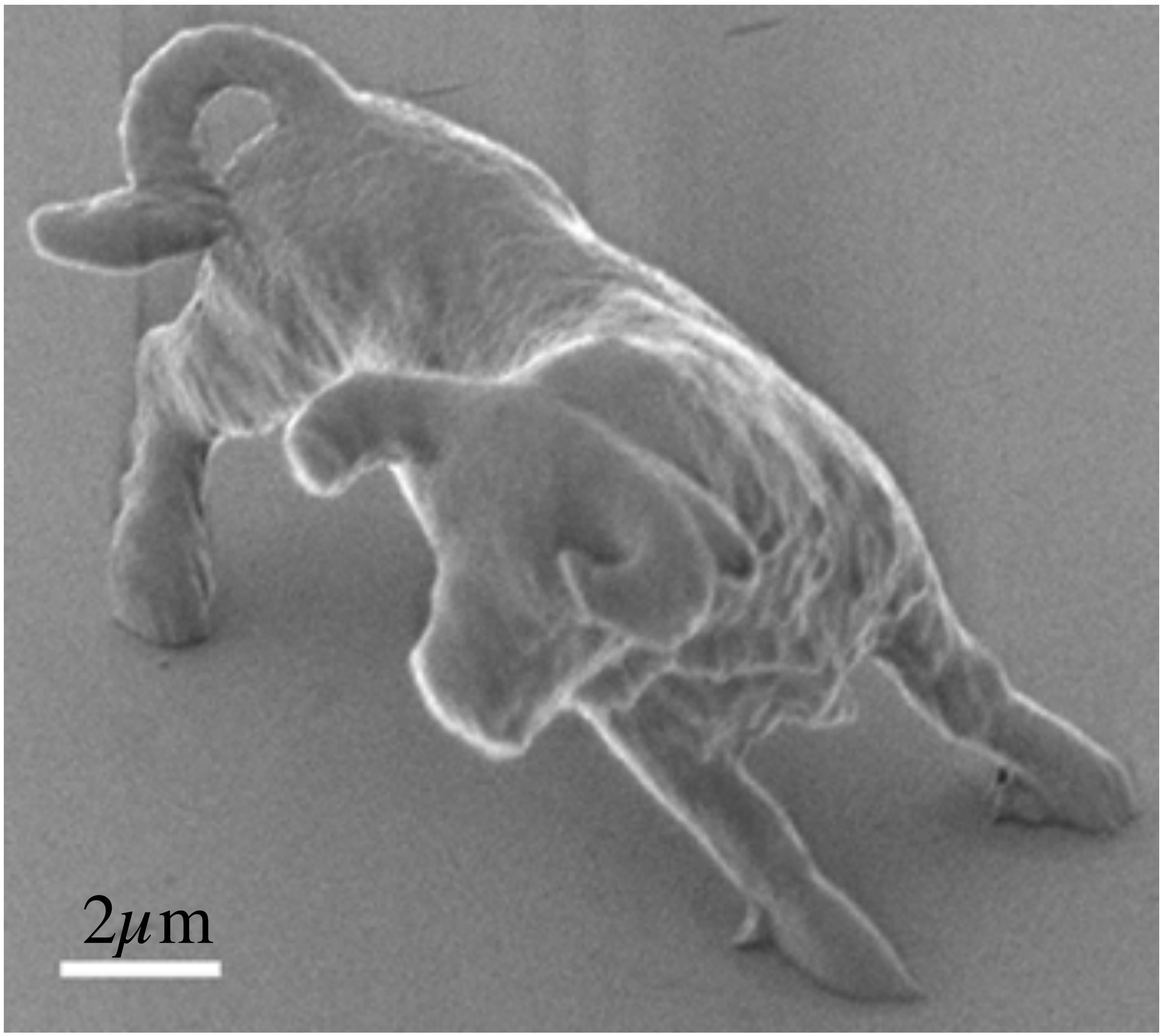


Two-Photon Lithography



Lithography is a Beast

[S. Kawata *et al*., *Nature* **412**, 697 (2001)]



$\lambda = 780\text{nm}$

resolution = 150nm

7μm

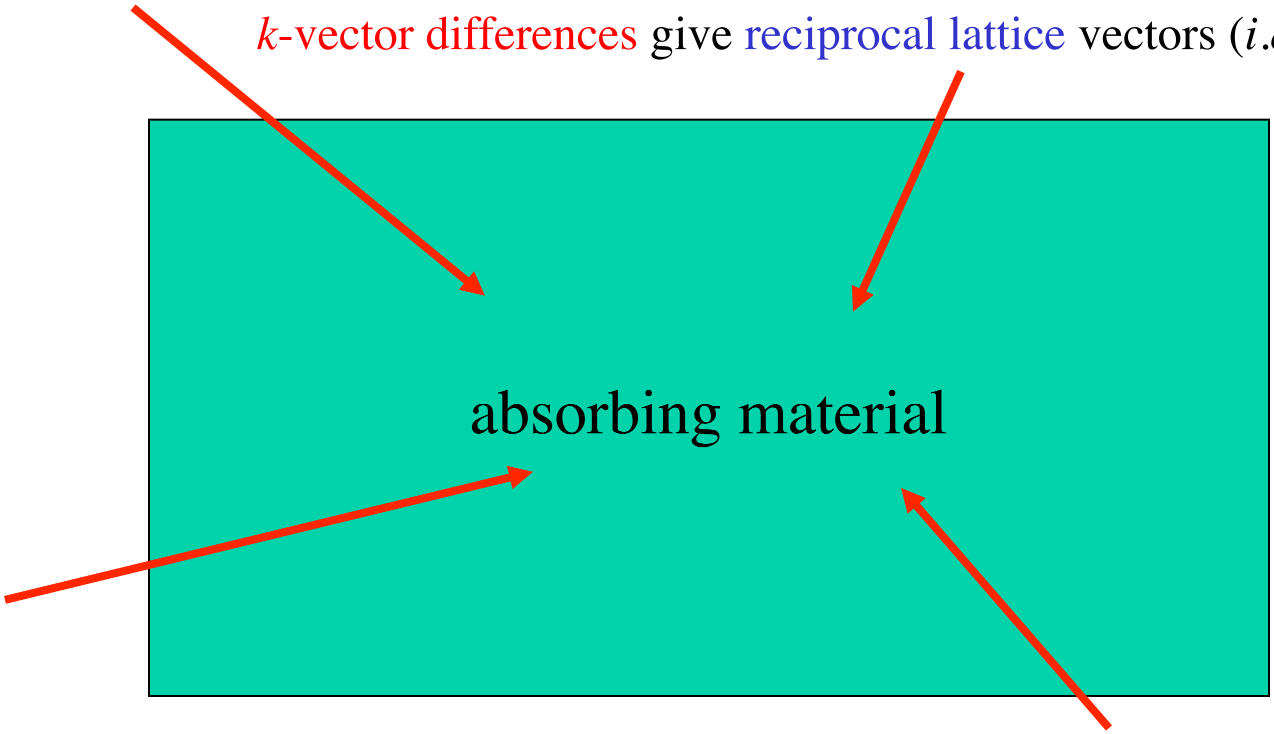
(3 hours to make)

Holographic Lithography

[D. N. Sharp *et al*., *Opt. Quant. Elec.* **34**, 3 (2002)]

Four beams make 3d-periodic interference pattern

k-vector differences give reciprocal lattice vectors (*i.e.* periodicity)

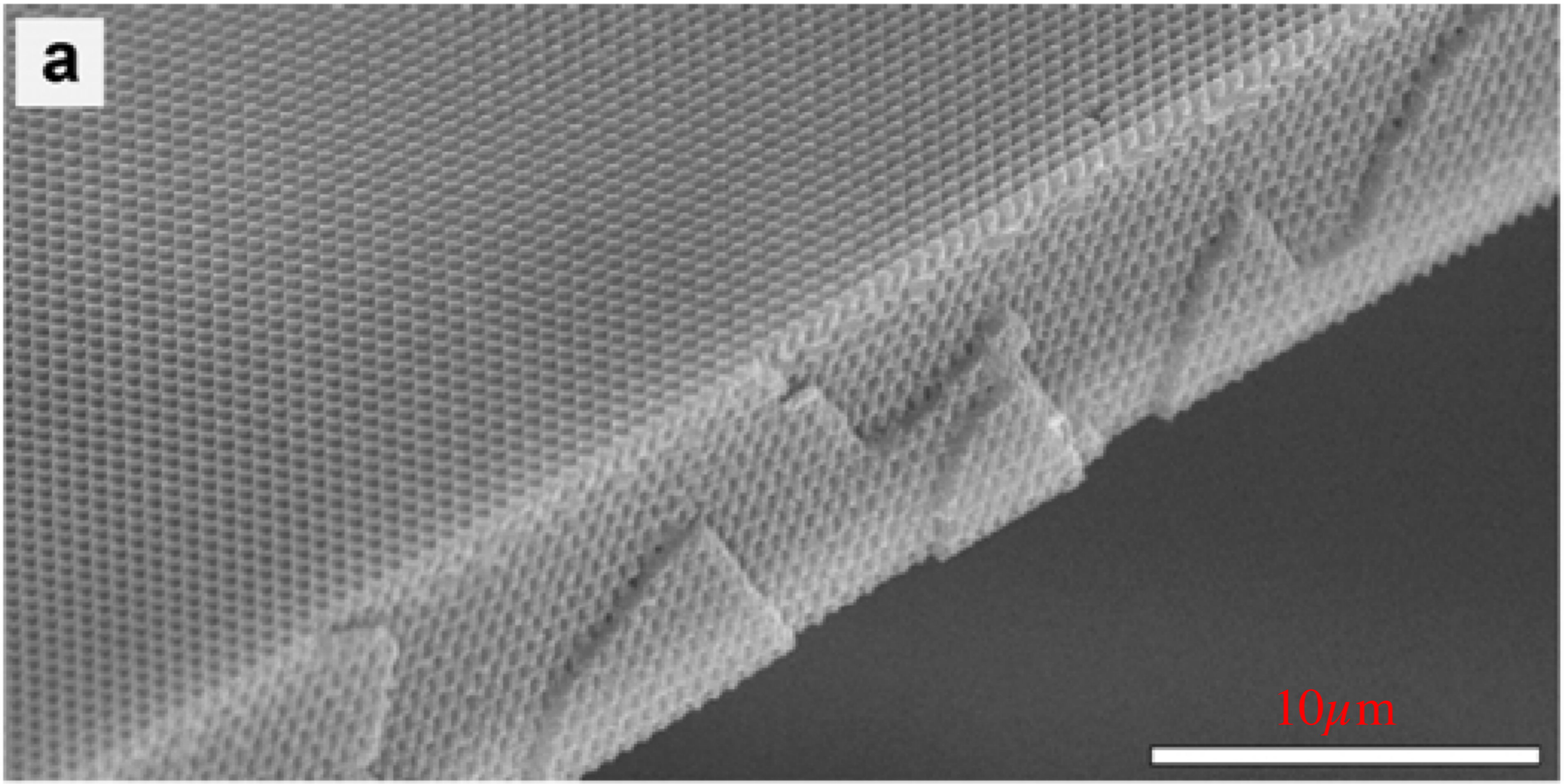


(1.4μm)

beam polarizations + amplitudes (8 parameters) give unit cell

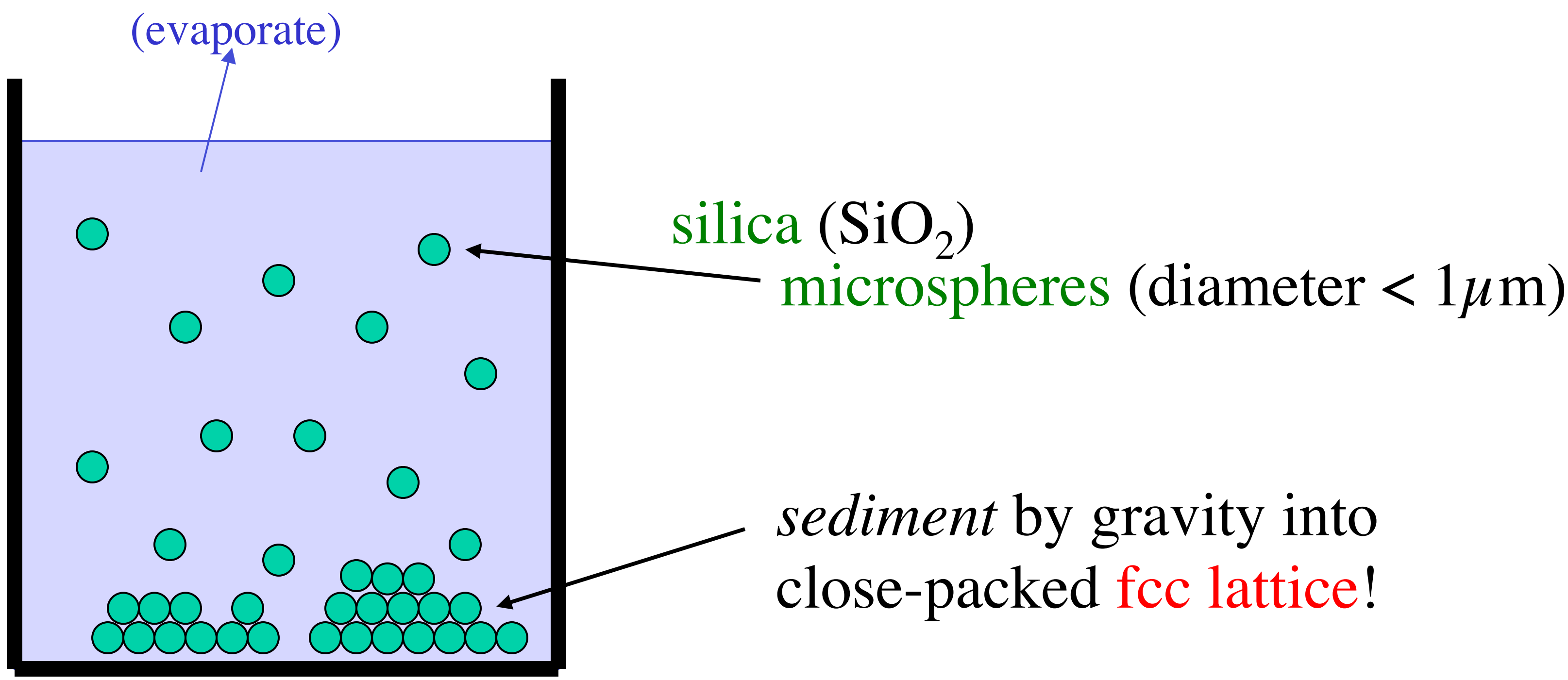
One-Photon
Holographic Lithography

[D. N. Sharp *et al*., *Opt. Quant. Elec.* **34**, 3 (2002)]

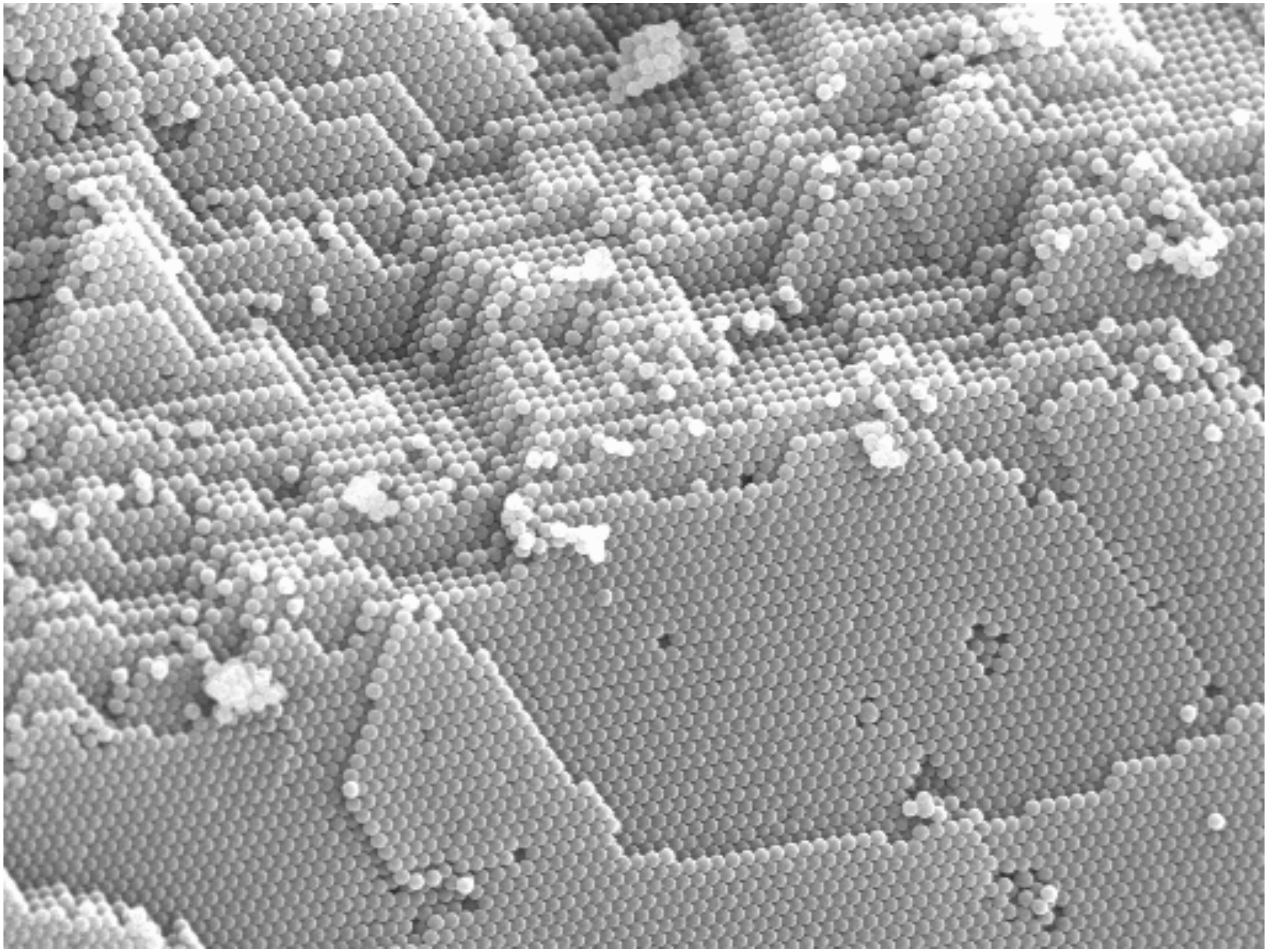


huge volumes, long-range periodic, fcc lattice…backfill for high contrast

Mass-production II: Colloids



Mass-production II: Colloids



http://www.icmm.csic.es/cefe/

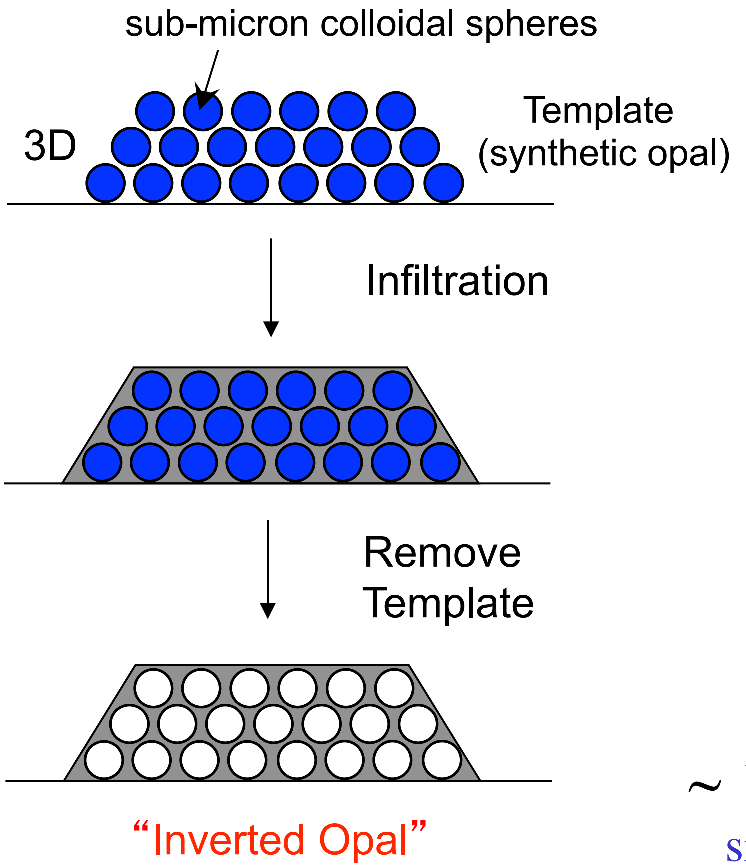
Inverse Opals

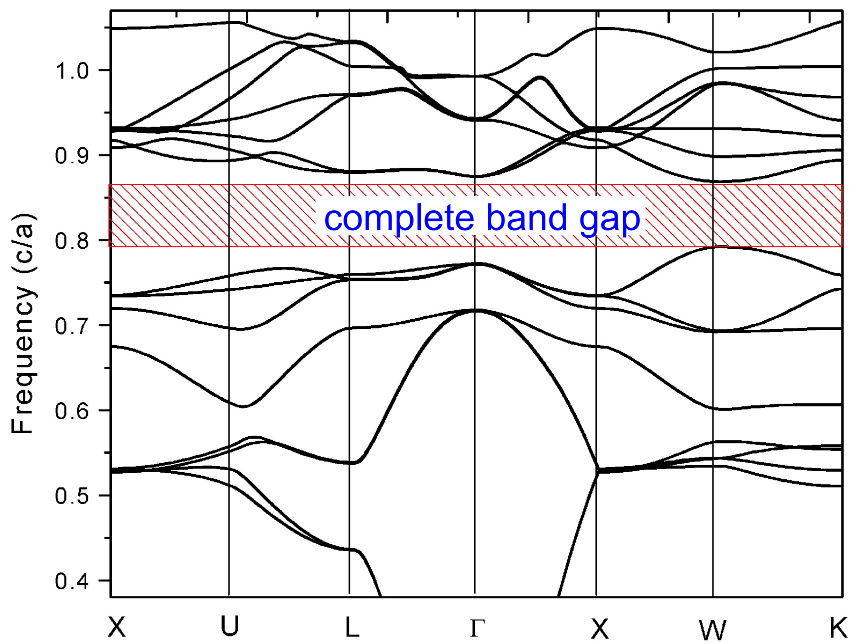
[figs courtesy D. Norris, UMN]

[H. S. Sözüer, *PRB* **45**, 13962 (1992)]

fcc solid spheres do not have a gap…

…but fcc spherical holes in Si *do* have a gap



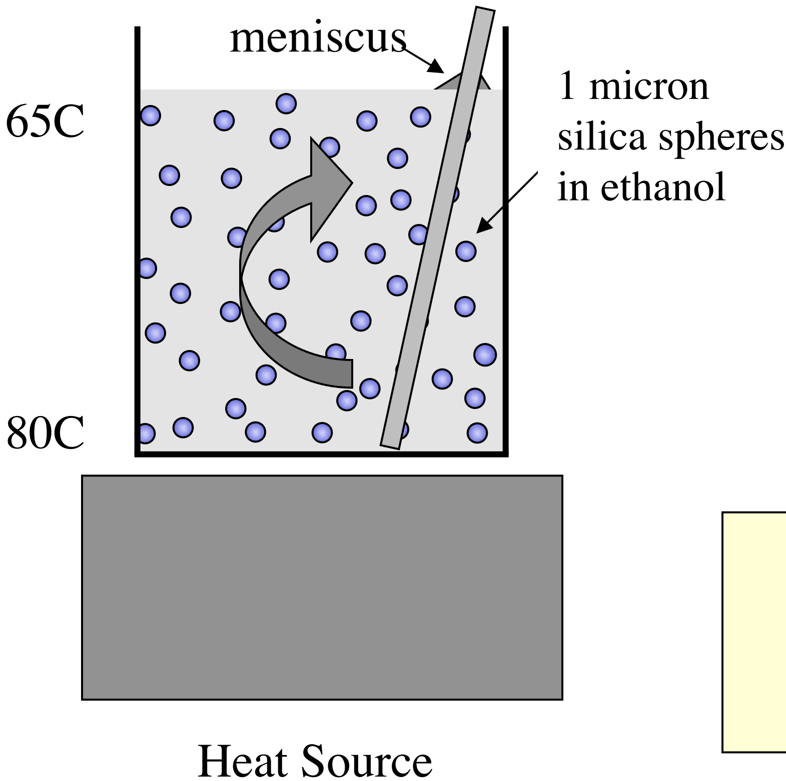


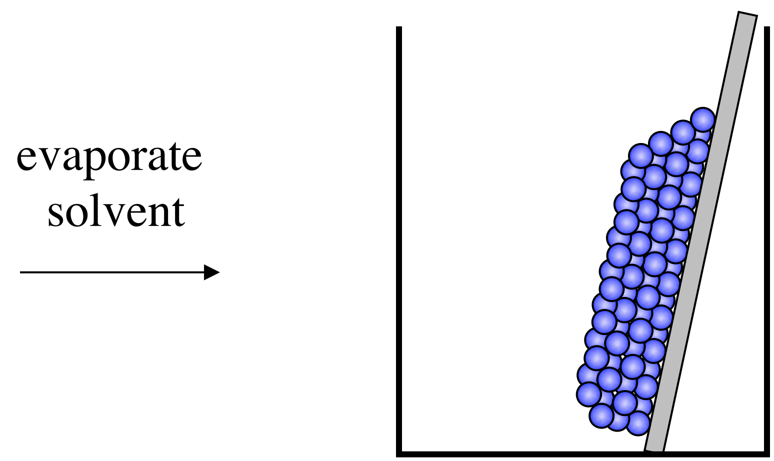
~ 10% gap between 8th & 9th bands

small gap, upper bands: sensitive to disorder

In Order To Form a More Perfect Crystal…

[figs courtesy D. Norris, UMN]





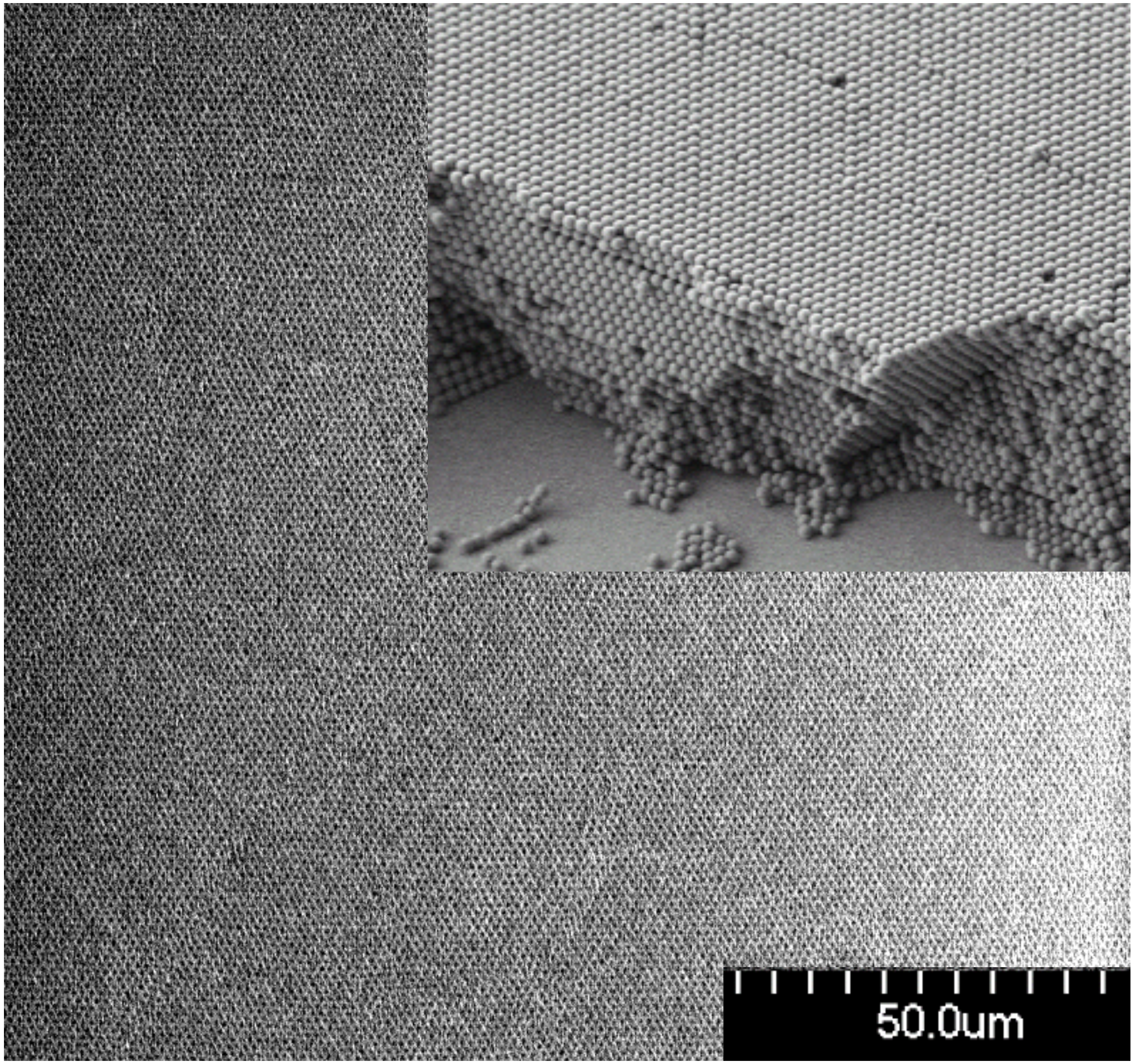
Convective Assembly

[Nagayama, Velev, *et al.*, *Nature* (1993)
Colvin *et al.*, *Chem. Mater.* (1999)]

- Capillary forces during drying cause assembly in the meniscus
- Extremely flat, large-area opals of controllable thickness

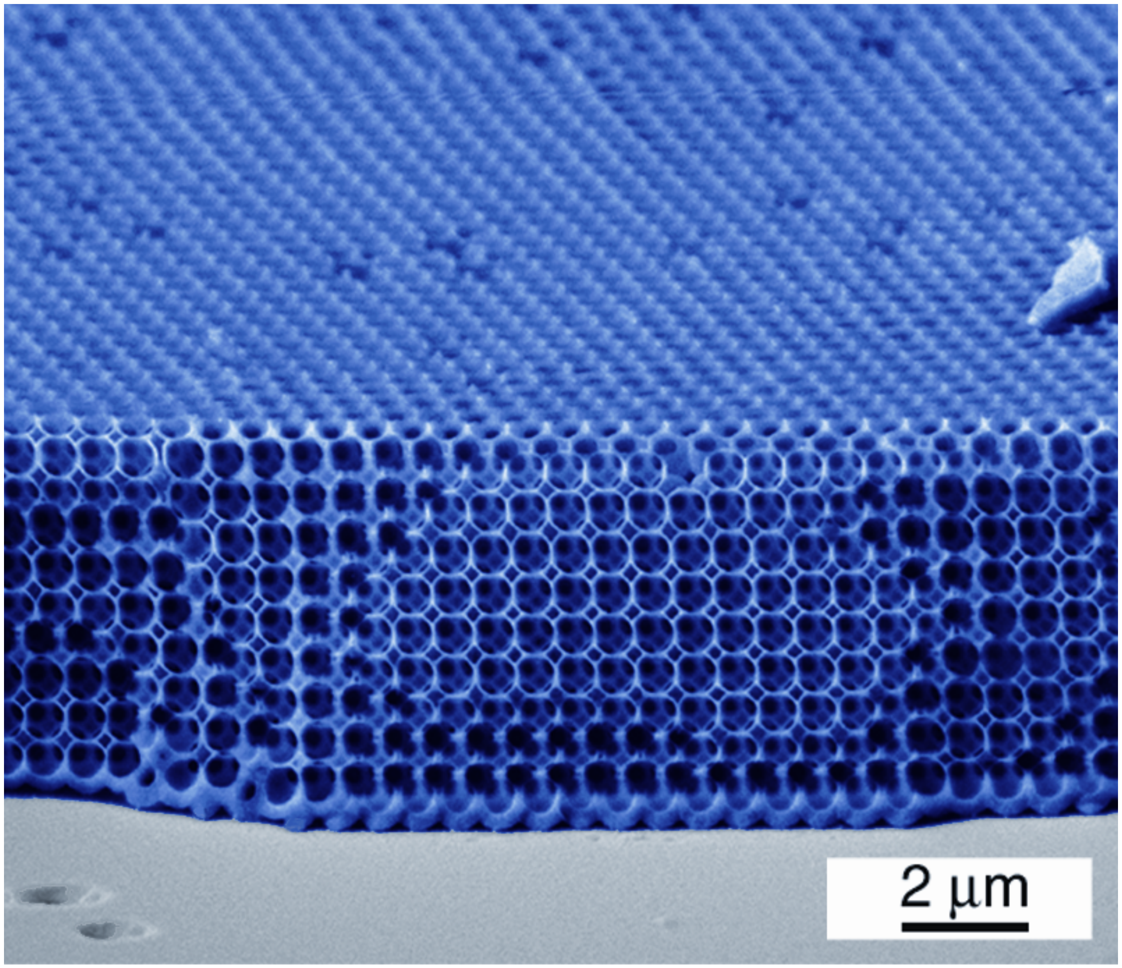
A Better Opal

[fig courtesy
D. Norris, UMN]



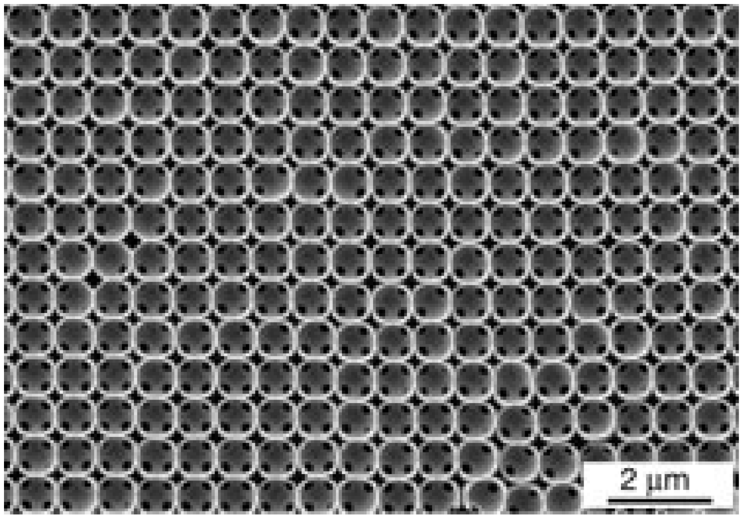
Inverse-Opal Photonic Crystal

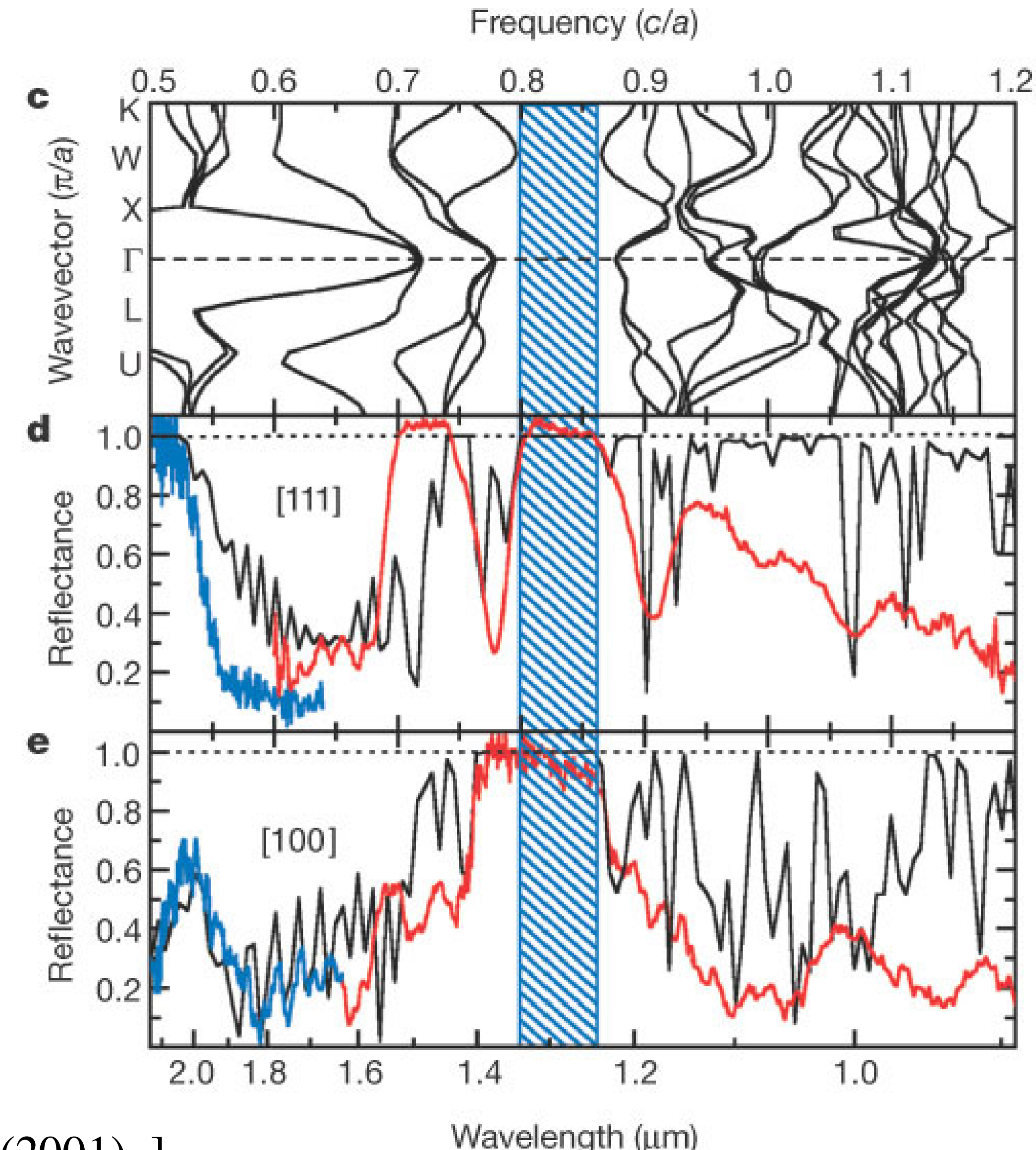
[fig courtesy
D. Norris, UMN]



[Y. A. Vlasov *et al*., *Nature* **414**, 289 (2001).]

Inverse-Opal Band Gap



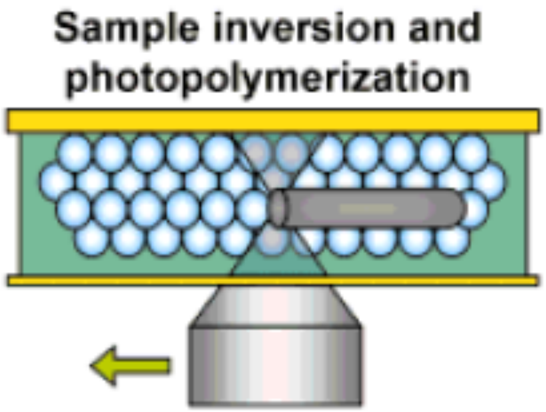


good agreement
between **theory** (black)
& experiment (red/blue)

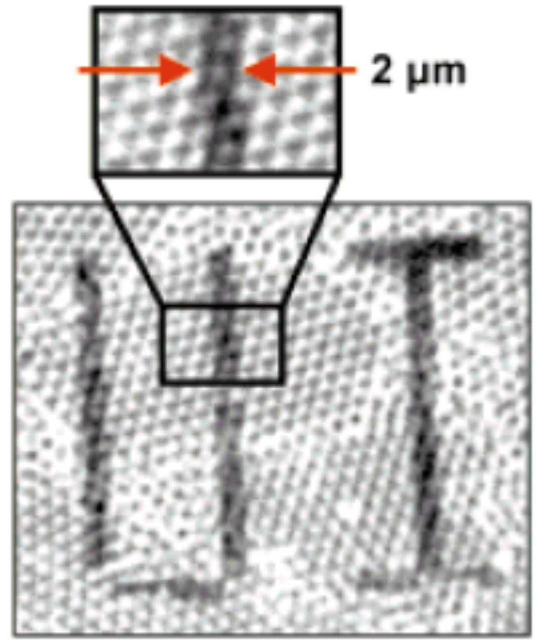
[Y. A. Vlasov *et al.*, *Nature* **414**, 289 (2001).]

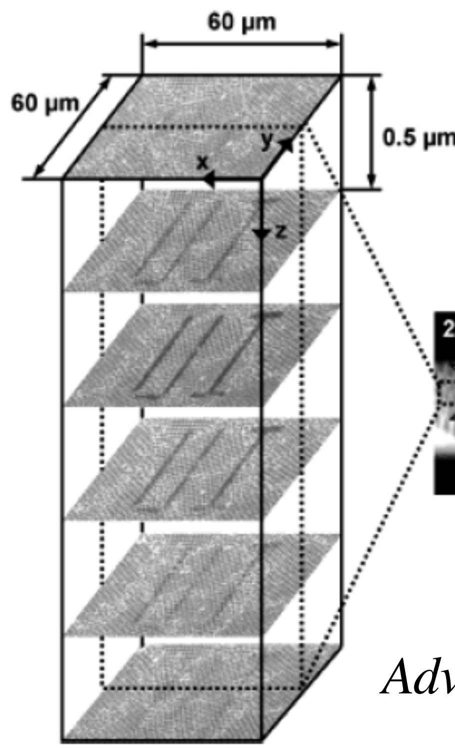
Inserting Defects in Inverse Opals

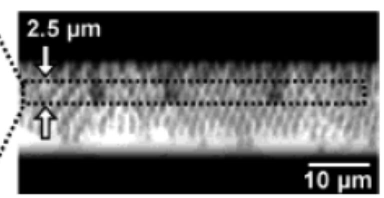
e.g., Waveguides



Three-photon lithography
with
laser scanning
confocal microscope
(LSCM)





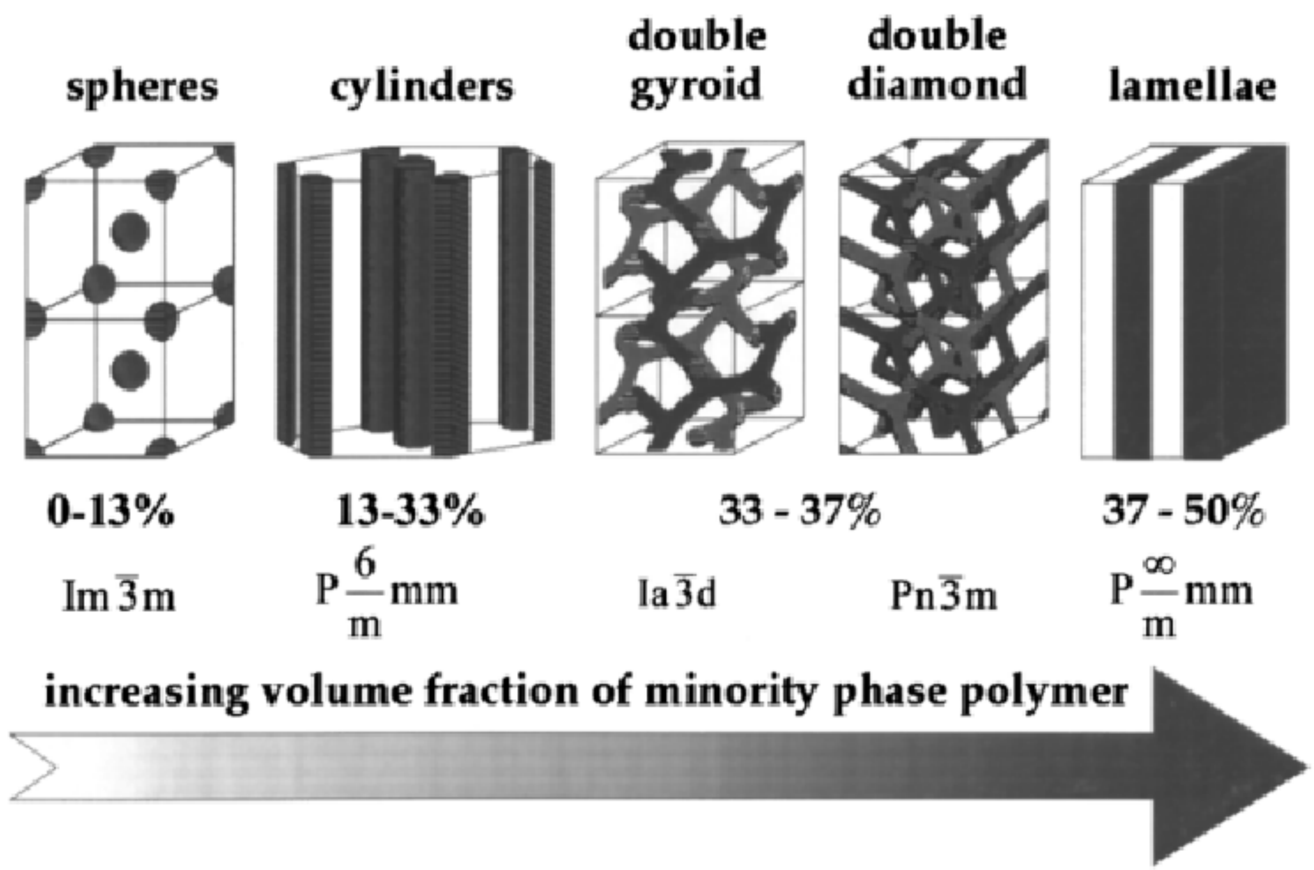


[Wonmok,
Adv. Materials **14**, 271 (2002)]

Mass-Production III:
Block (not Bloch) Copolymers

two polymers can segregate, ordering into periodic arrays

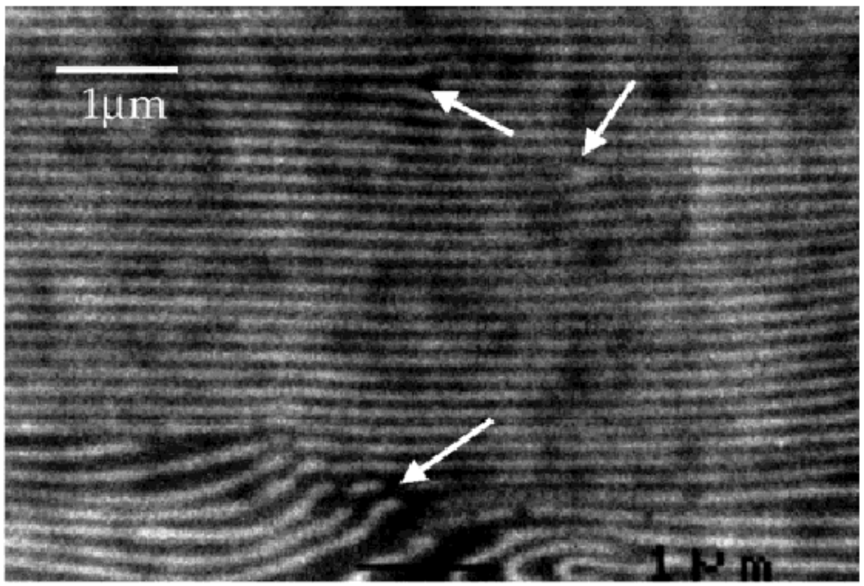
periodicity ~ polymer block size ~ 50nm (possibly bigger)



[Y. Fink, A. M. Urbas, M. G. Bawendi, J. D. Joannopoulos, E. L. Thomas, *J. Lightwave Tech*. **17**, 1963 (1999)]

Block-Copolymer 1d Visible Bandgap

/ homopolymer

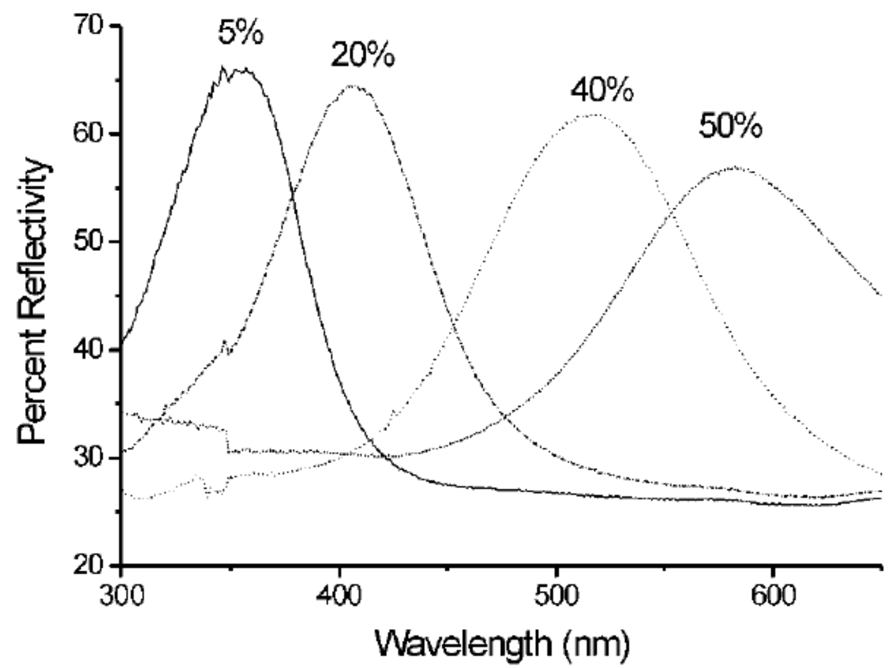


dark/light:
polystyrene/polyisoprene

n = 1.59/1.51

Flexible material:
bandgap can be
shifted by stretching it!

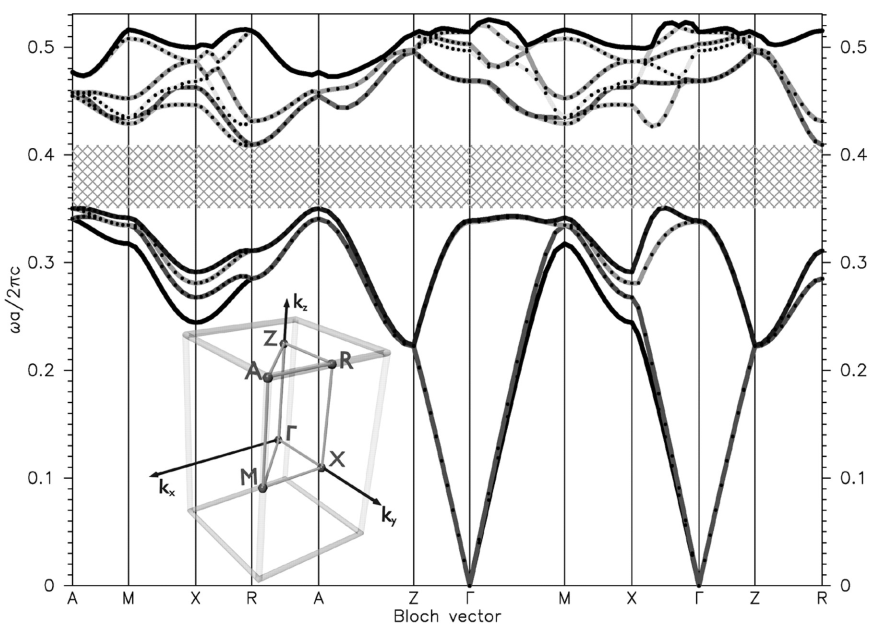
reflection for differing homopolymer %

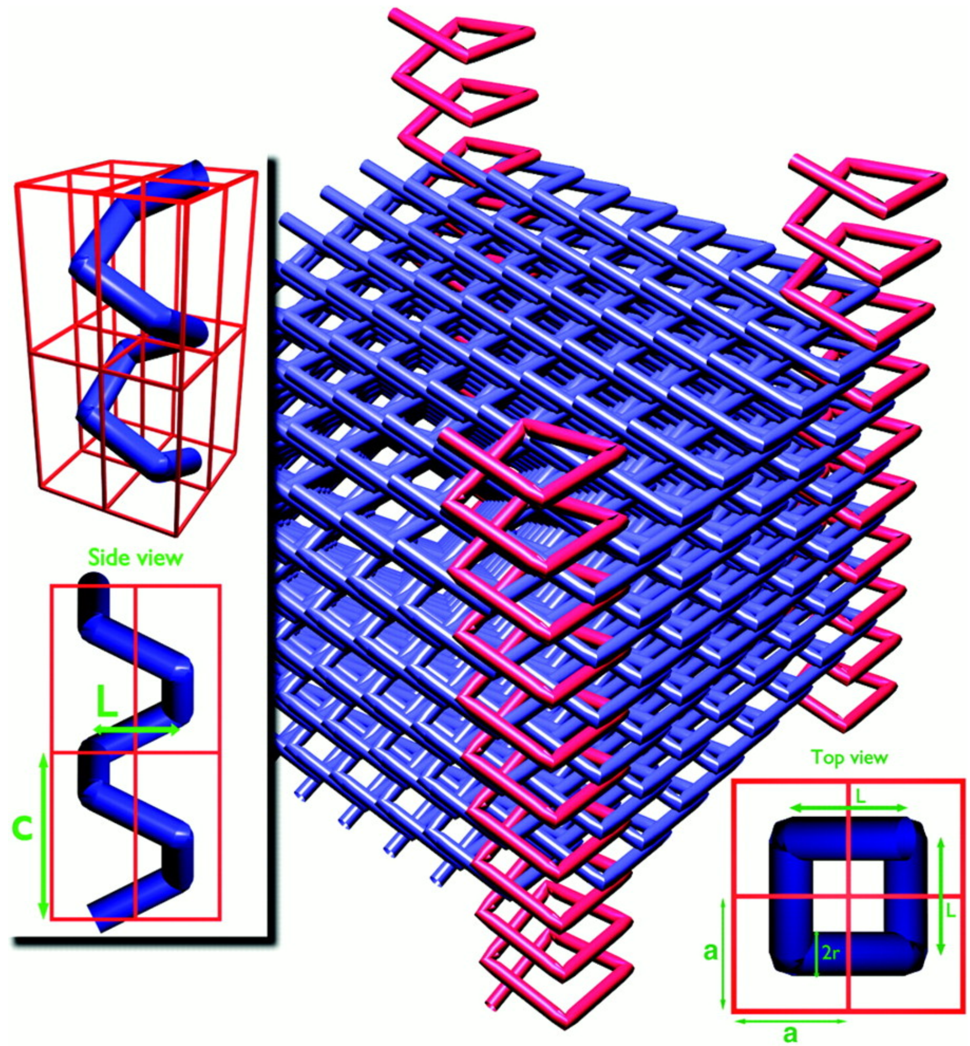


[A. Urbas *et al*., *Advanced Materials* **12**, 812 (2000)]

Be GLAD: Even more crystals!

"GLAD" = "GLancing Angle Deposition"





15% gap for Si/air

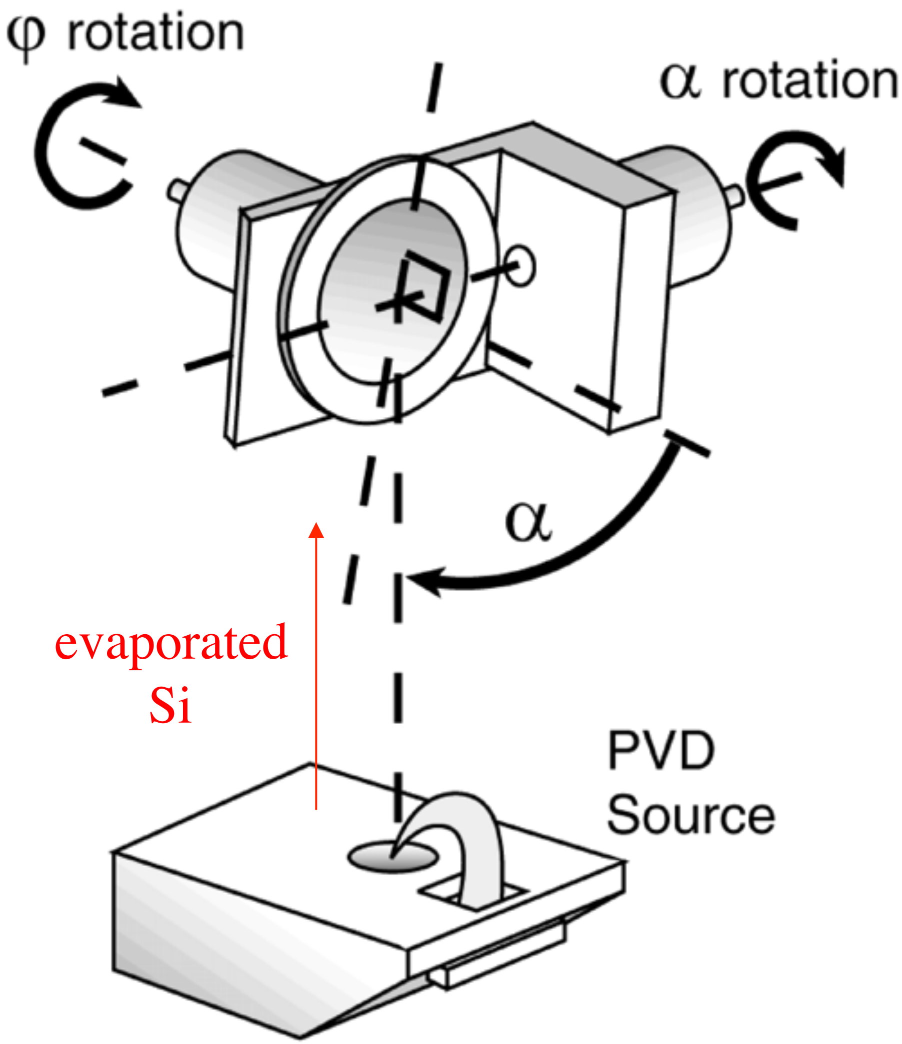
diamond-like

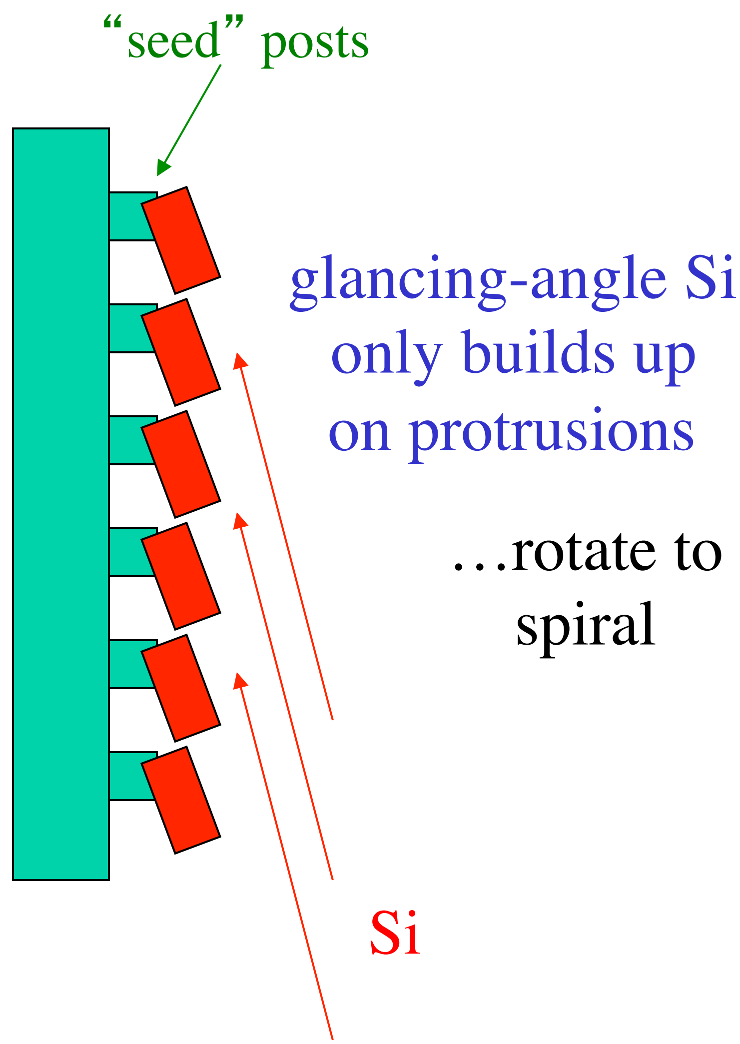
with "broken bonds"

doubled unit cell, so gap between 4th & 5th bands

[O. Toader and S. John, *Science* **292**, 1133 (2001)]

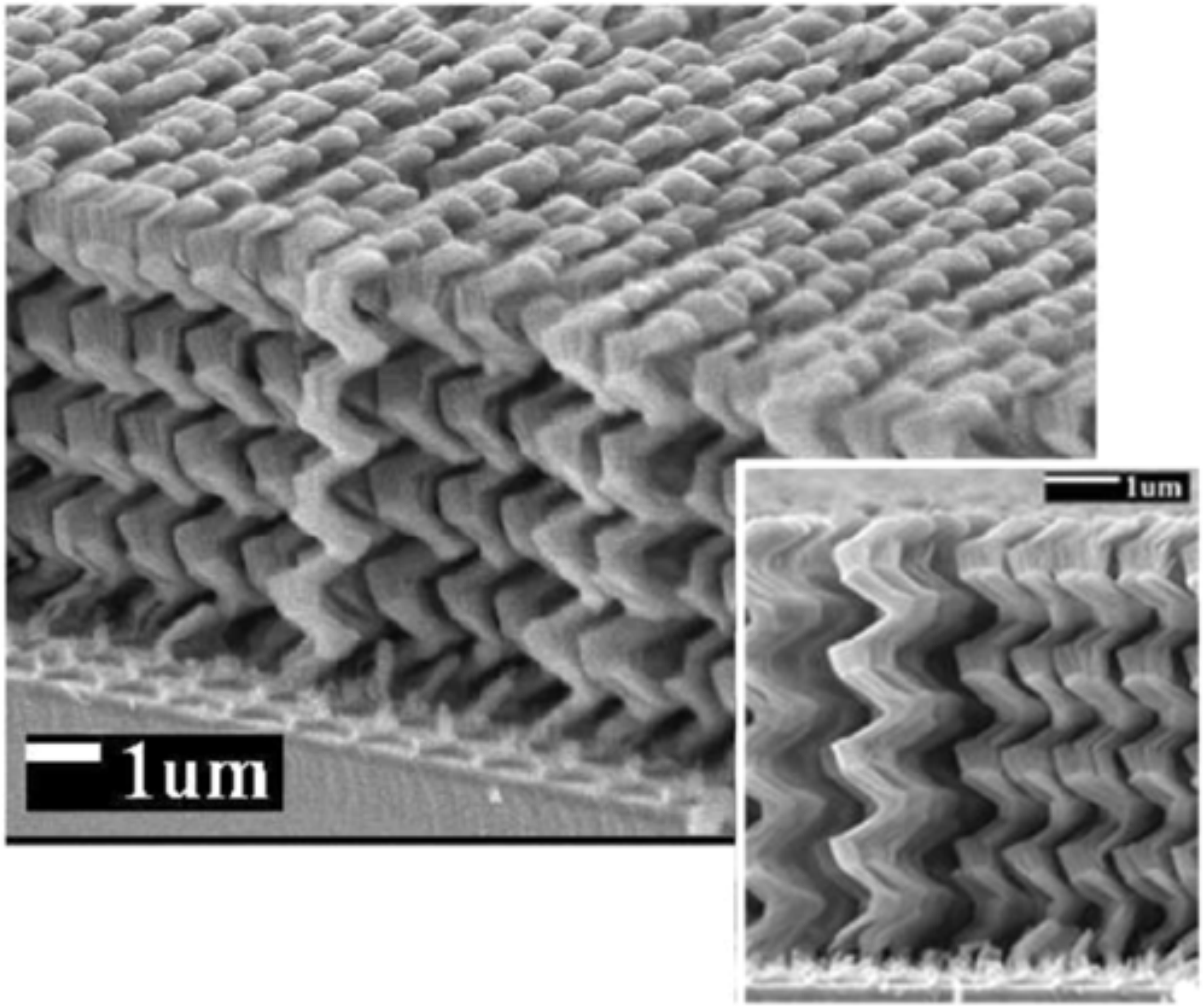
Glancing Angle Deposition





[S. R. Kennedy *et al*., *Nano Letters* **2**, 59 (2002)]

An Early GLAD Crystal



[S. R. Kennedy *et al*., *Nano Letters* **2**, 59 (2002)]

Outline

- Preliminaries: waves in periodic media
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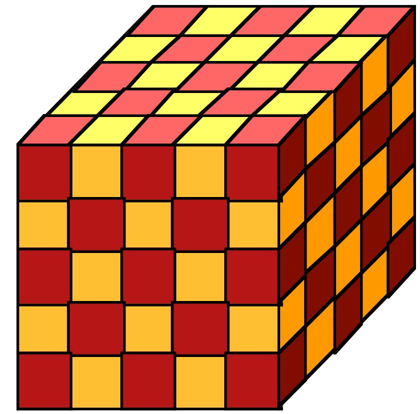
- Bulk crystal properties
- Intentional defects and devices
- Index-guiding and incomplete gaps
- Photonic-crystal fibers
- Perturbations, tuning, and disorder

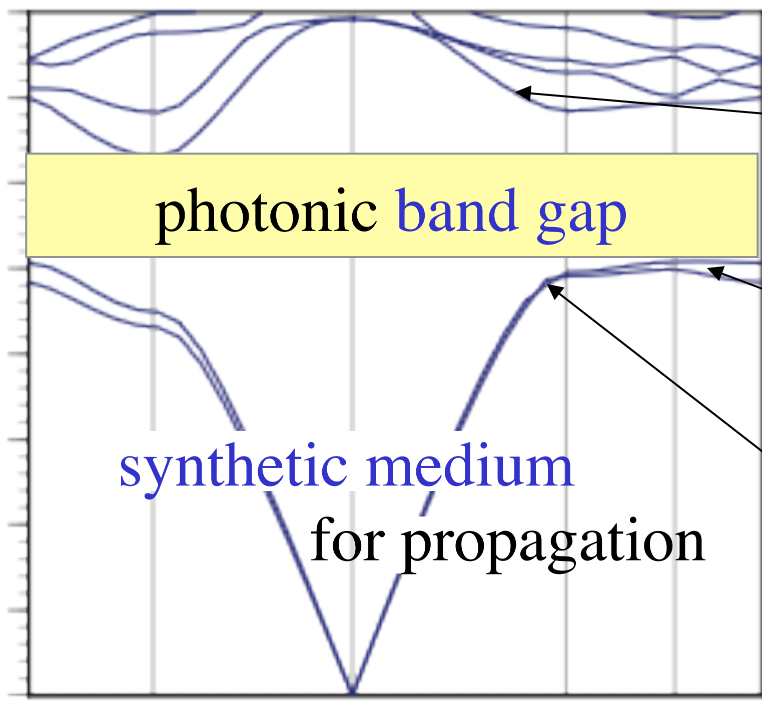
Properties of Bulk Crystals

by Bloch's theorem



(cartoon)

band diagram (dispersion relation)



conserved frequency ω

conserved wavevector k

backwards slope:
negative refraction

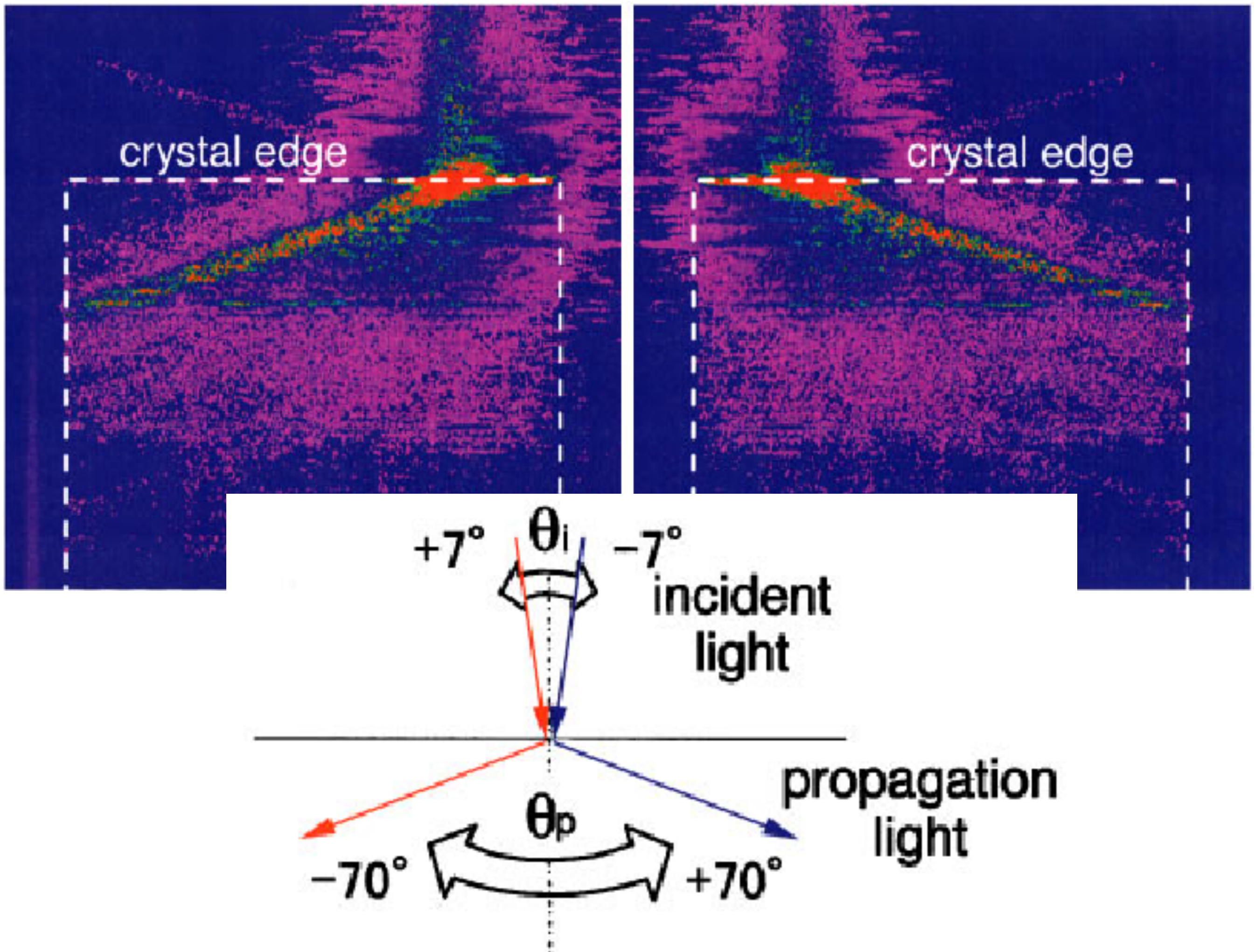
$d\omega/dk \approx 0$: slow light
(e.g. DFB lasers)

strong curvature:
super-prisms, …
(+ negative refraction)

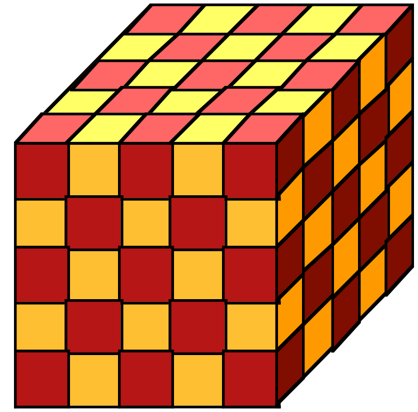
Superprisms

from divergent dispersion (band curvature)

[Kosaka, *PRB* **58**, R10096 (1998).]

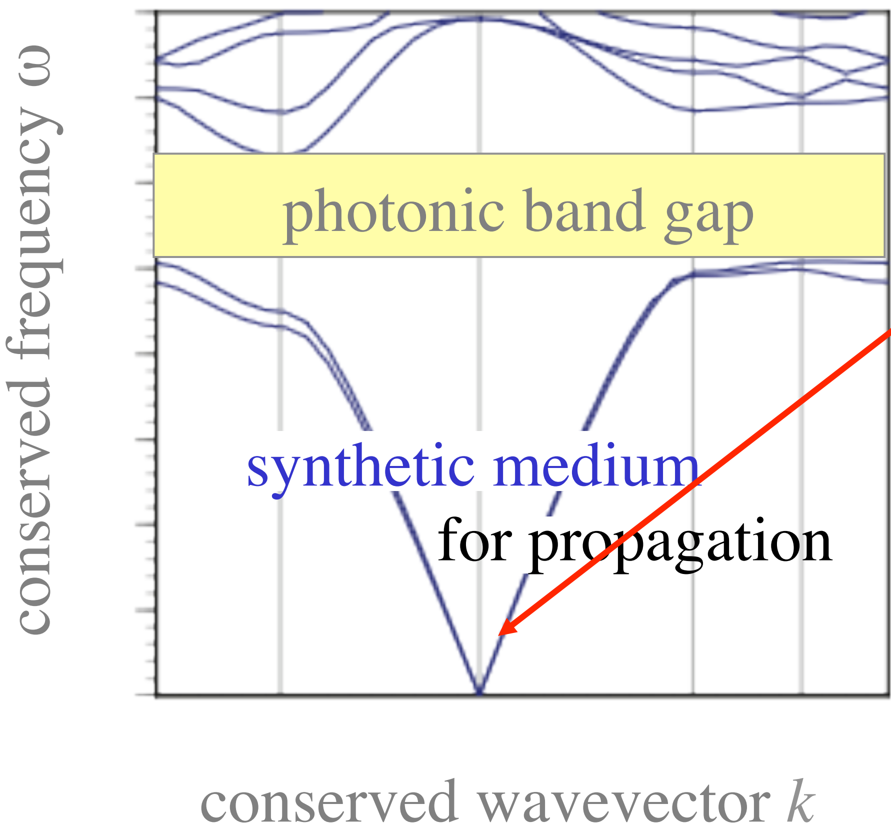


Photonic Crystals & Metamaterials



(cartoon)

band diagram (dispersion relation)



at small ω

(long wavelengths $\lambda >> a$)

$\omega(k)$ ~ straight line

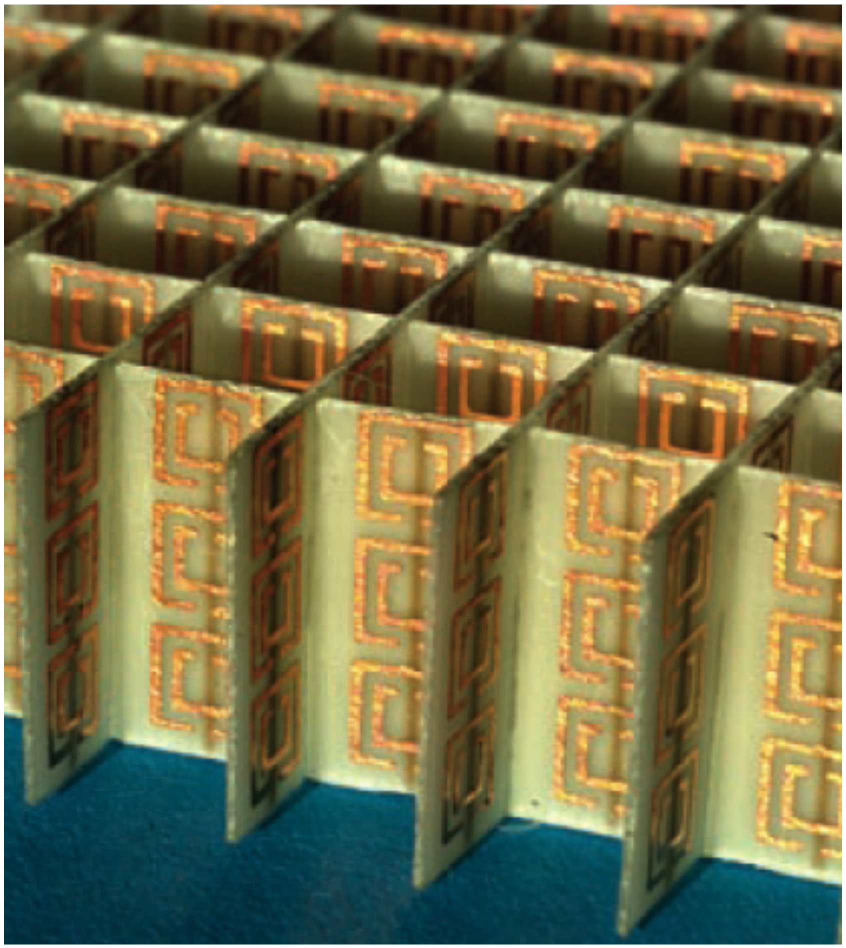
~ *effectively homogeneous* material

= **metamaterials**

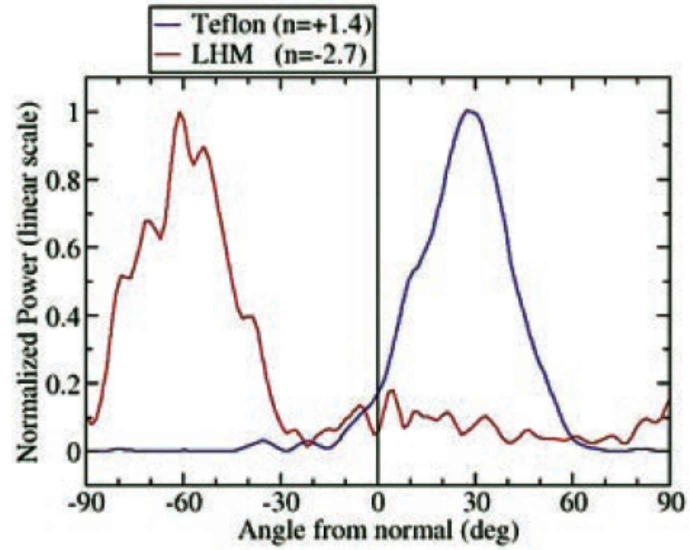
Microwave negative refraction

[D. R. Smith, J. B. Pendry, M. C. K. Wiltshire, *Science* **305**, 788 (2004)]

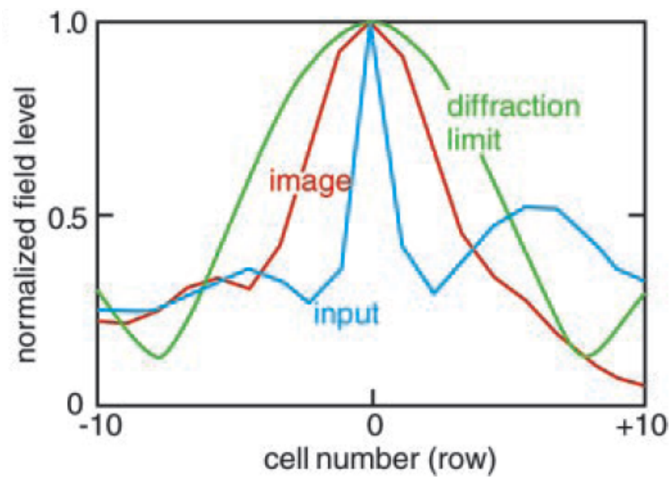
1 cm



negative refraction



superlensing

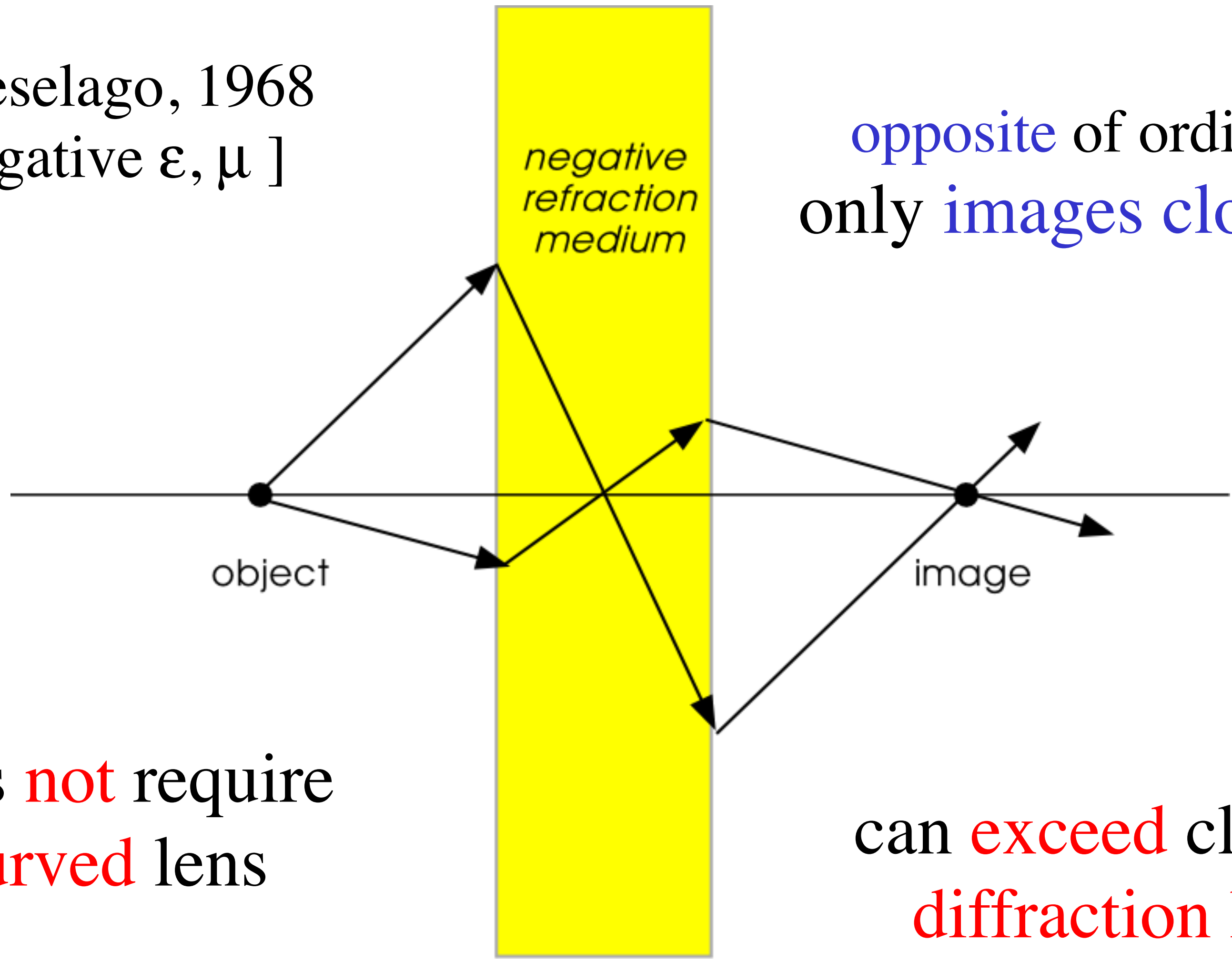


Magnetic (ring) + Electric (strip) resonances

Negative Indices & Refraction

[Veselago, 1968
negative ε, μ]

opposite of ordinary lens:
only images close objects

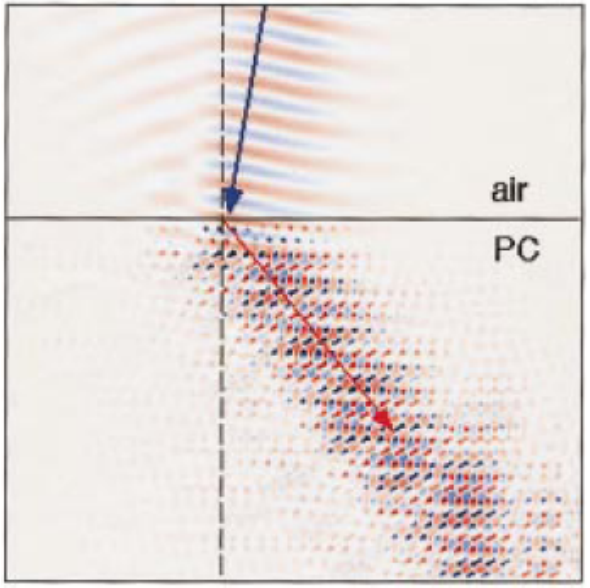


does not require
curved lens

can exceed classical
diffraction limit

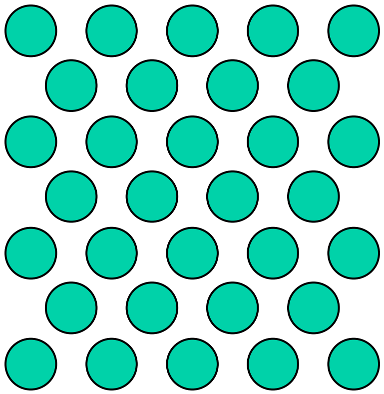
Negative-refractive all-dielectric photonic crystals

negative refraction



focussing

(2d rods in air, TE)



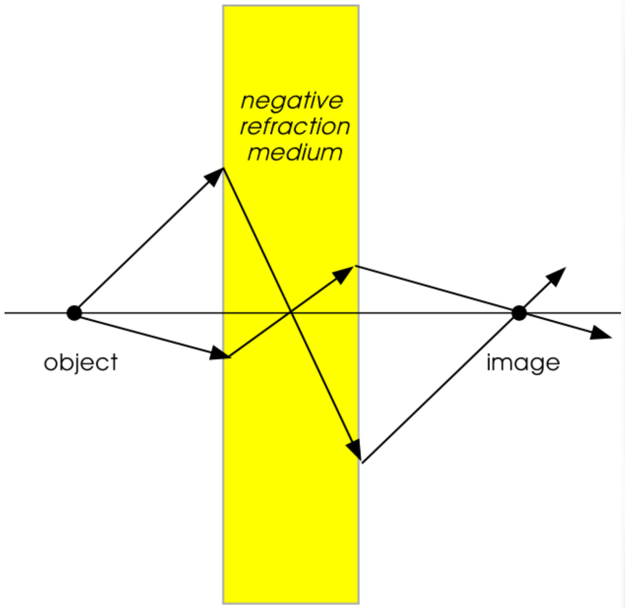
[M. Notomi, *PRB* **62**, 10696 (2000).]

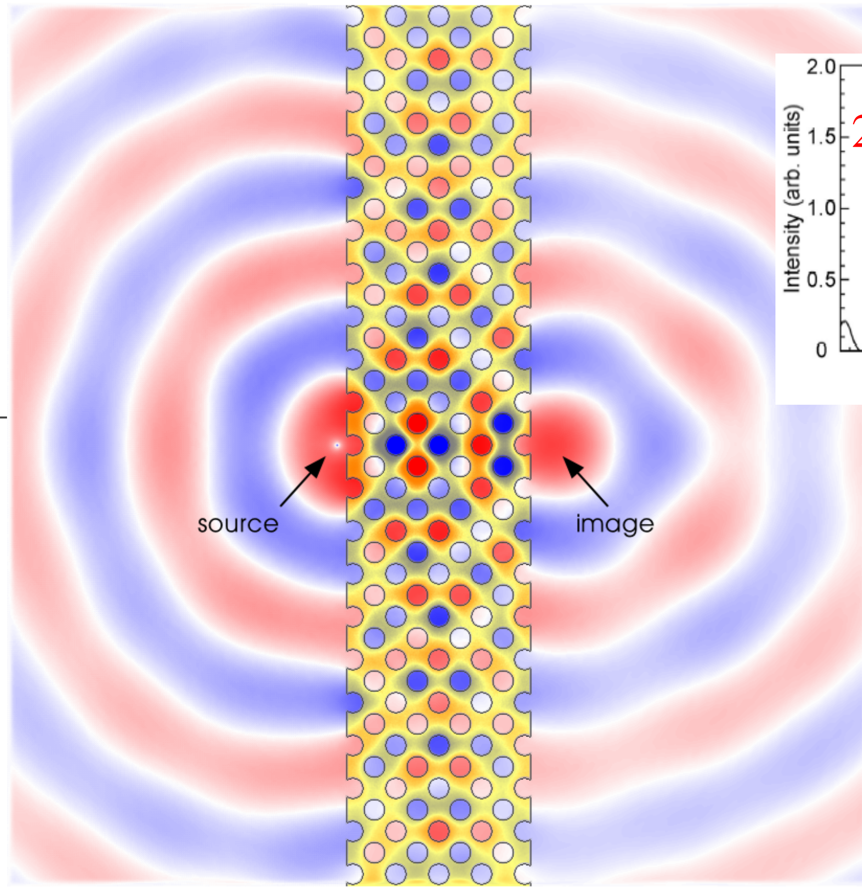
not metamaterials: wavelength ~ a,

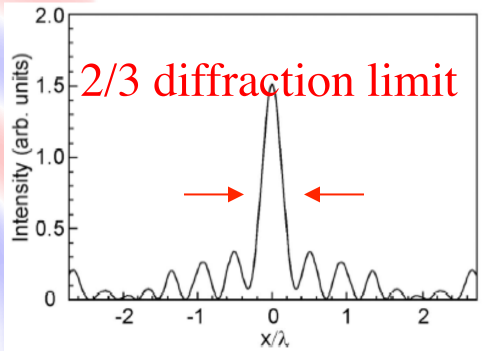
no homogeneous material can reproduce *all* behaviors

Superlensing with Photonic Crystals

[Luo *et al*, *PRB* **68**, 045115 (2003).]

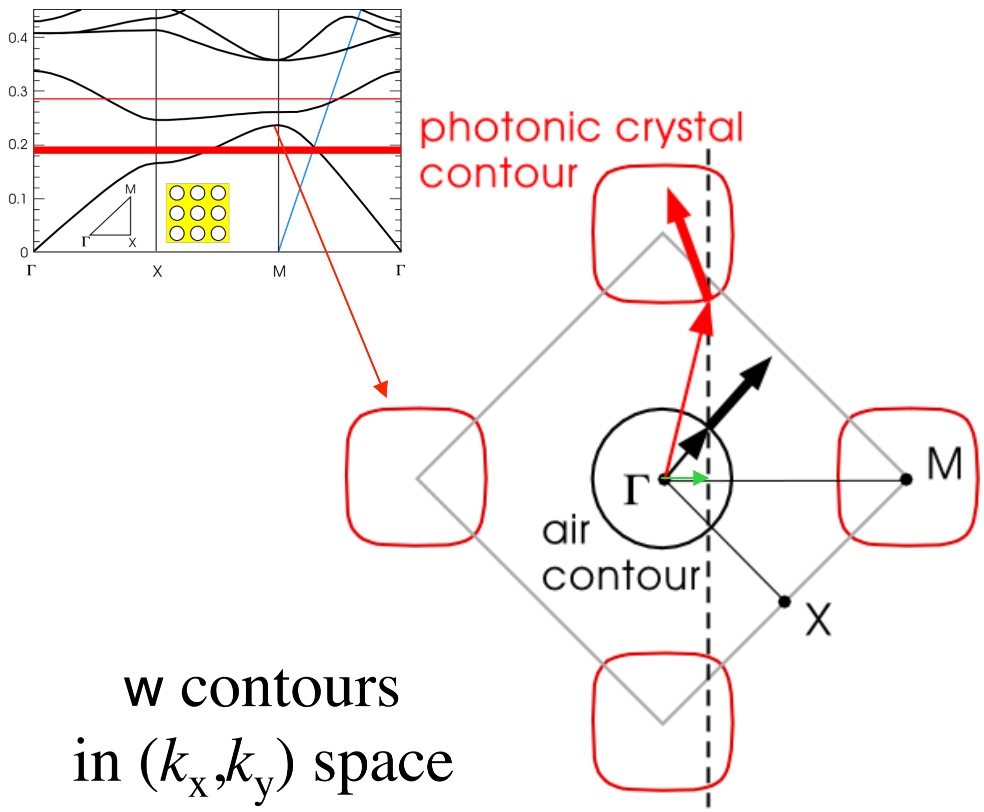




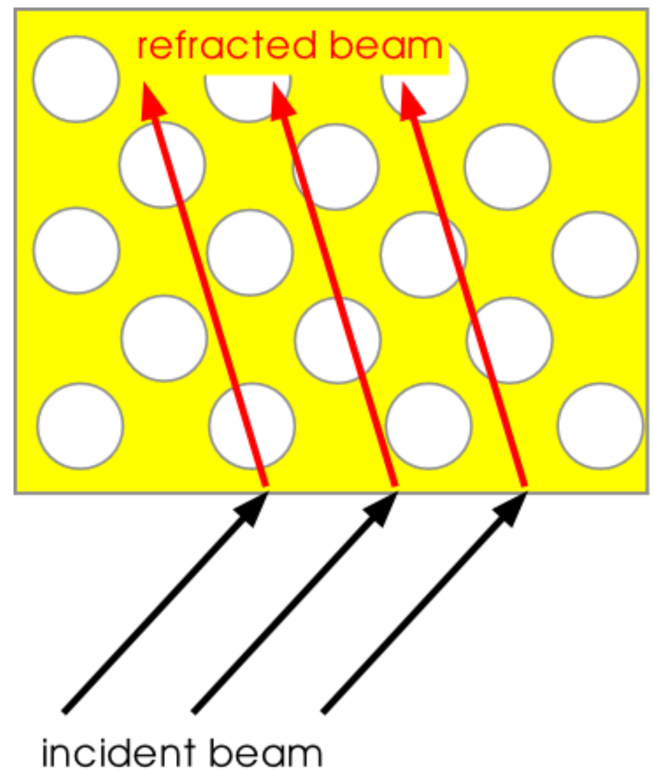


Negative Refraction and wavevector diagrams

[Luo *et al*, *PRB* **65**, 2001104 (2002).]



w contours in (k_x, k_y) space



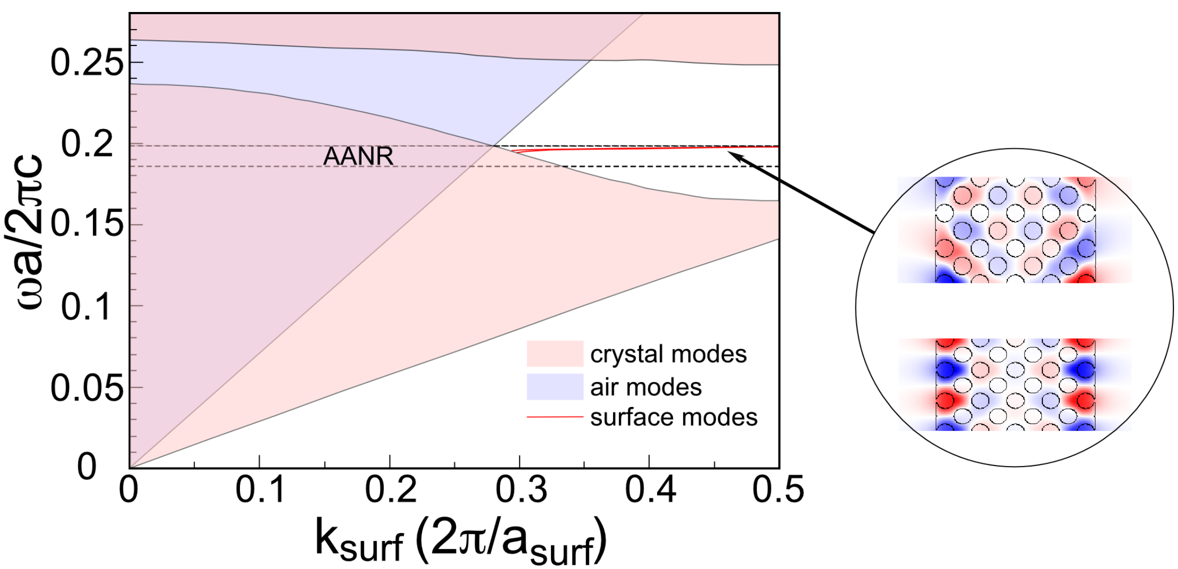
→ $k_{||}$ is conserved

Super-lensing

[Luo, *PRB* **68**, 045115 (2003).]

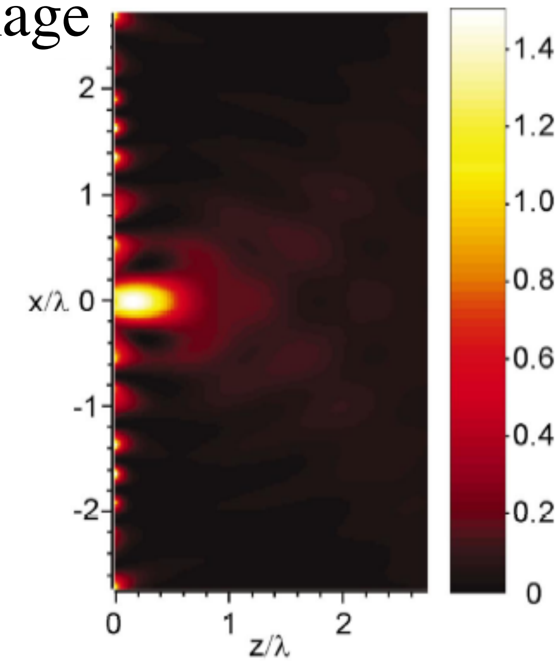
Classical diffraction limit comes from
loss of evanescent waves

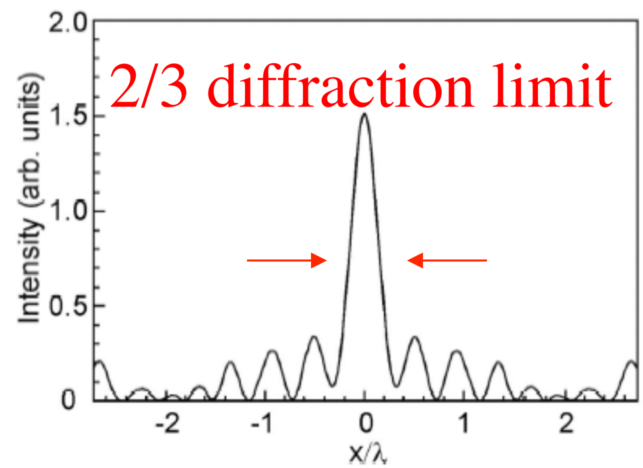
… can be recovered by
resonant coupling to surface states



(needs band gap)

image





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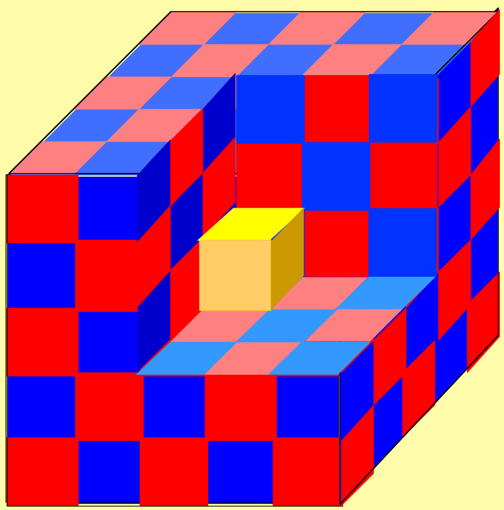
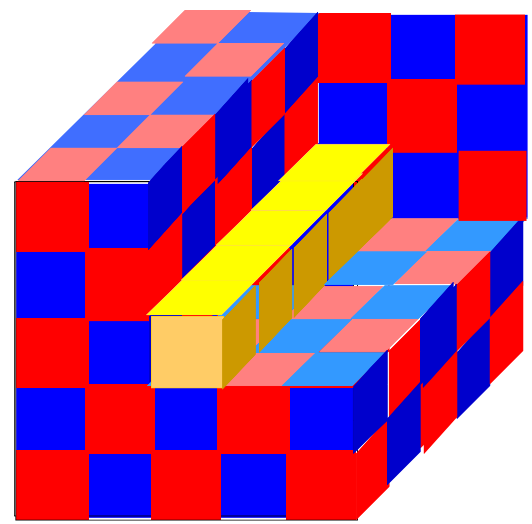
Intentional "defects" are good

microcavities



waveguides ("wires")



Resonance

an oscillating mode trapped for a long time in some volume

(of light, sound, …)

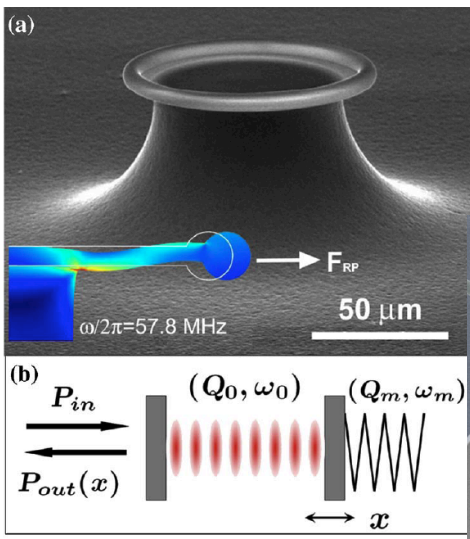
lifetime $\tau \gg 2\pi/\omega_0$

frequency ω_0

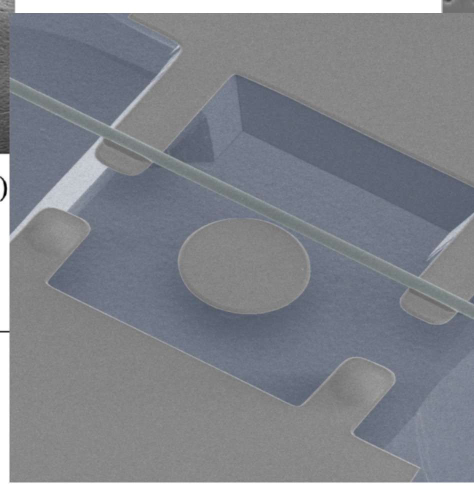
quality factor $Q = \omega_0\tau/2$

energy $\sim e^{-\omega_0\tau/Q}$

modal volume V

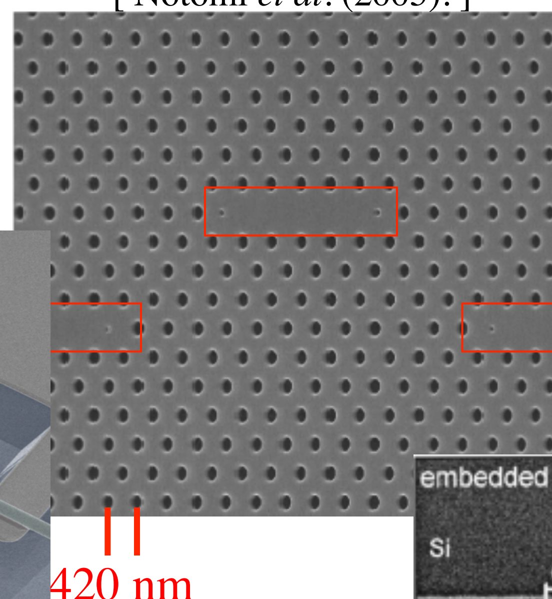


[Schliesser et al., *PRL* **97**, 243905 (2006)]

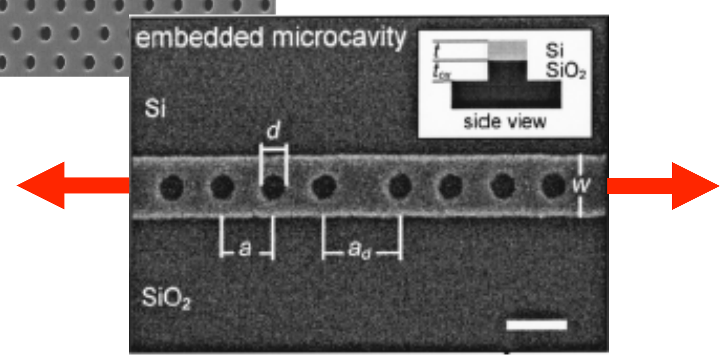


[Eichenfield et al. *Nature Photonics* **1**, 416 (2007)]

[Notomi *et al*. (2005).]



[C.-W. Wong, *APL* **84**, 1242 (2004).]



Why Resonance?

an oscillating mode trapped for a long time in some volume

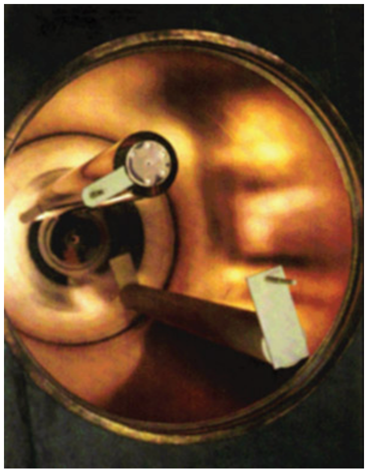
- long time = narrow bandwidth … filters (WDM, etc.)
  — $1/Q$ = fractional bandwidth

- resonant processes allow one to "impedance match" hard-to-couple inputs/outputs

- long time, small V … enhanced wave/matter interaction
  — lasers, nonlinear optics, opto-mechanical coupling, sensors, LEDs, thermal sources, …

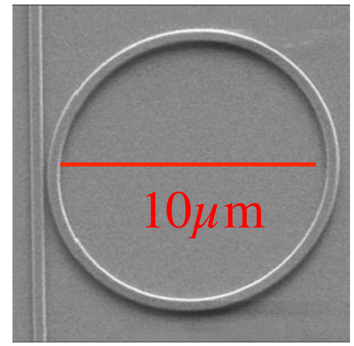
How Resonance?

need mechanism to trap light for long time



[llnl.gov]

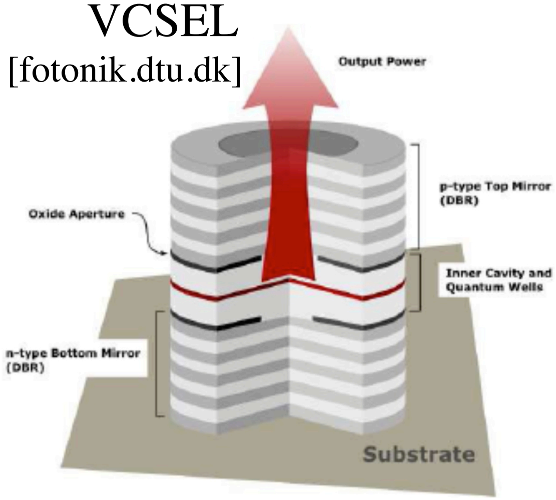
metallic cavities:
good for microwave,
dissipative for infrared

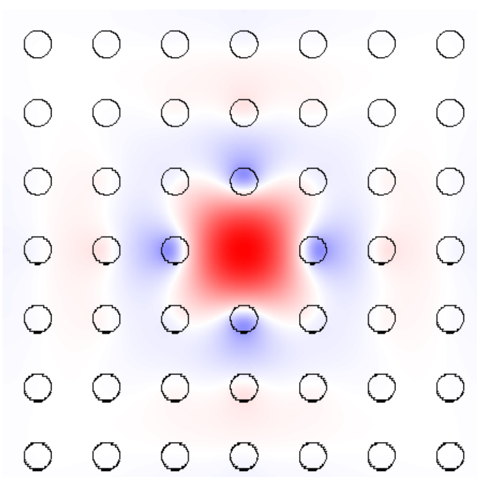


[Xu & Lipson (2005)]

ring/disc/sphere resonators:
a waveguide bent in circle,
bending loss ~ exp(–radius)

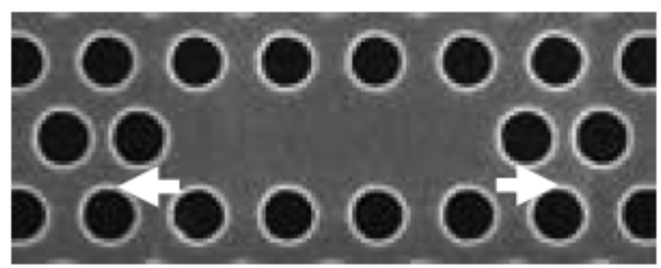
VCSEL
[fotonik.dtu.dk]





photonic bandgaps
(complete or partial
+ index-guiding)

[Akahane, *Nature* **425**, 944 (2003)]

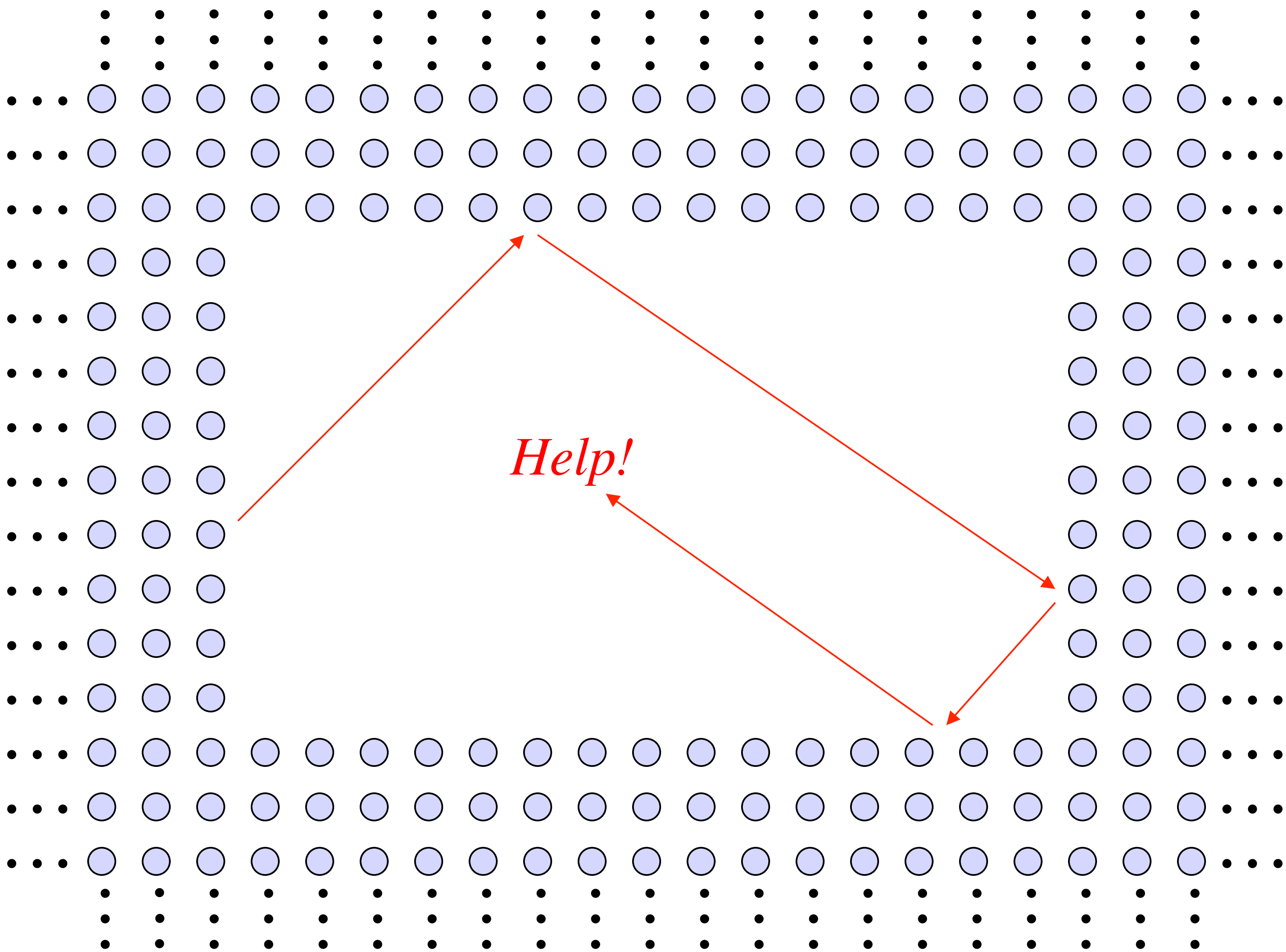


(planar Si slab)

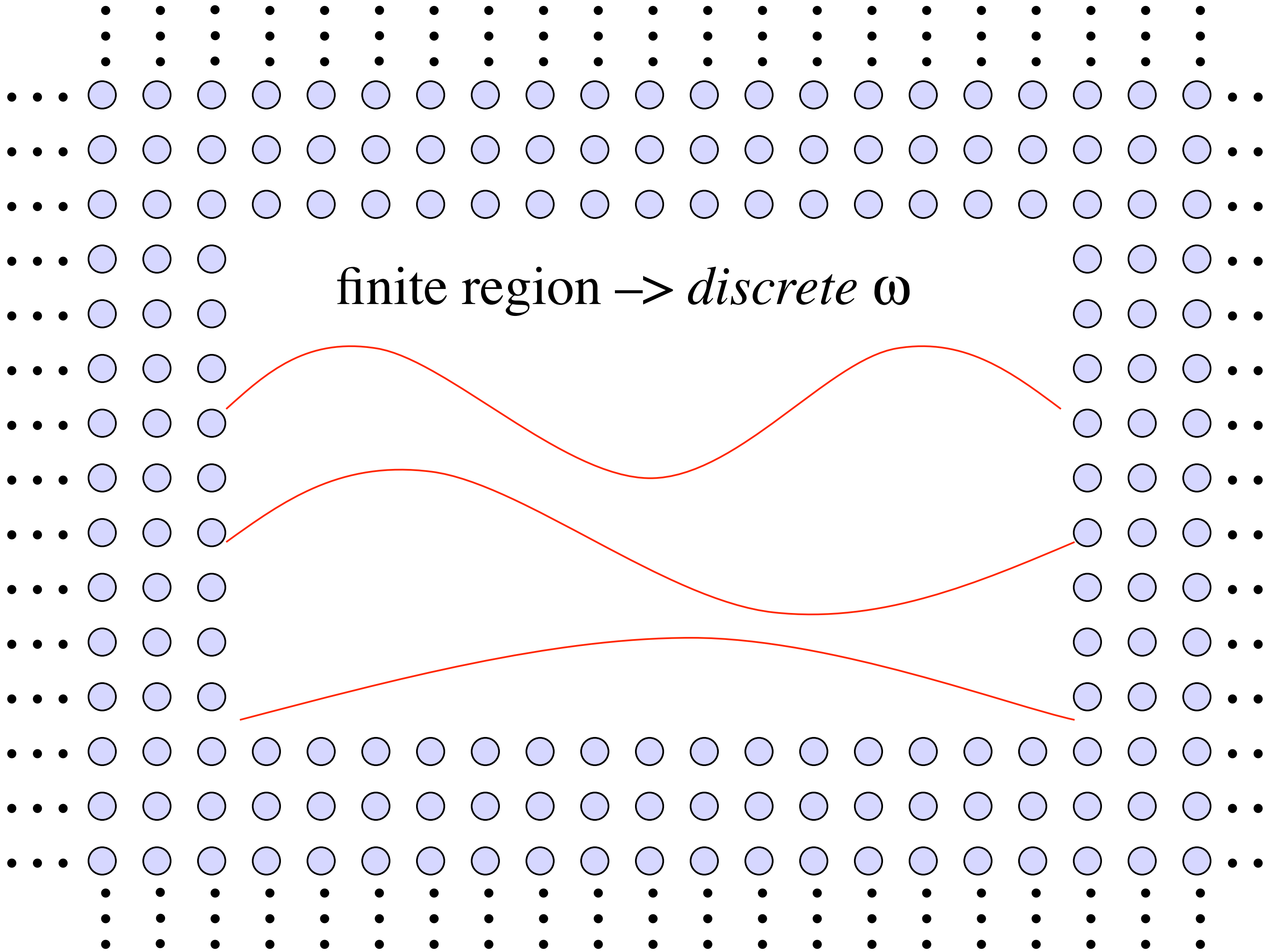
Why do defects in crystals
trap resonant modes?

What do the modes look like?

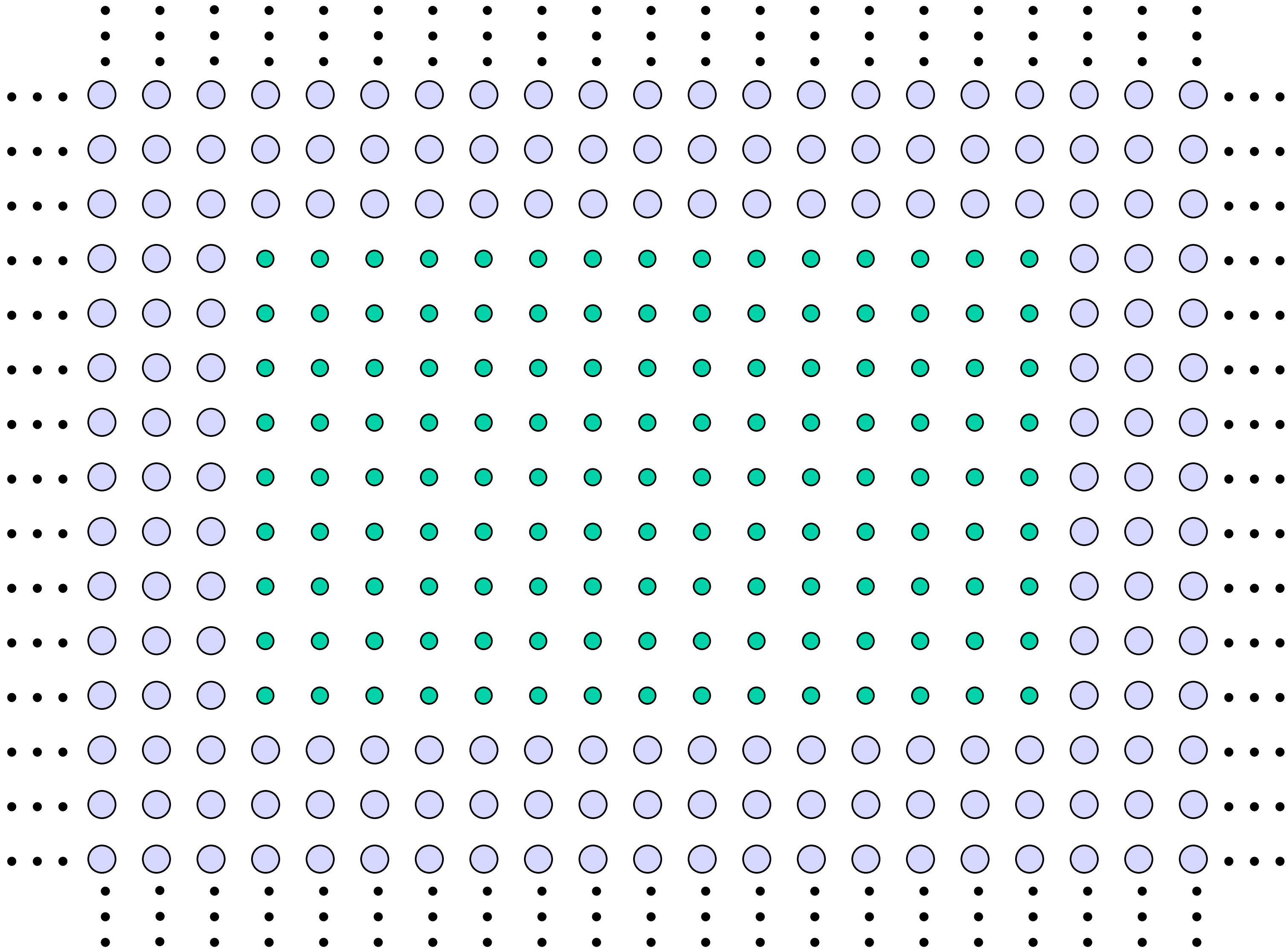
Cavity Modes



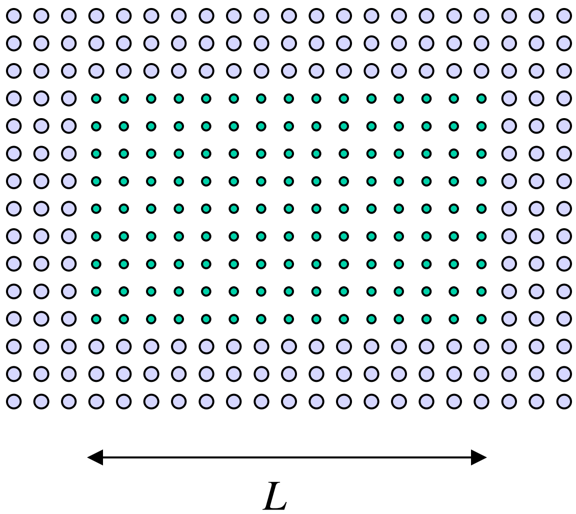
Cavity Modes



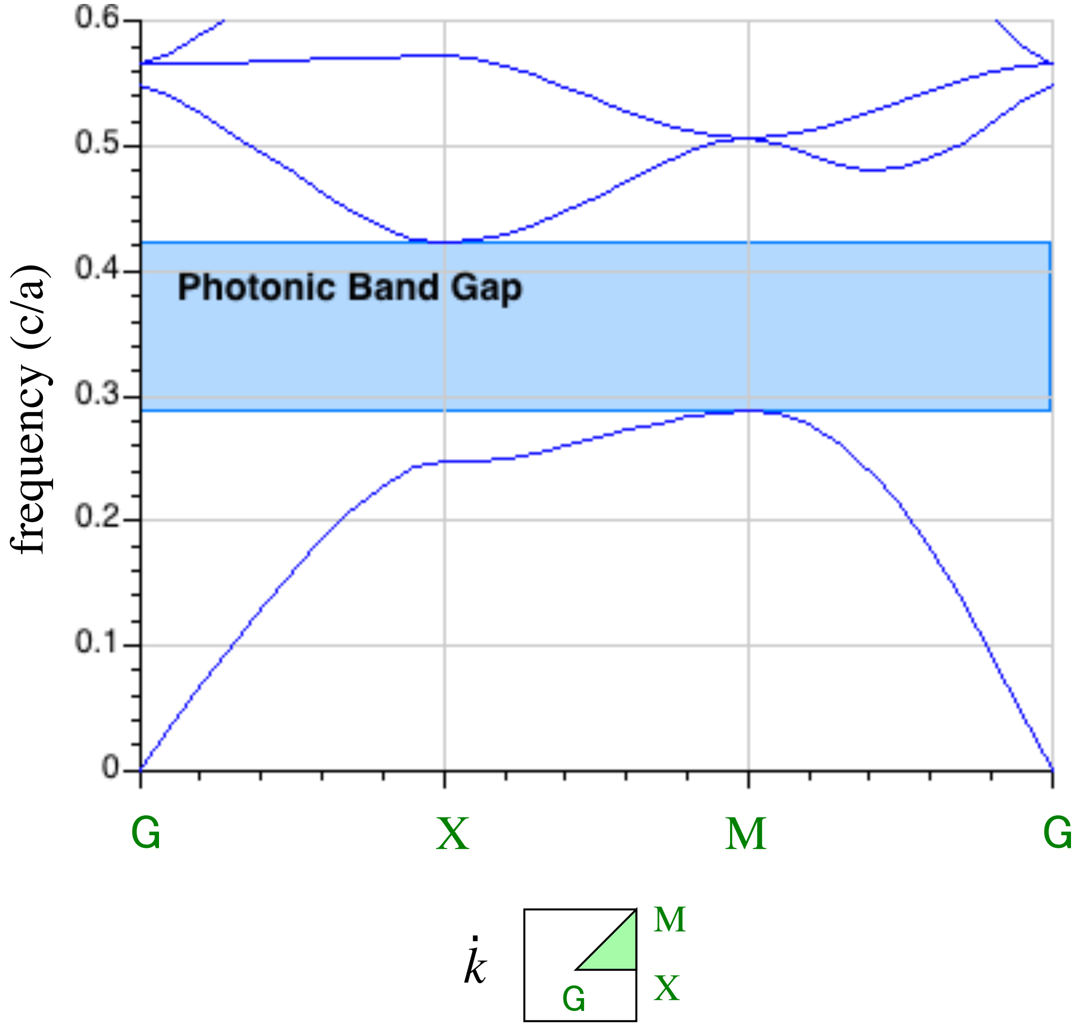
Cavity Modes: Smaller Change



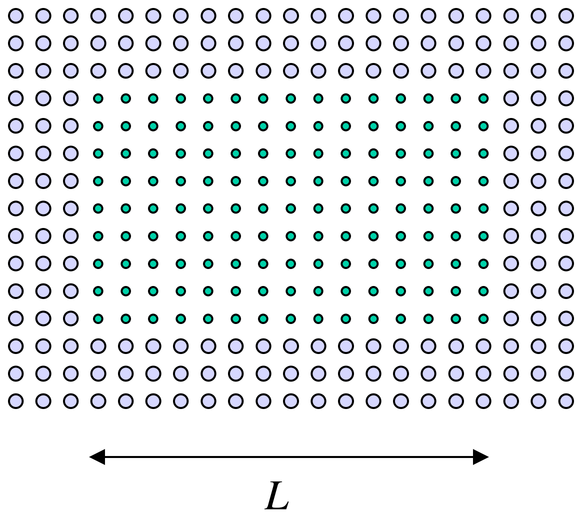
Cavity Modes: Smaller Change



Bulk Crystal Band Diagram



Cavity Modes: Smaller Change

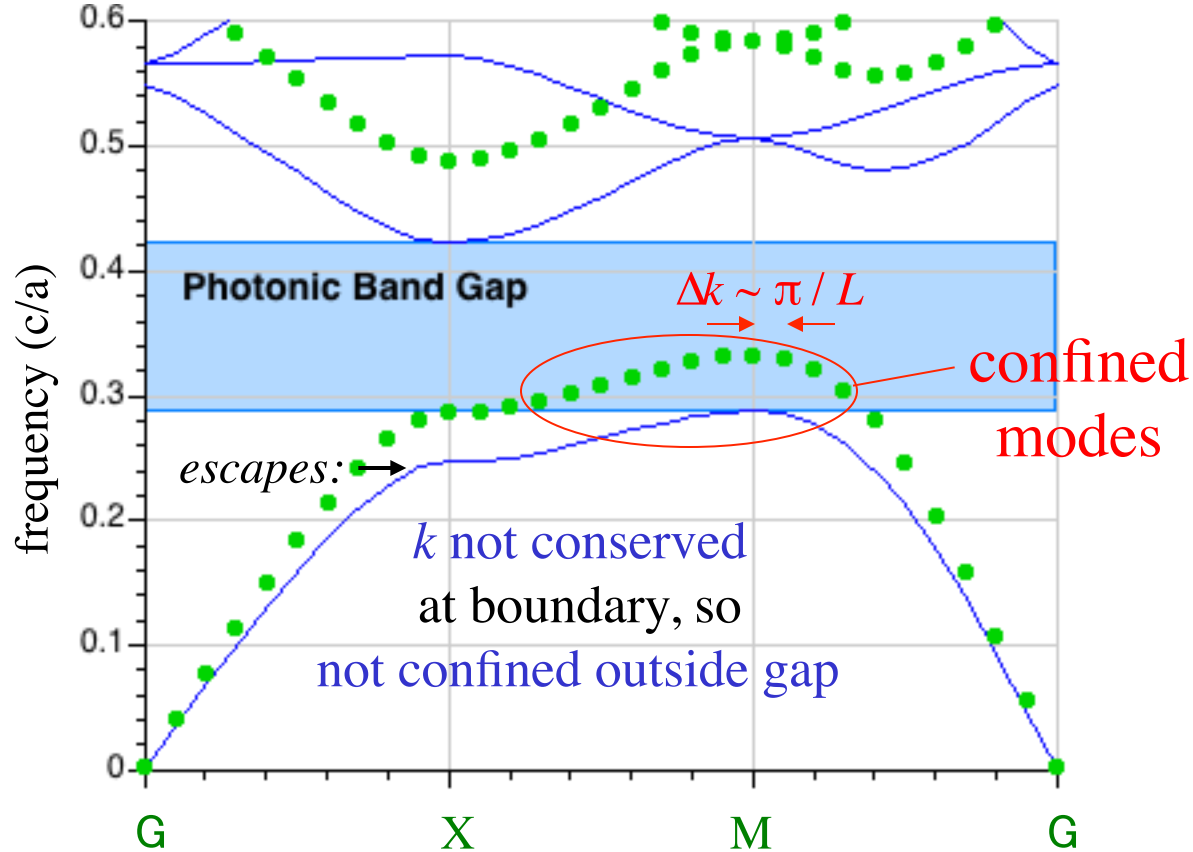


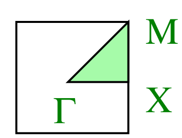
Defect bands are shifted *up* (less ε)

with *discrete k*

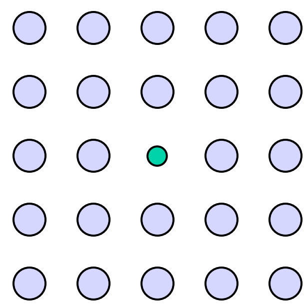
$$\# \cdot \frac{\lambda}{2} \sim L \qquad (k \sim 2\pi / \lambda)$$

Defect Crystal Band Diagram





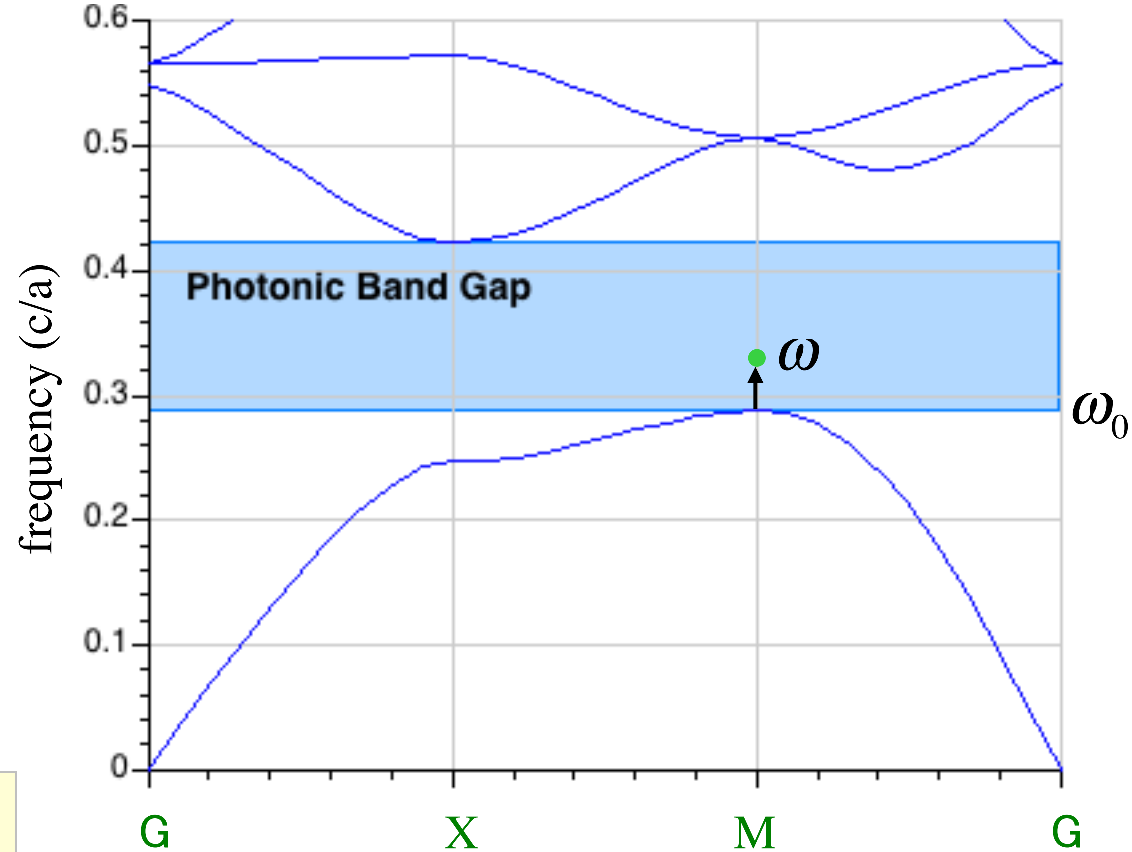
Single-Mode Cavity

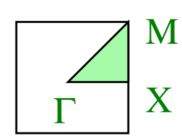


A *point defect*
can push up
a single mode
from the band edge

$$\text{field decay} \sim \sqrt{\frac{\omega - \omega_0}{\text{curvature}}}$$

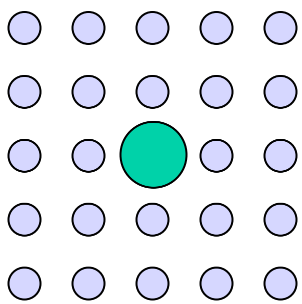
Bulk Crystal Band Diagram





(k not conserved)

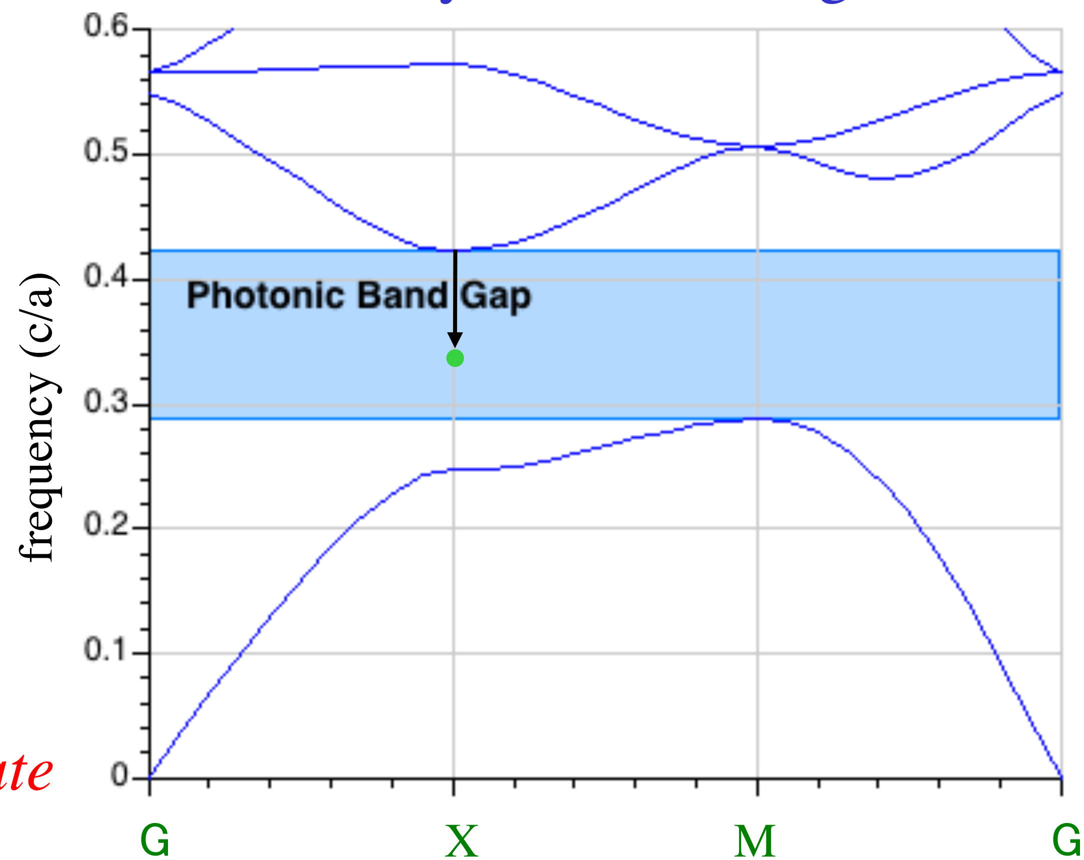
“Single”-Mode Cavity

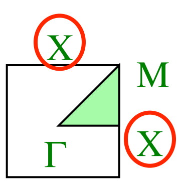


A *point defect* can pull down a “single” mode

…here, doubly-*degenerate* (*two* states at *same* ω)

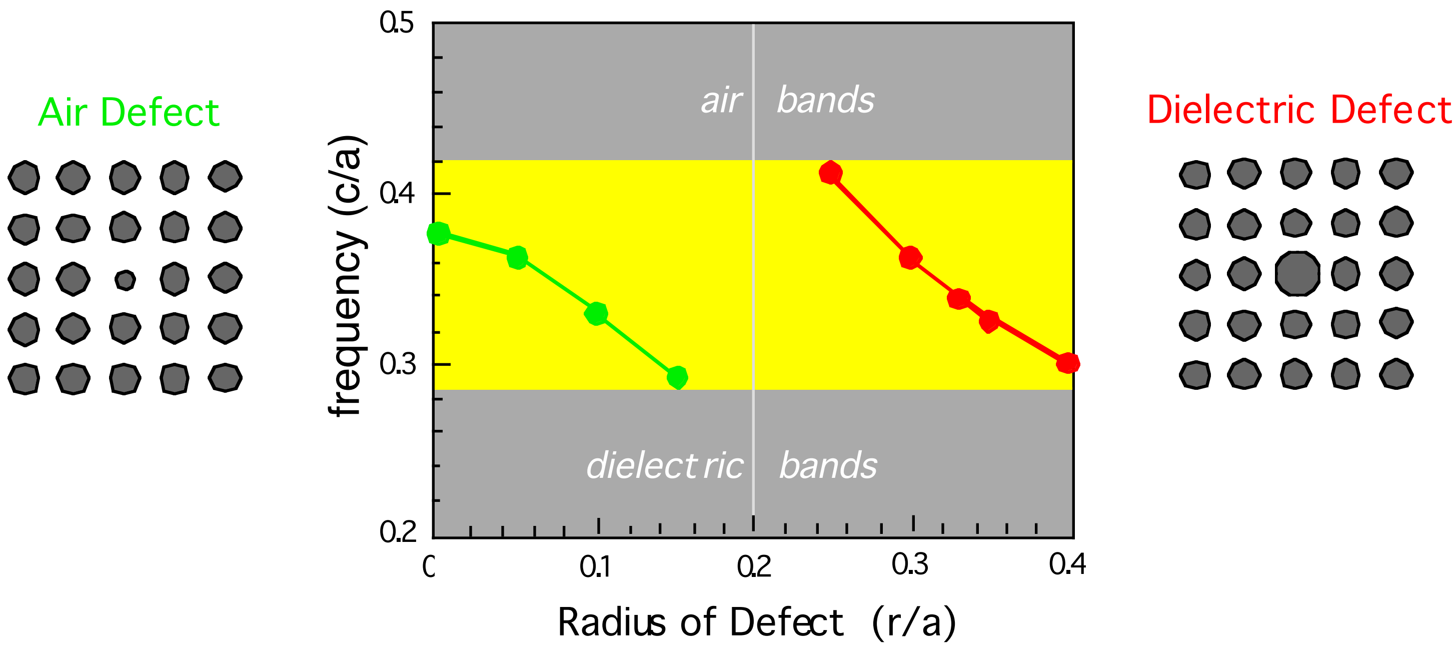
Bulk Crystal Band Diagram



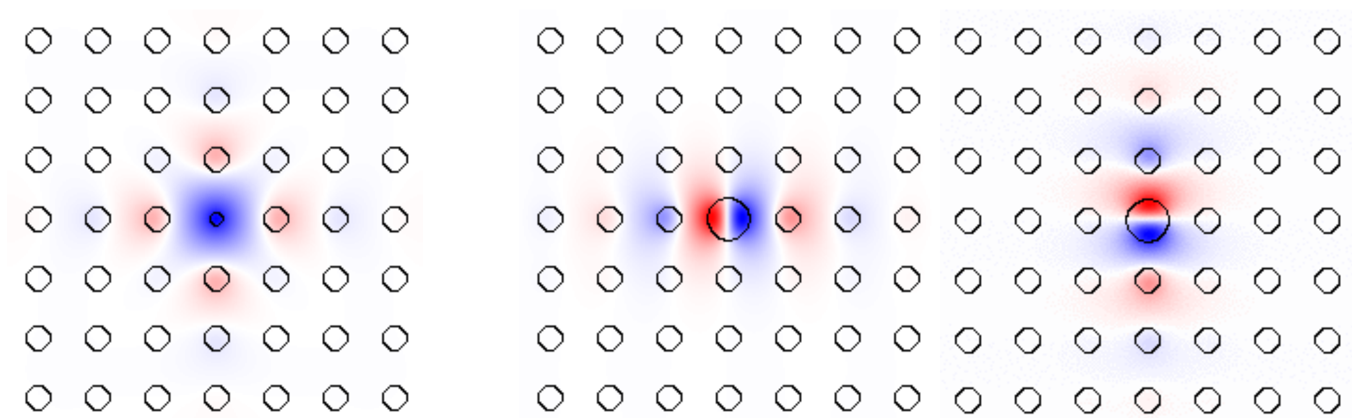


(k not conserved)

Tunable Cavity Modes



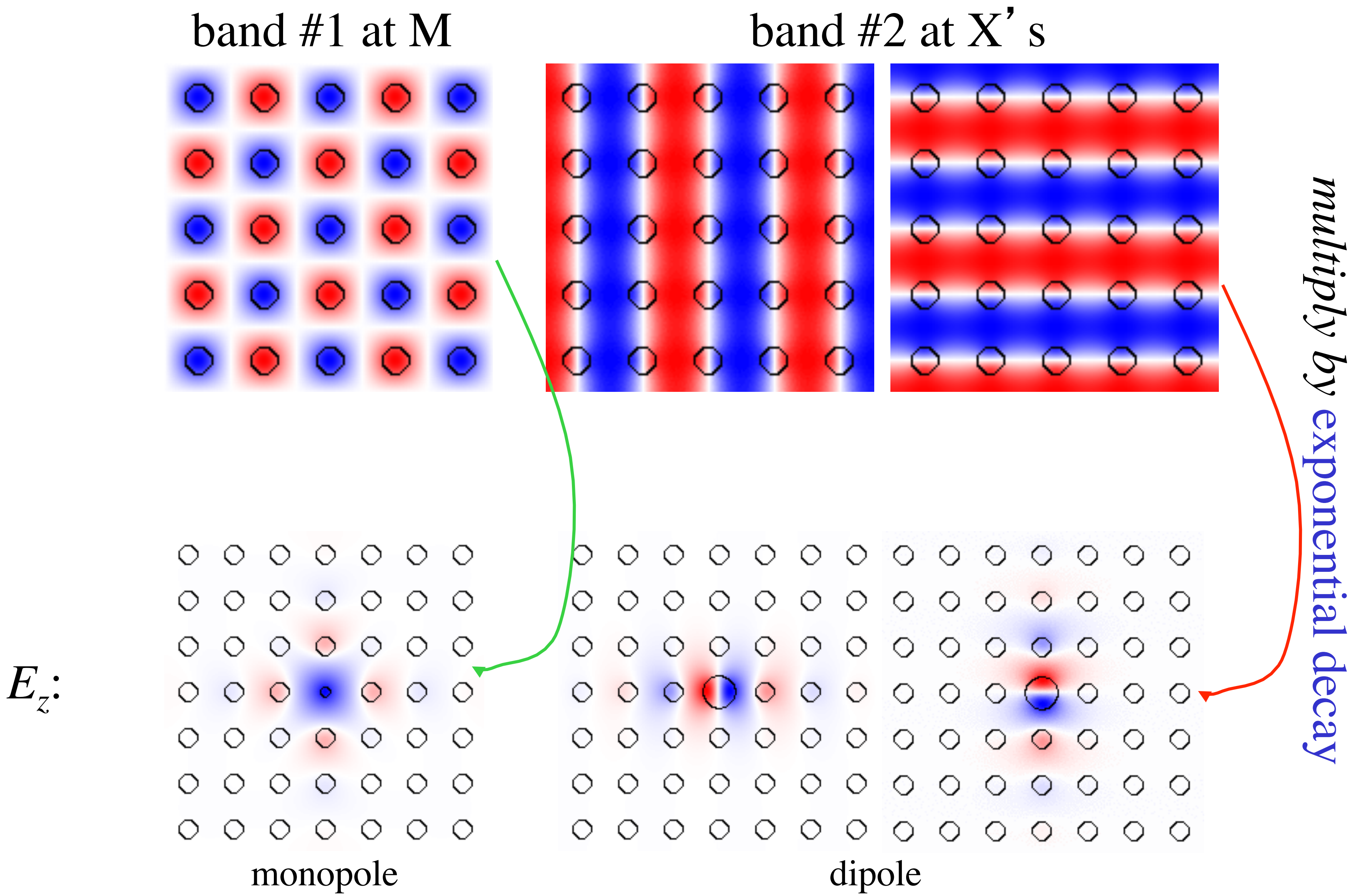
E_z:



monopole

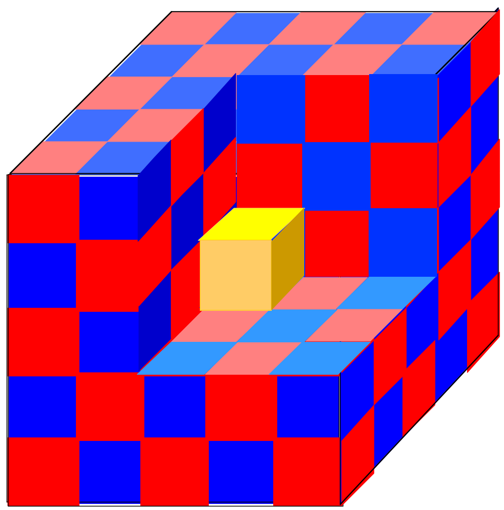
dipole

Tunable Cavity Modes

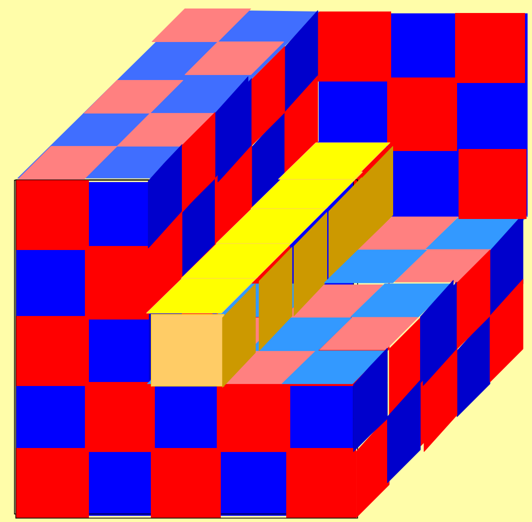


Intentional "defects" are good

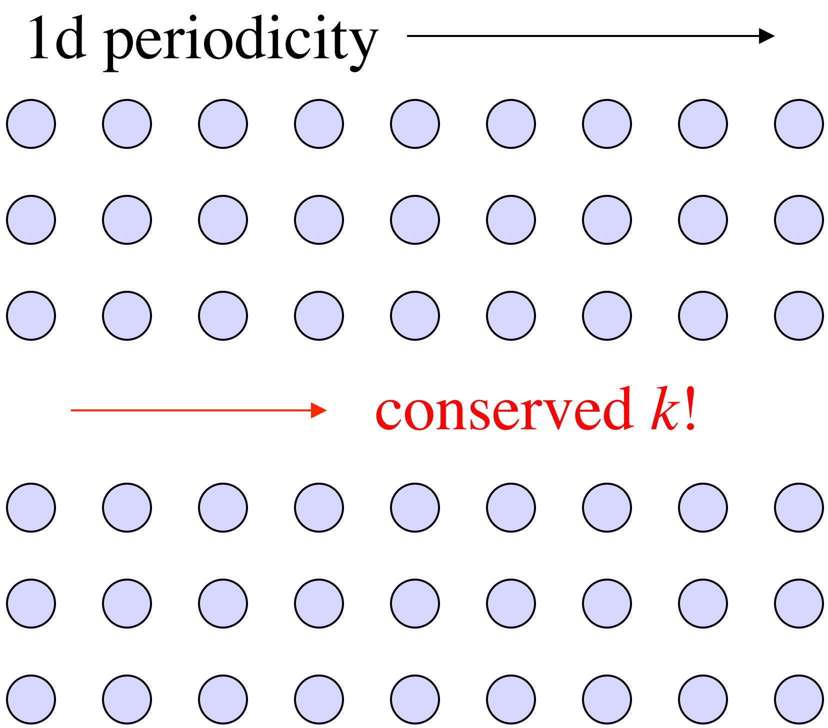
microcavities

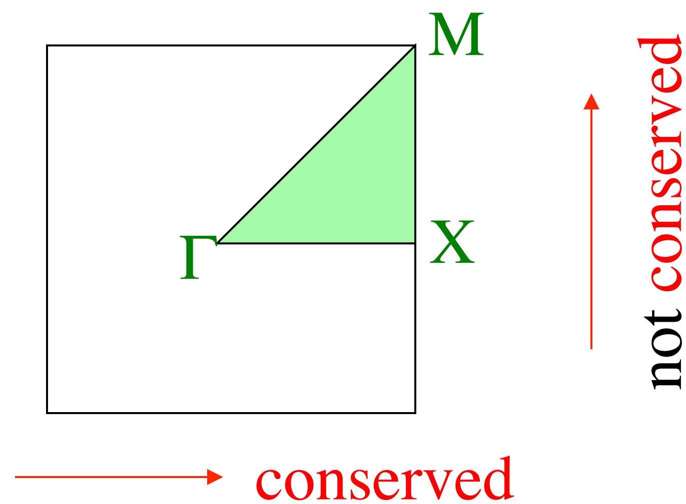


waveguides ("wires")

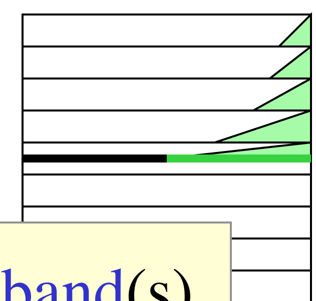


Projected Band Diagrams



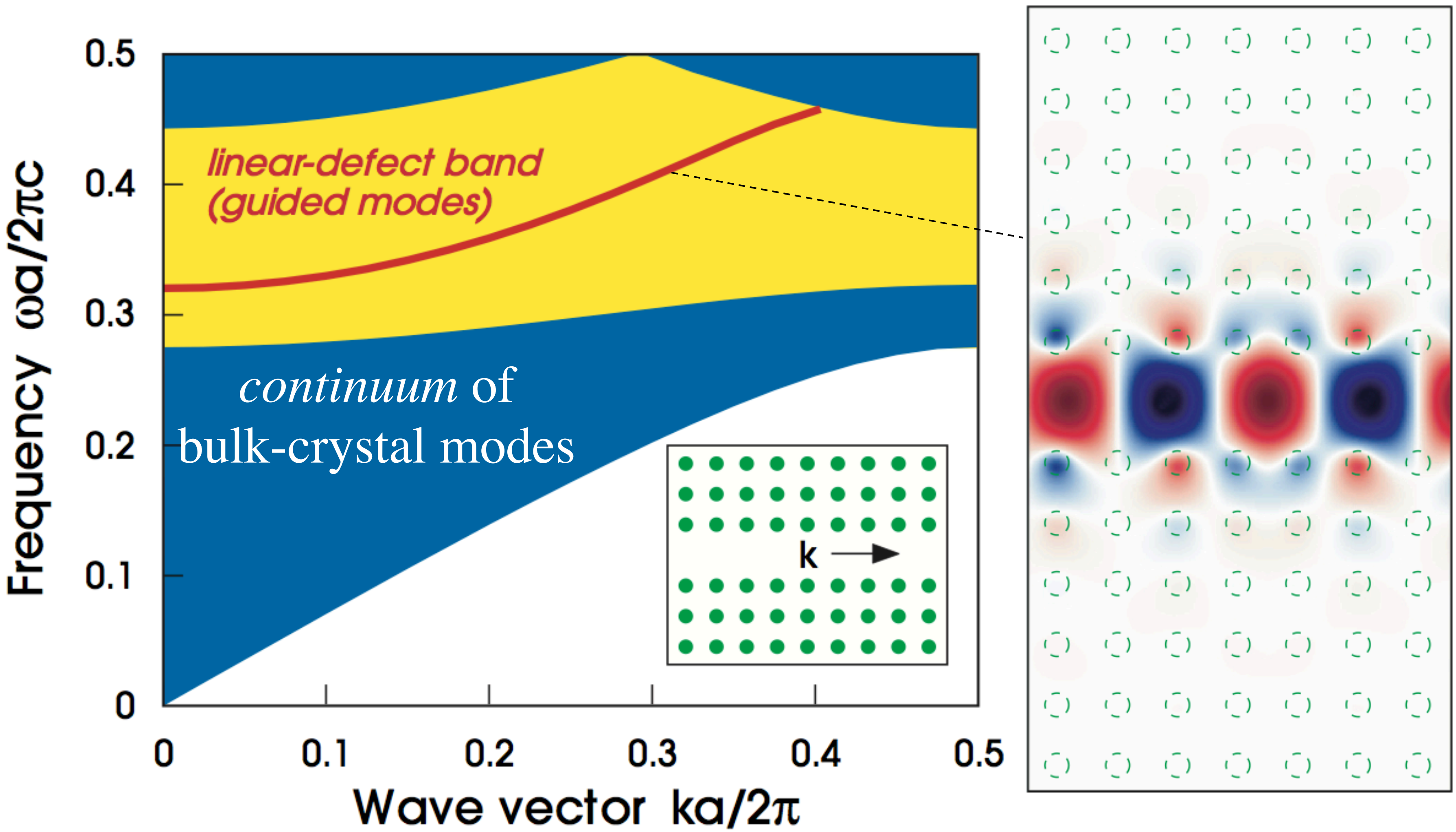


So, plot ω vs. k_x only…project Brillouin zone onto Γ–X:



gives continuum of bulk states + discrete guided band(s)

Air-waveguide Band Diagram



any state in the gap cannot couple to bulk crystal ⇒ localized

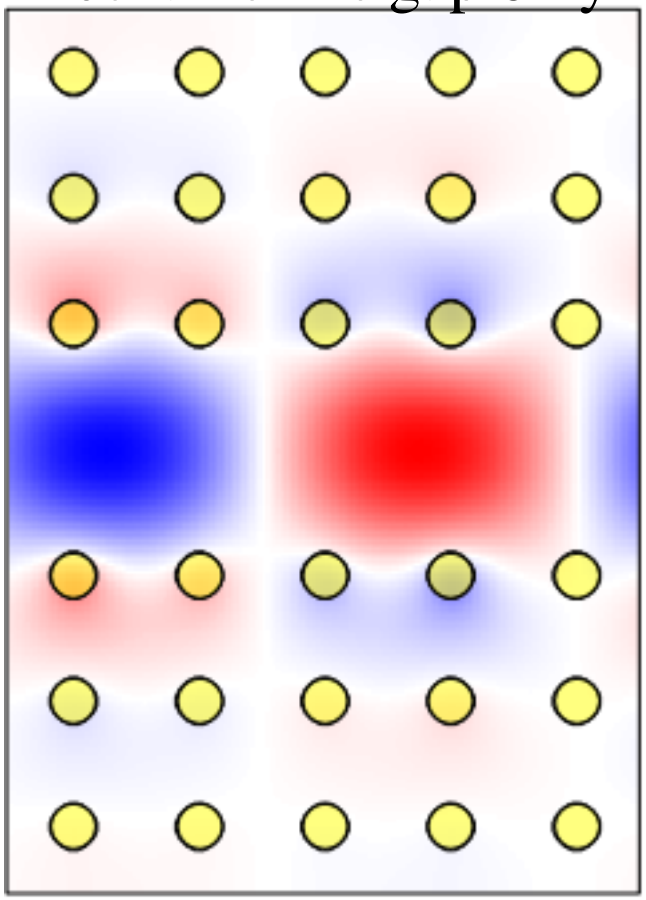
(Waveguides don't really need a *complete* gap)

Fabry-Perot waveguide:

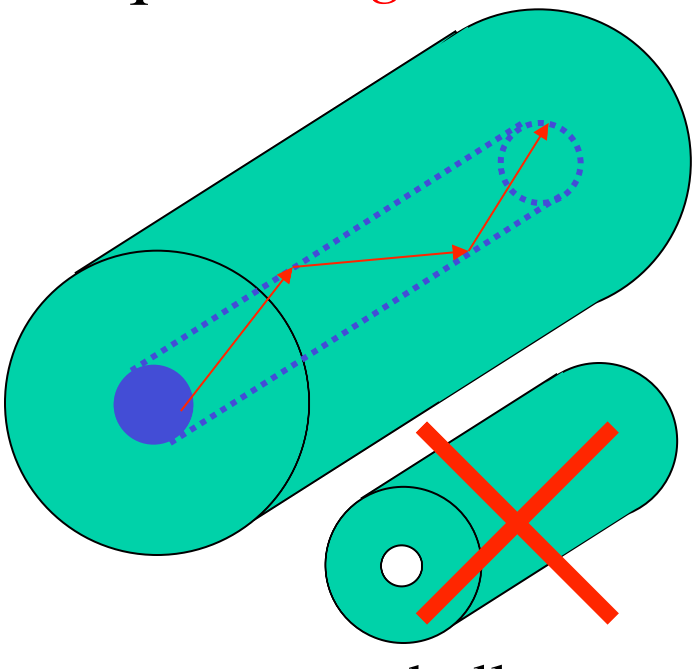
This is exploited *e.g.* for photonic-crystal fibers…

Guiding Light in Air!

mechanism is gap only



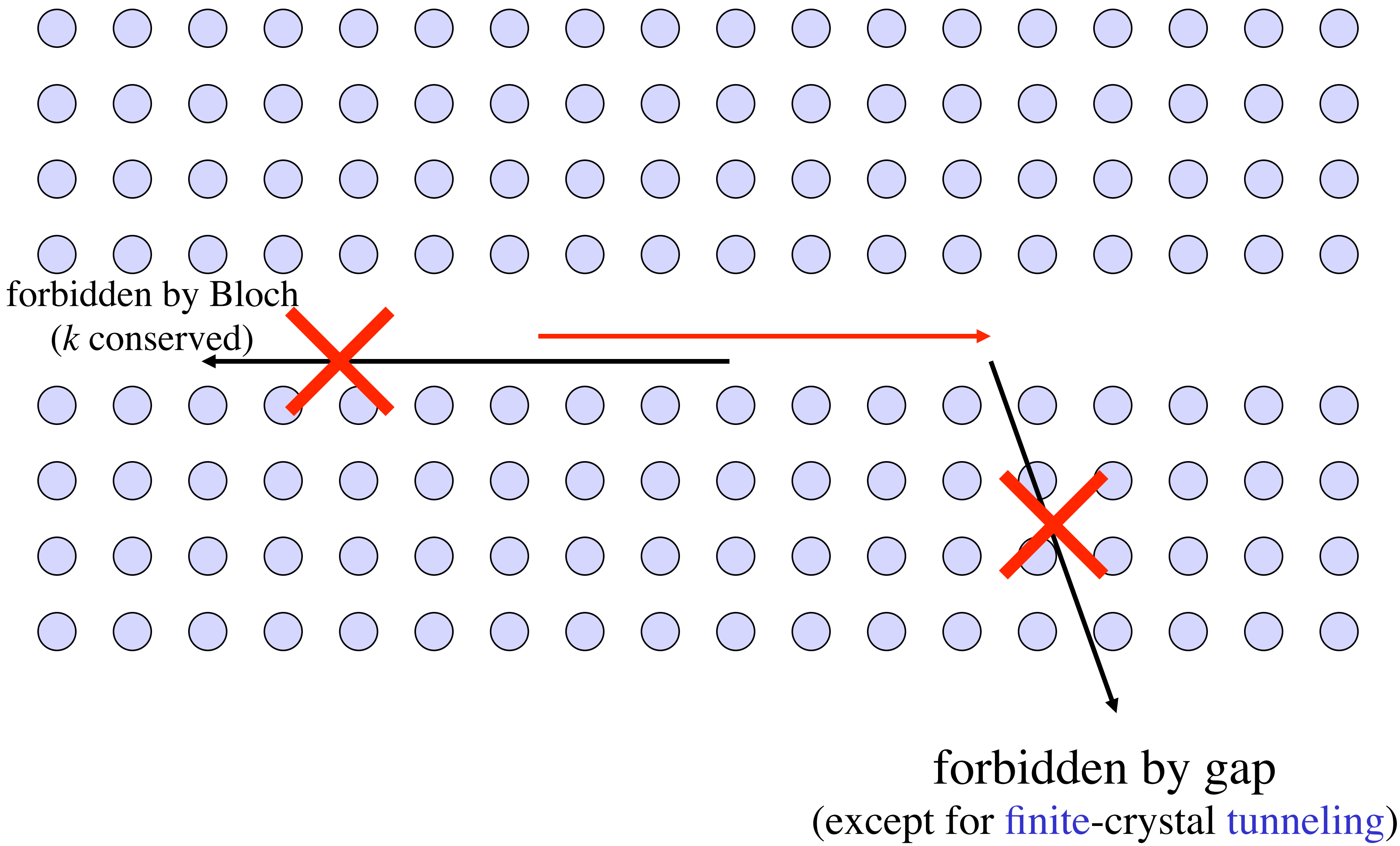
vs. standard optical fiber:
"total internal reflection"
— requires *higher*-index core



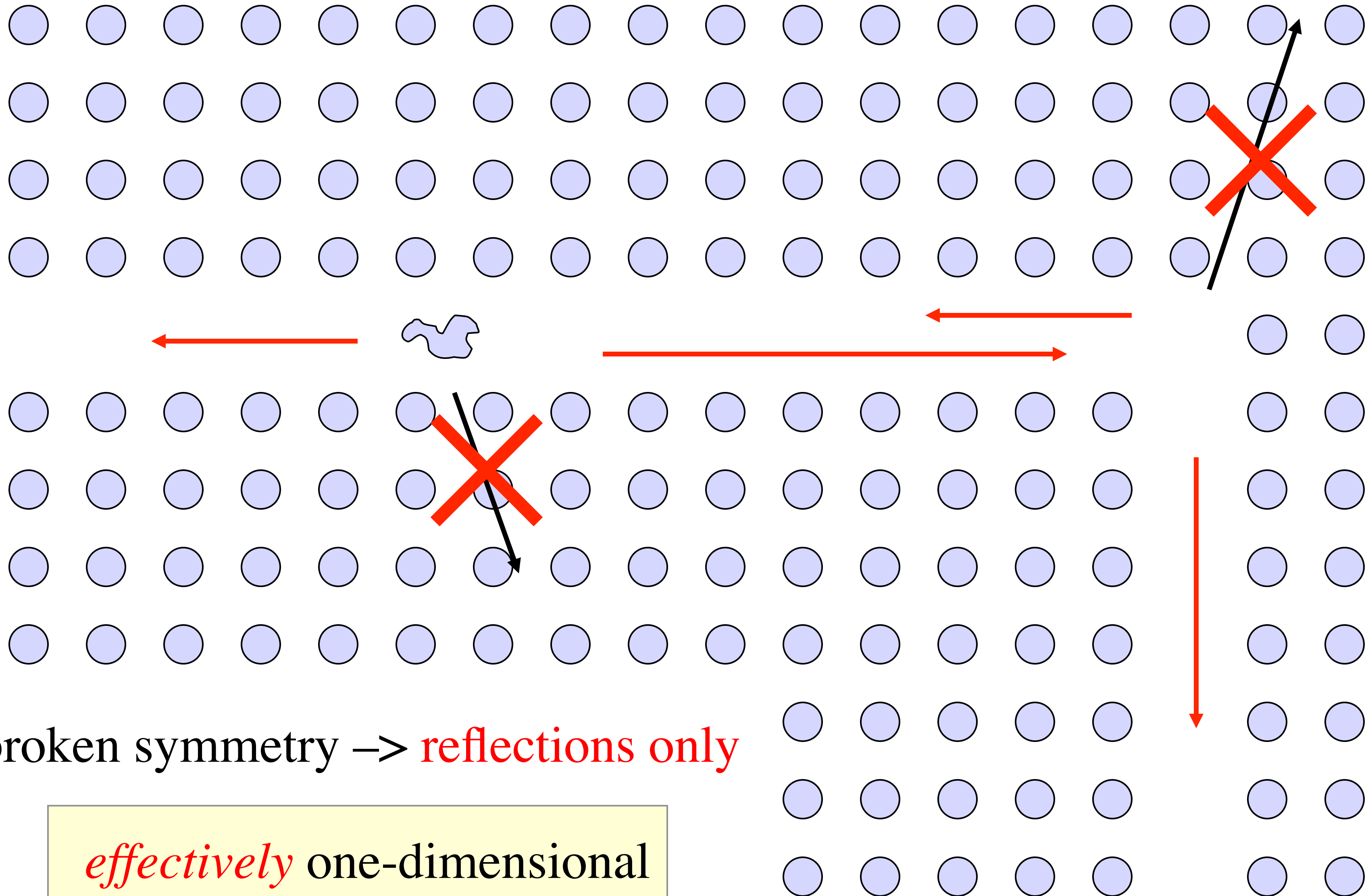
no hollow core!

hollow = lower absorption, lower nonlinearities, higher power

Review: Why no scattering?



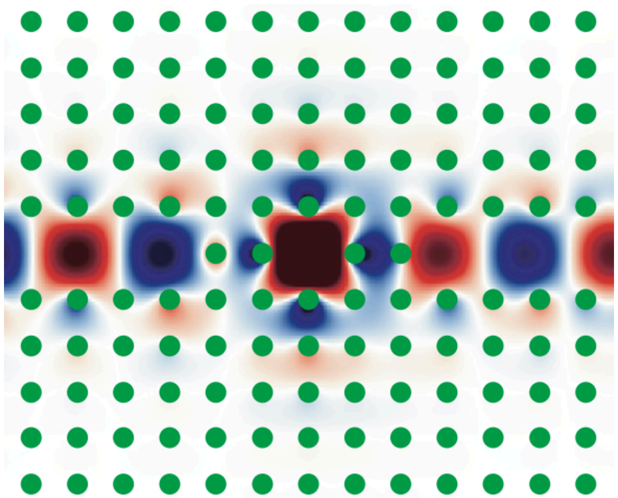
Benefits of a complete gap…



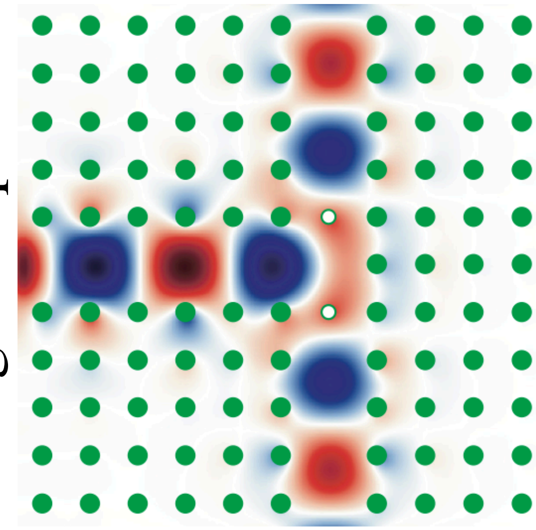
broken symmetry –> reflections only

effectively one-dimensional

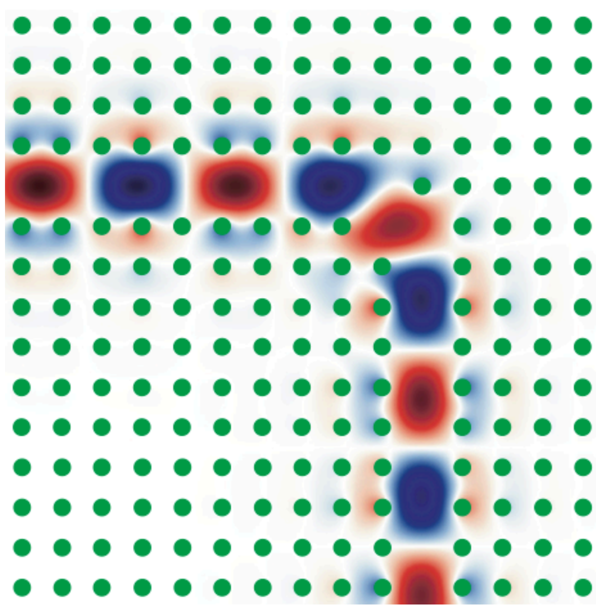
“1d” Waveguides + Cavities = Devices



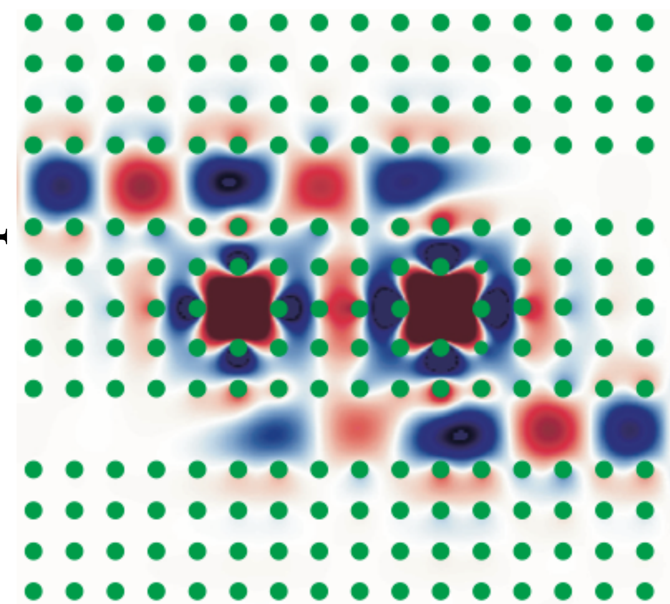
resonant filters



waveguide splitters

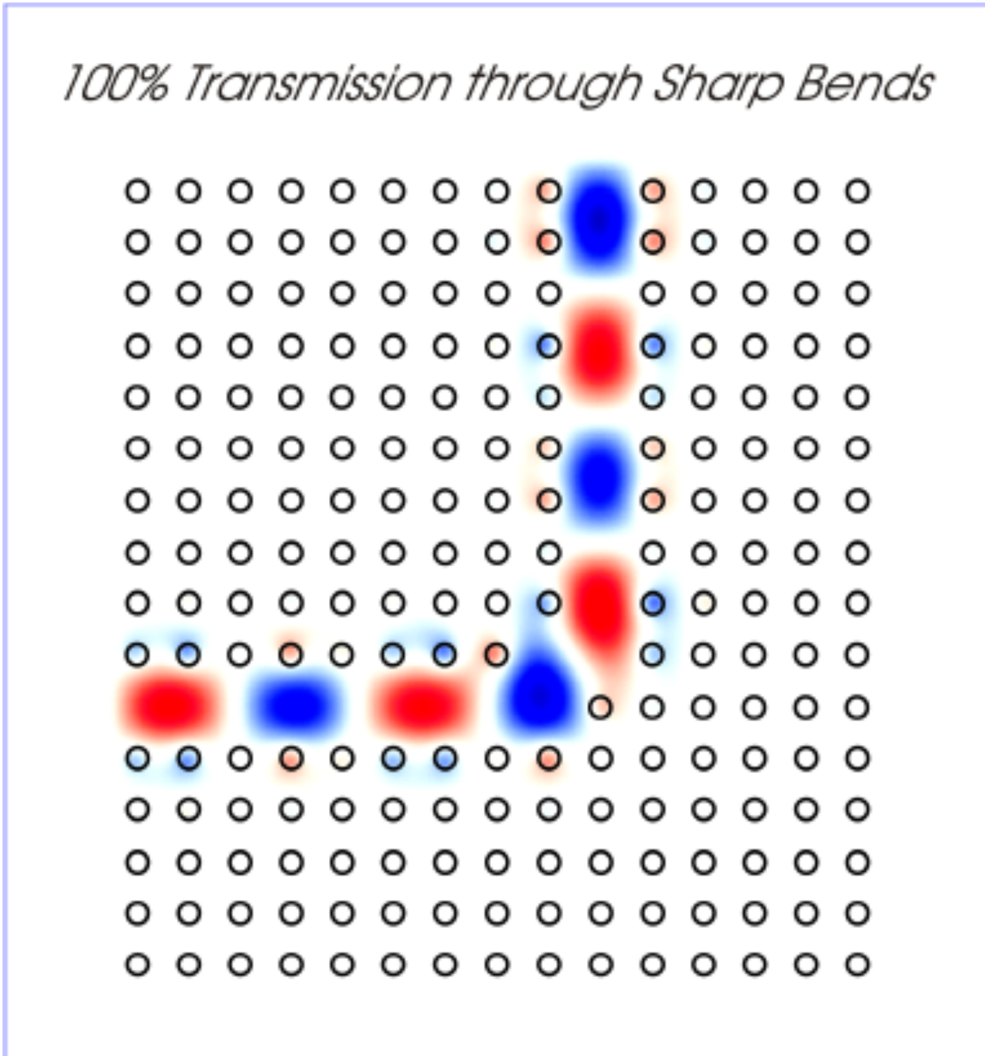


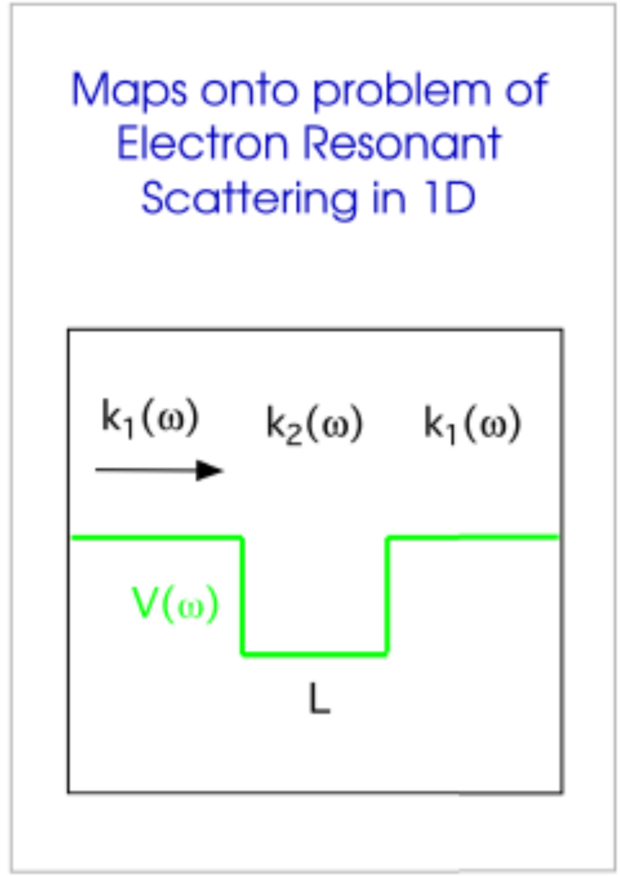
high-transmission
sharp bends



channel-drop filters

Lossless Bends

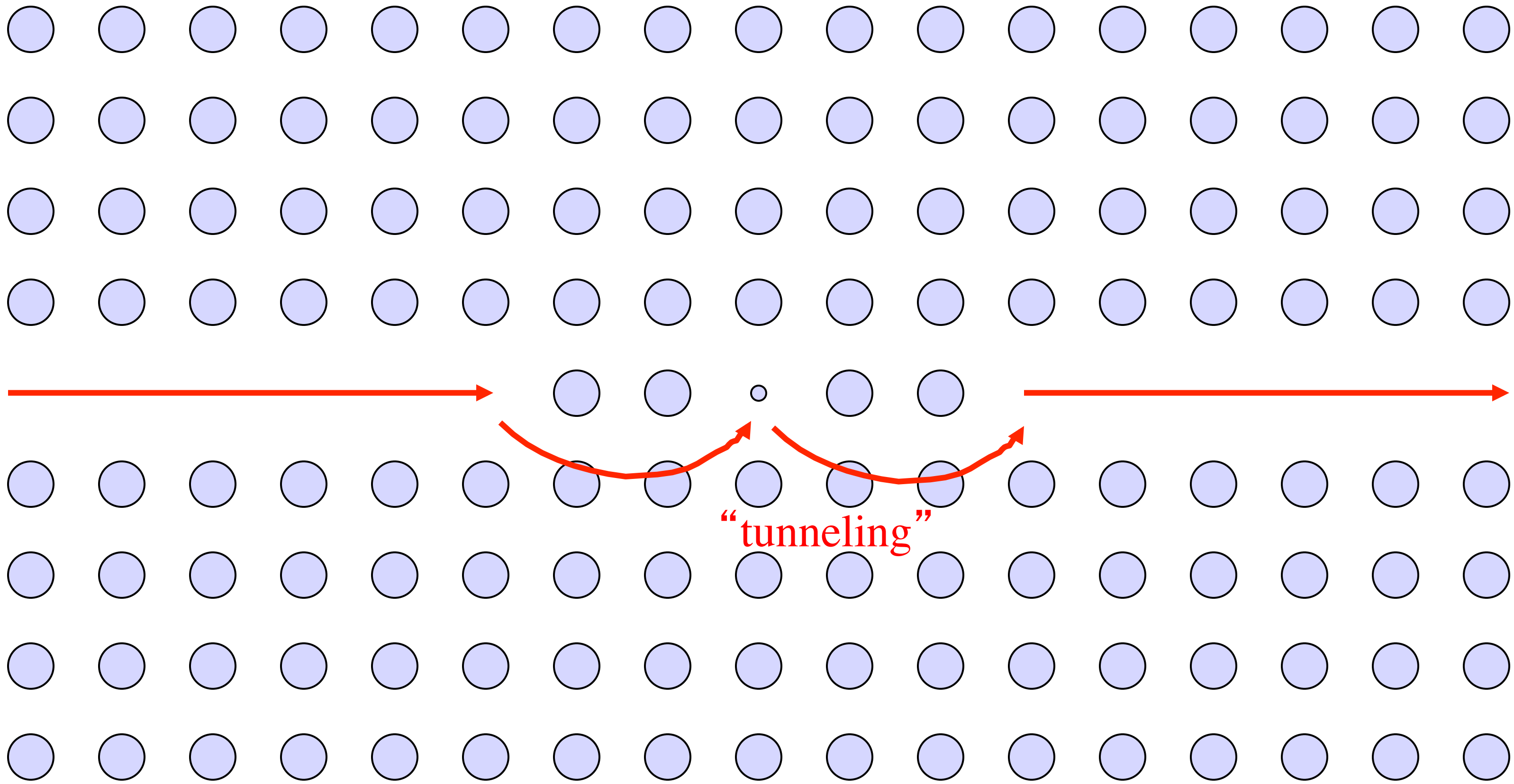




[A. Mekis *et al*.,
Phys. Rev. Lett. **77**, 3787 (1996)]

symmetry + single-mode + "1d" = resonances of 100% transmission

Waveguides + Cavities = Devices

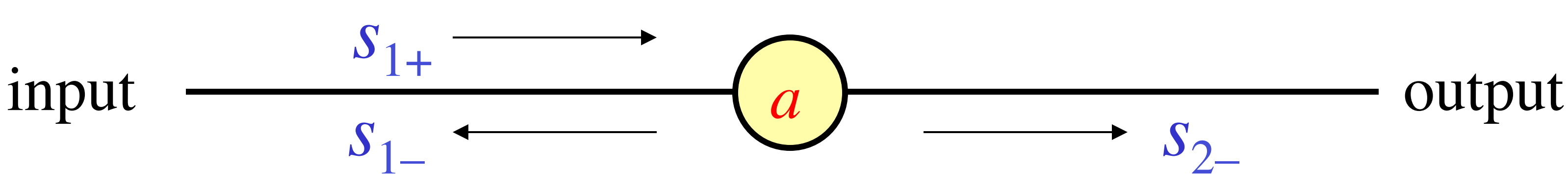


Ugh, must we simulate this to get the basic behavior?

Temporal Coupled-Mode Theory

(one of several things called of "coupled-mode theory")

[H. Haus, *Waves and Fields in Optoelectronics*]



resonant cavity
frequency ω_0, lifetime τ

$|s|^2$ = power

$|a|^2$ = energy

$$\frac{da}{dt} = -i\omega_0 a - \frac{2}{\tau} a + \sqrt{\frac{2}{\tau}} s_{1+}$$

$$s_{1-} = -s_{1+} + \sqrt{\frac{2}{\tau}} a, \quad s_{2-} = \sqrt{\frac{2}{\tau}} a$$

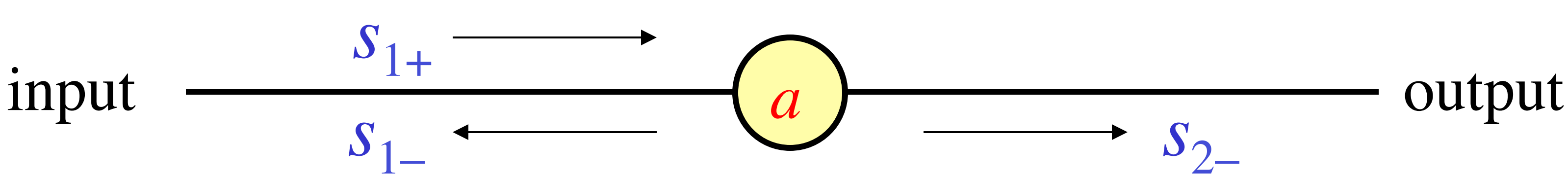
assumes only:

- exponential decay
  (strong confinement)
- conservation of energy
- time-reversal symmetry

Temporal Coupled-Mode Theory

(one of several things called of "coupled-mode theory")

[H. Haus, *Waves and Fields in Optoelectronics*]



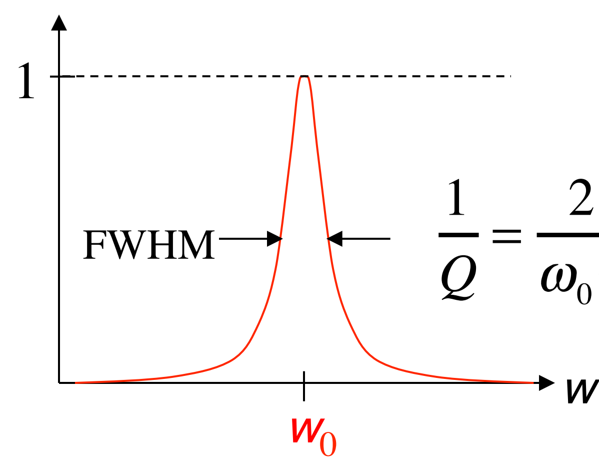
resonant cavity
frequency ω_0, lifetime τ

$|s|^2$ = flux

$|a|^2$ = energy

transmission T

$= | s_{2-} |^2 / | s_{1+} |^2$



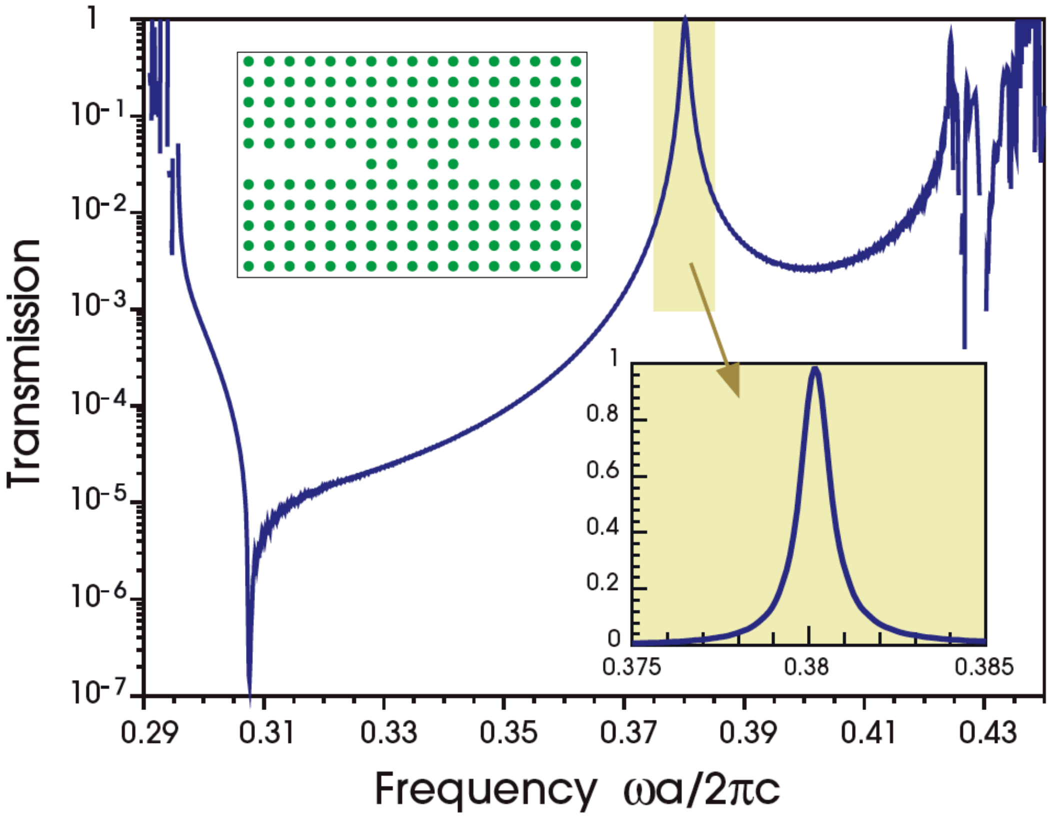
$$\frac{1}{Q} = \frac{2}{\omega_0 \tau}$$

…quality factor Q

T = Lorentzian filter

$$= \frac{\frac{4}{\tau^2}}{(\omega - \omega_0)^2 + \frac{4}{\tau^2}}$$

Resonant Filter Example

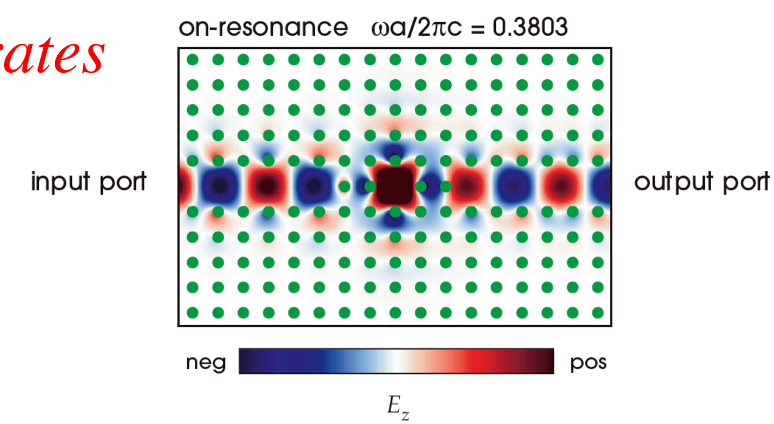


Lorentzian peak, as predicted.

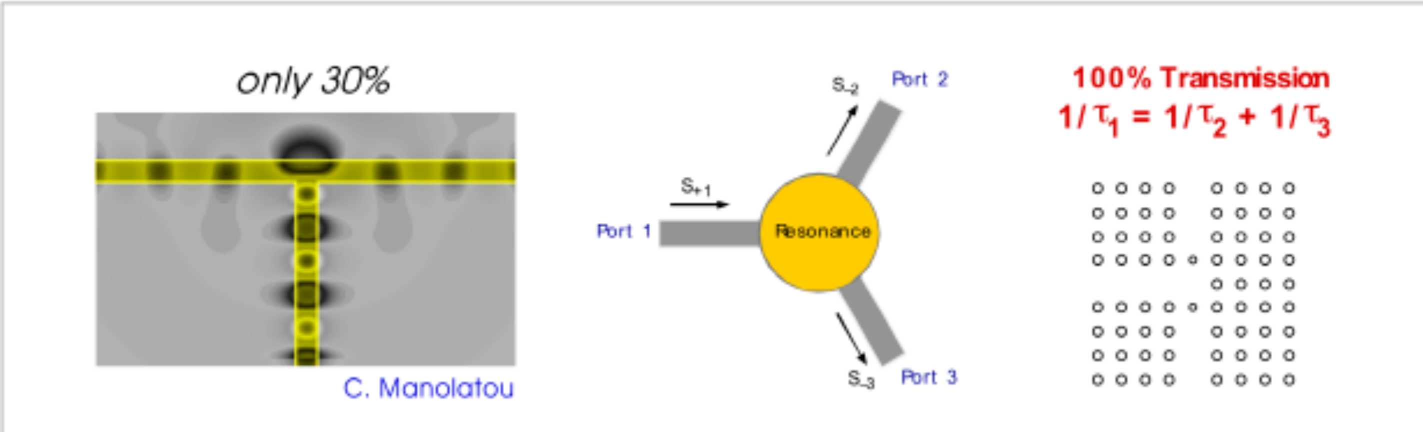
An apparent miracle:

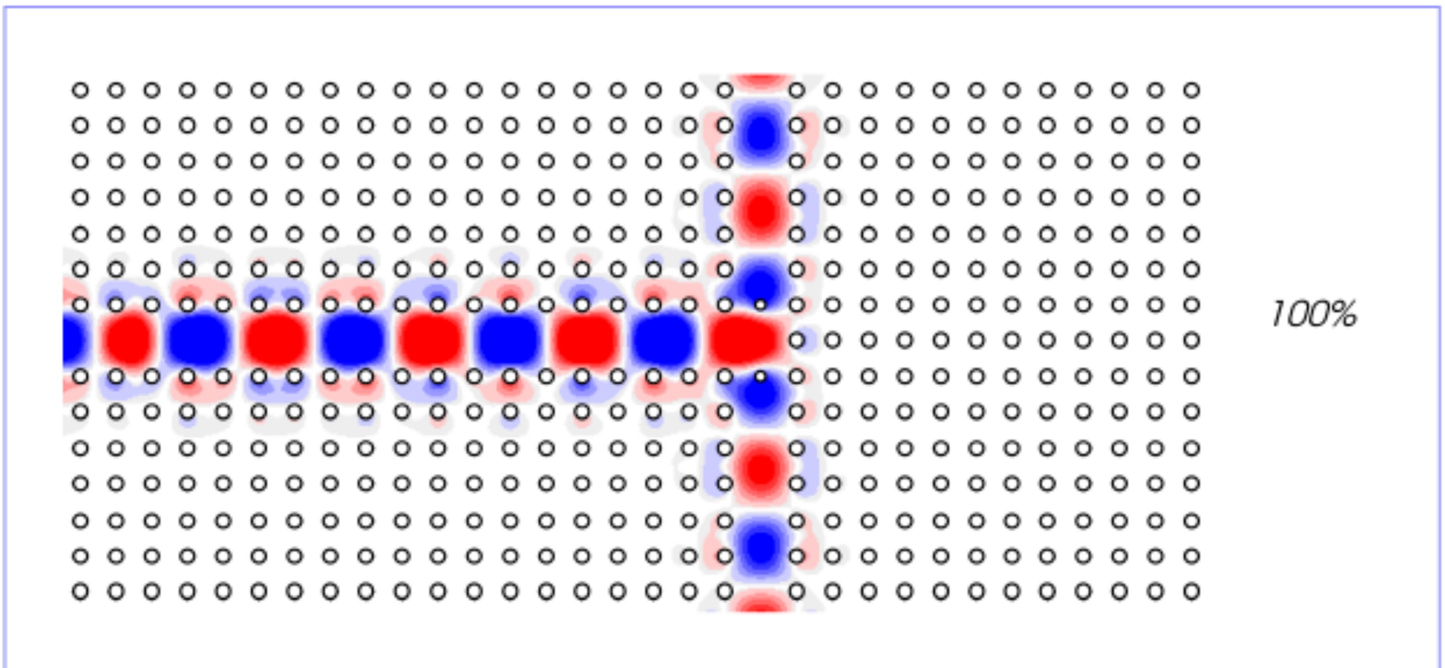
~ 100% transmission
at the resonant frequency

cavity decays to input/output with *equal rates*
⇒ At resonance, reflected wave
destructively interferes
with backwards-decay from cavity
& the two *exactly cancel*.



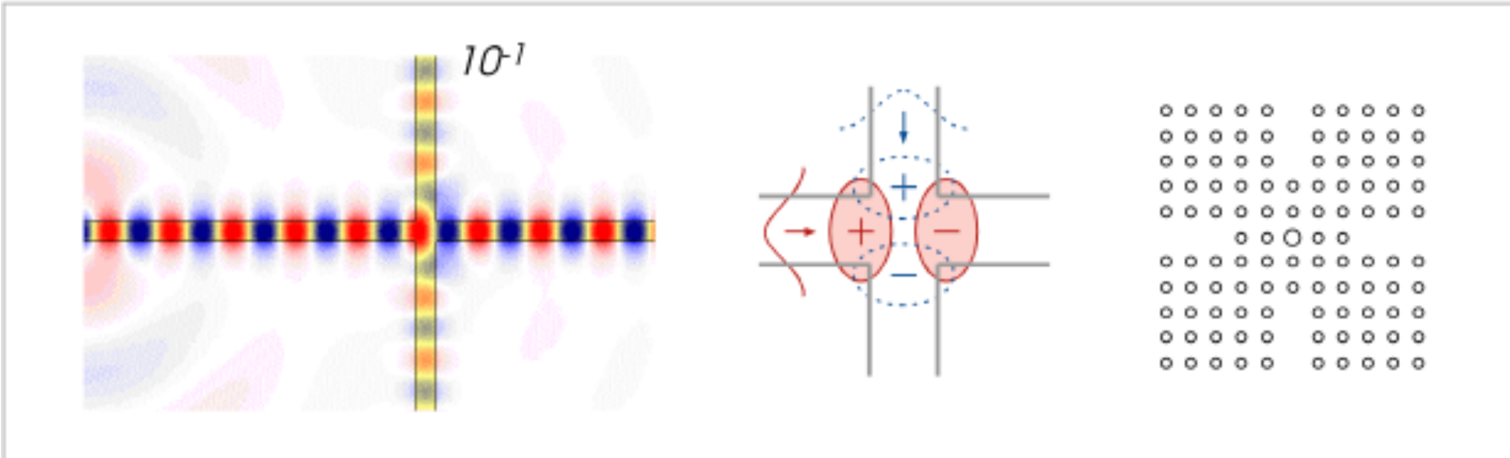
Wide-angle Splitters

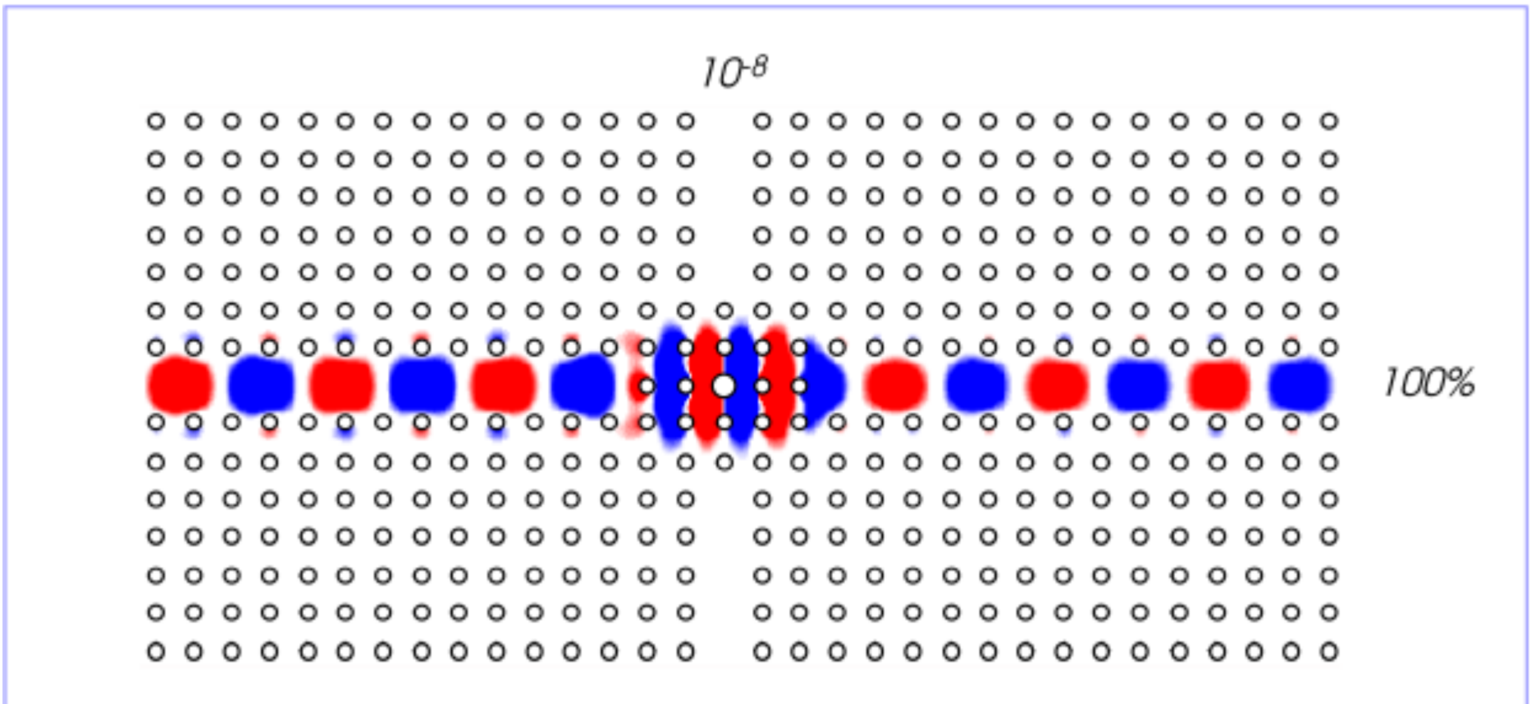




[S. Fan *et al*., *J. Opt. Soc. Am. B* **18**, 162 (2001)]

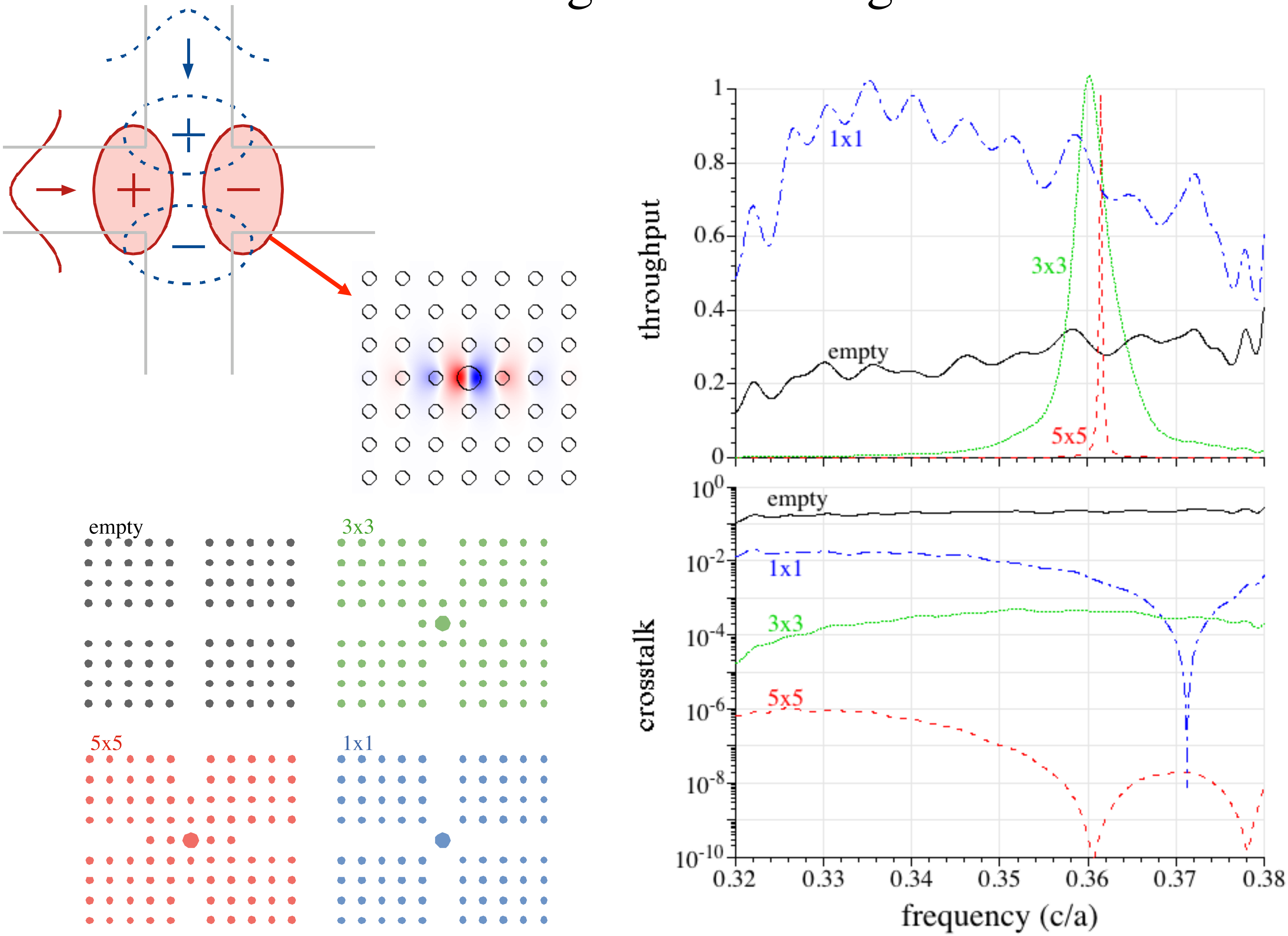
Waveguide Crossings



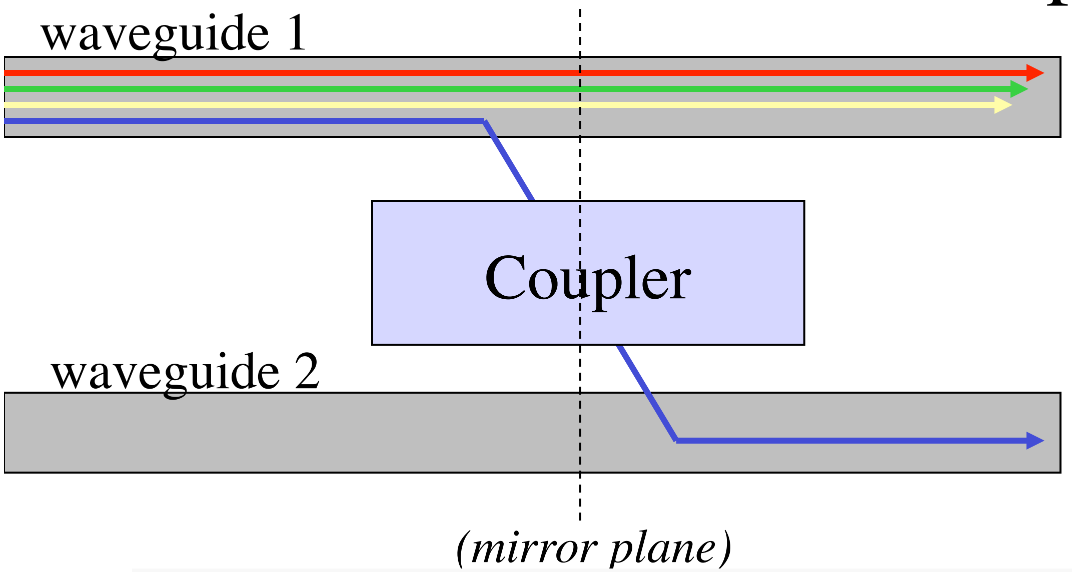


[S. G. Johnson *et al*., *Opt*. *Lett*. **23**, 1855 (1998)]

Waveguide Crossings



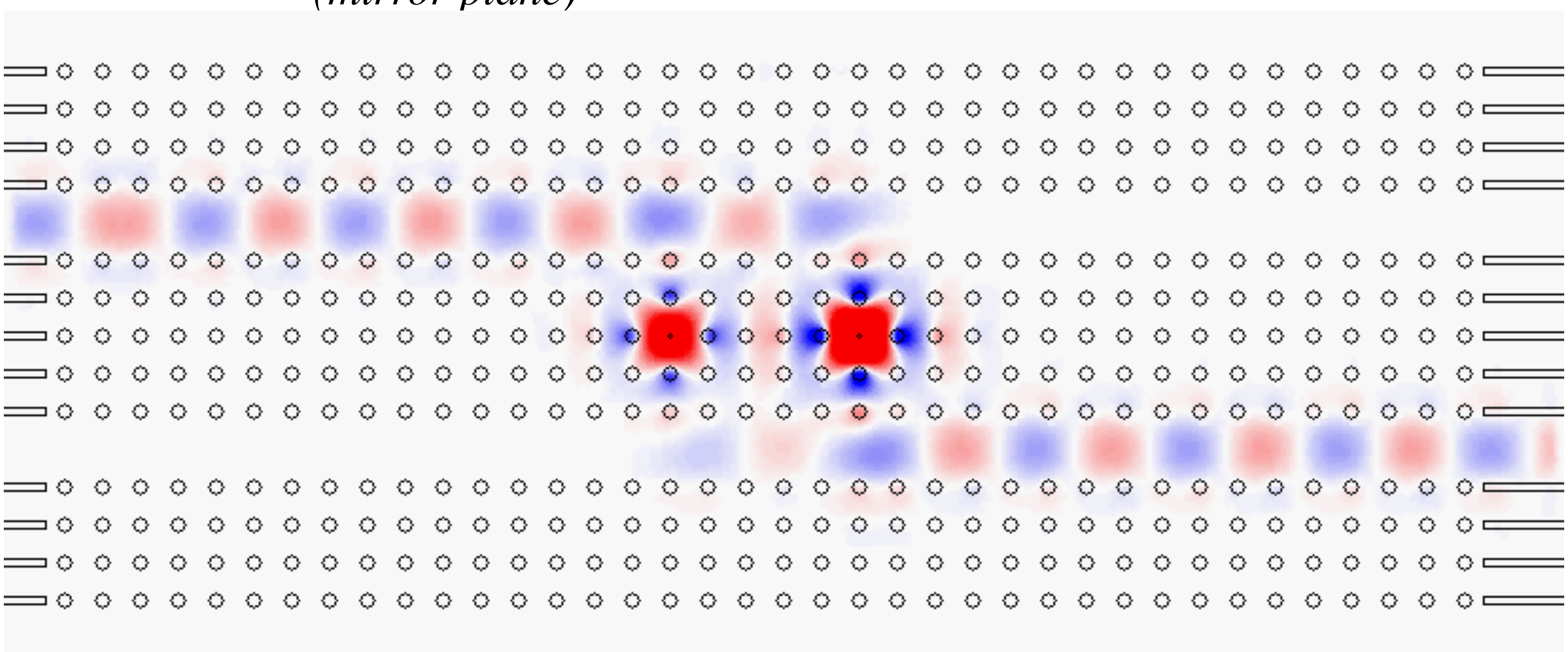
Channel-Drop Filters



Perfect channel-dropping if:

Two resonant modes with:

- even and odd symmetry
- equal frequency (degenerate)
- equal decay rates



[S. Fan *et al.*, *Phys. Rev. Lett.* **80**, 960 (1998)]

Enough passive, linear devices…

Photonic crystal cavities:

tight confinement (~ l/2 diameter)

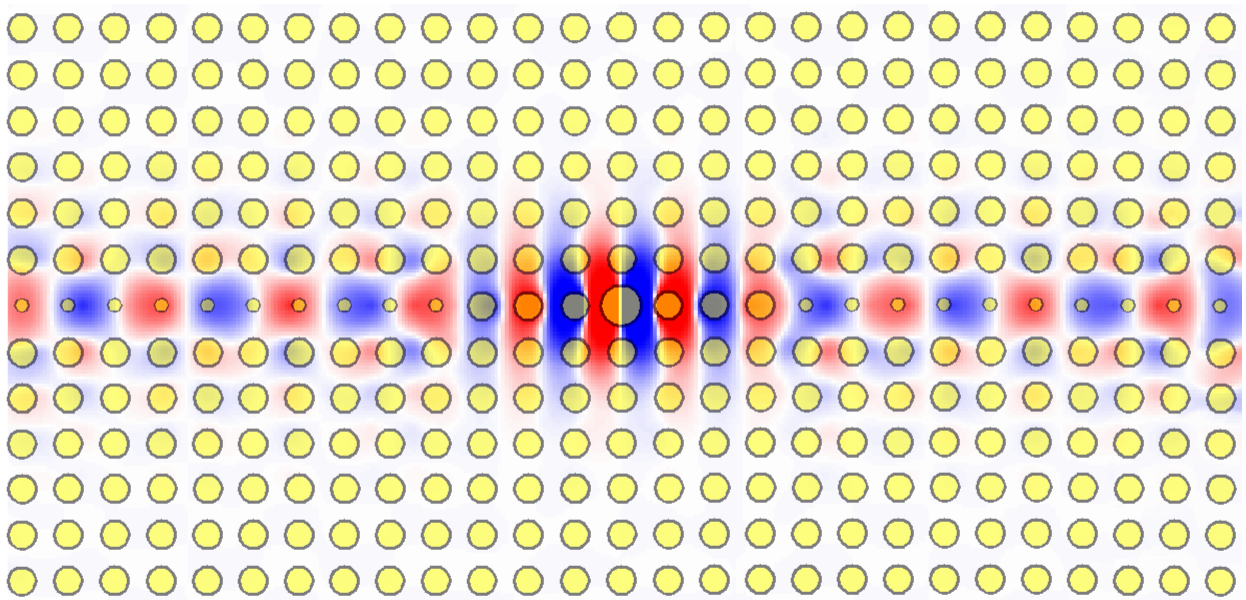
+ long lifetime (high *Q* independent of size)

= enhanced nonlinear effects

e.g. Kerr nonlinearity, Δn ~ intensity

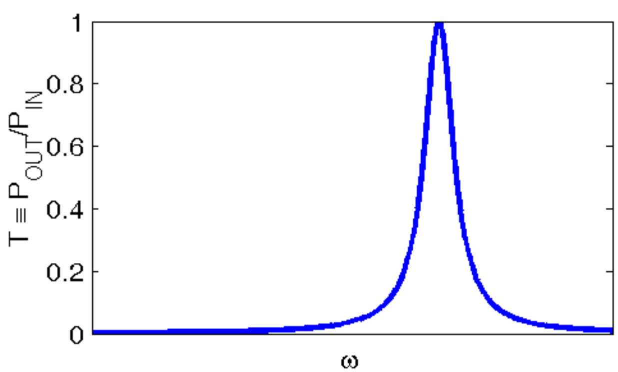
A ~~Linear~~ *Nonlinear* Filter

in



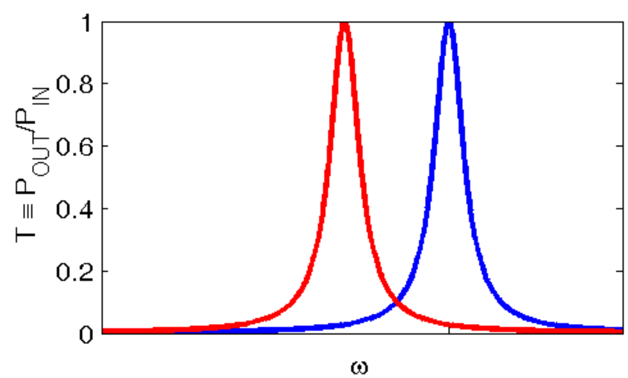
out

Linear response:
Lorenzian Transmisson



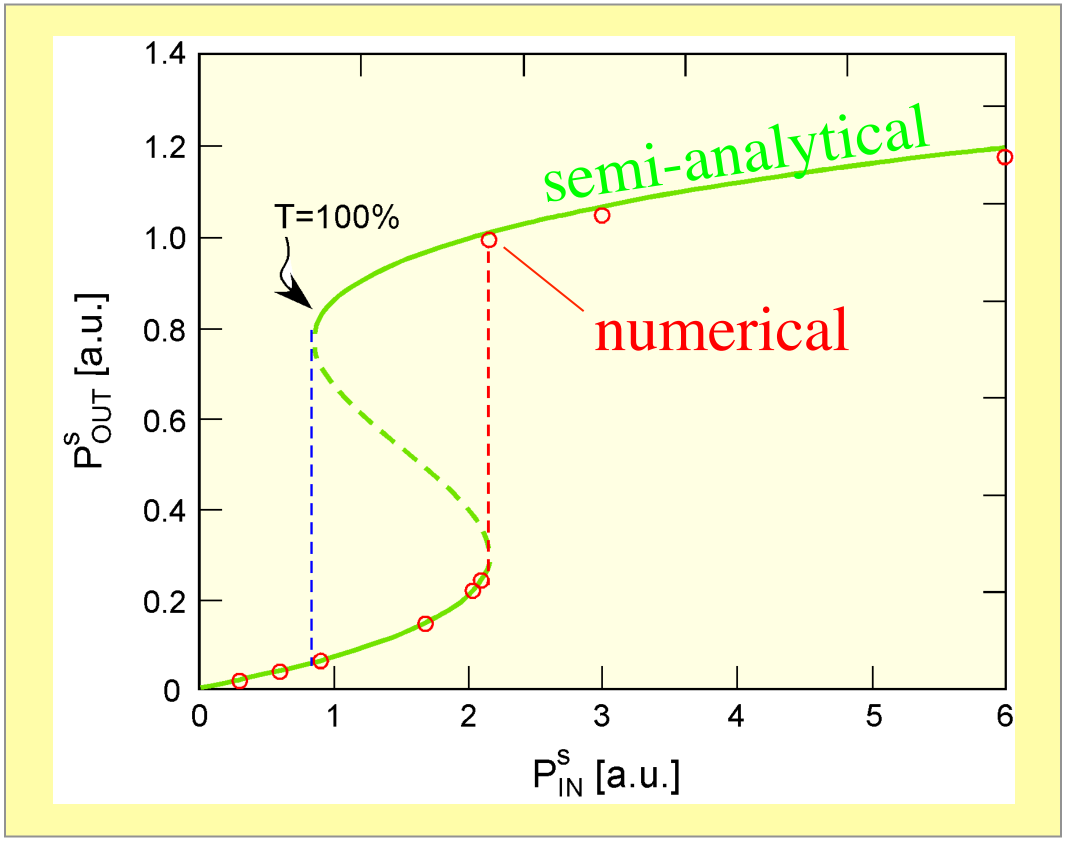
+ nonlinear
index shift

shifted peak



A ~~Linear~~ *Nonlinear* "Transistor"

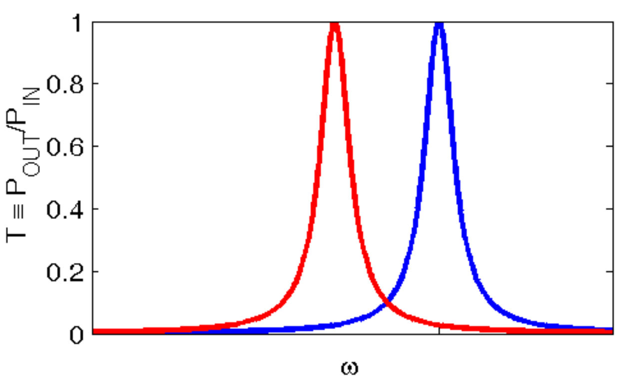
[Soljacic *et al*., *PRE Rapid*. *Comm*. **66**, 055601 (2002).]



Logic gates, switching, rectifiers, amplifiers, isolators, …

+ feedback

shifted peak

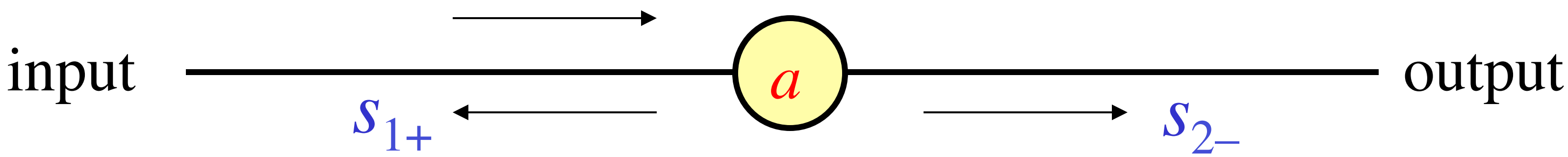


Bistable (hysteresis) response

Power threshold ~ V/Q^2 is near optimal
(~mW for Si and telecom bandwidth)

TCMT for Bistability

[Soljacic *et al*., *PRE Rapid*. *Comm*. **66**, 055601 (2002).]



resonant cavity
frequency ω_0, lifetime τ,
SPM coefficient $\alpha \sim \chi^{(3)}$
(computed from perturbation theory)

$|s|^2$ = power

$|a|^2$ = energy

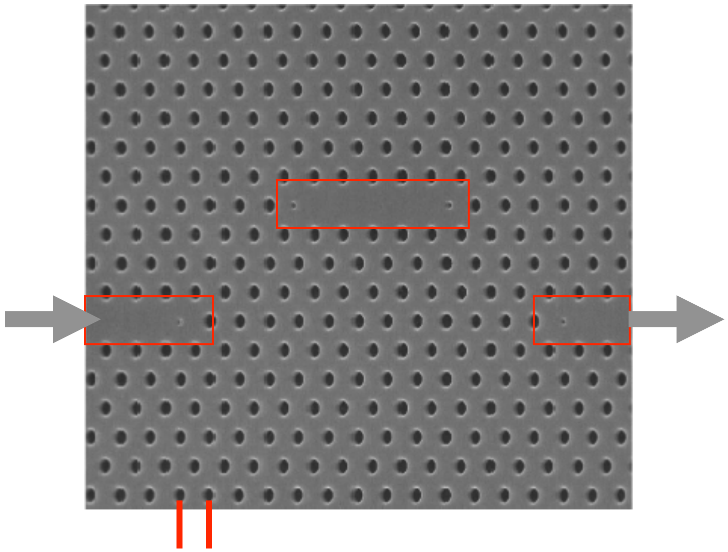
$$\frac{da}{dt} = -i(\omega_0 - \alpha|a|^2)a - \frac{2}{\tau}a + \sqrt{\frac{2}{\tau}}s_{1+}$$

$$s_{1-} = -s_{1+} + \sqrt{\frac{2}{\tau}}a, \quad s_{2-} = \sqrt{\frac{2}{\tau}}a$$

gives cubic equation
for transmission
… bistable curve

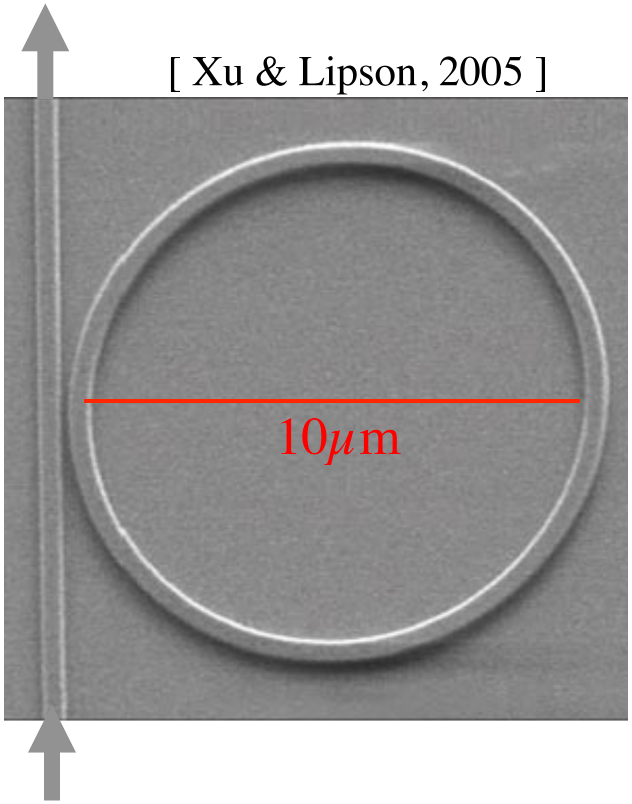
Experimental Nonlinear Switches

[Notomi *et al*. (2005).]



$Q \sim 30{,}000$
$V \sim 10$ optimum
Power threshold $\sim 40\ \mu W$

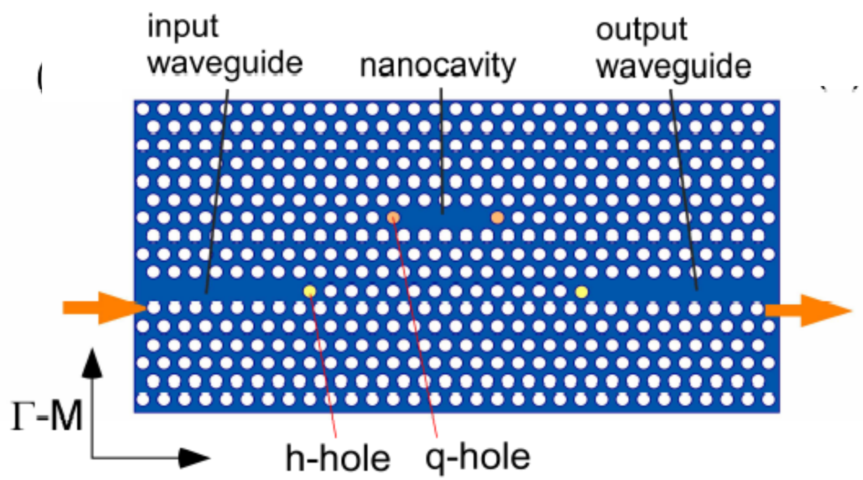
[Xu & Lipson, 2005]

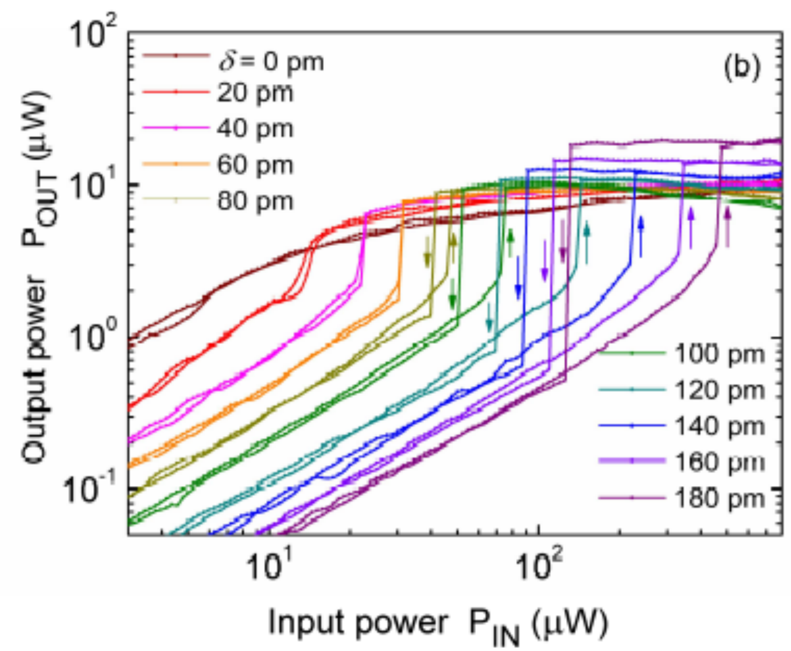


$Q \sim 10{,}000$
$V \sim 300$ optimum
Power threshold ~ 10 mW

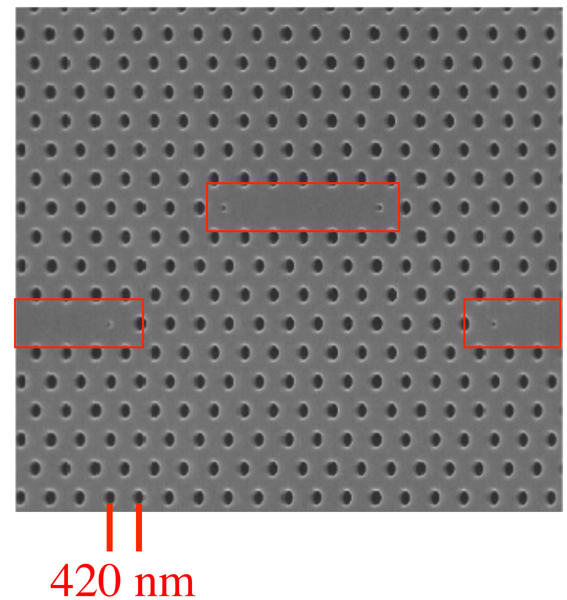
Experimental Bistable Switch

[Notomi *et al*., *Opt*. *Express* **13** (7), 2678 (2005).]





Silicon-on-insulator

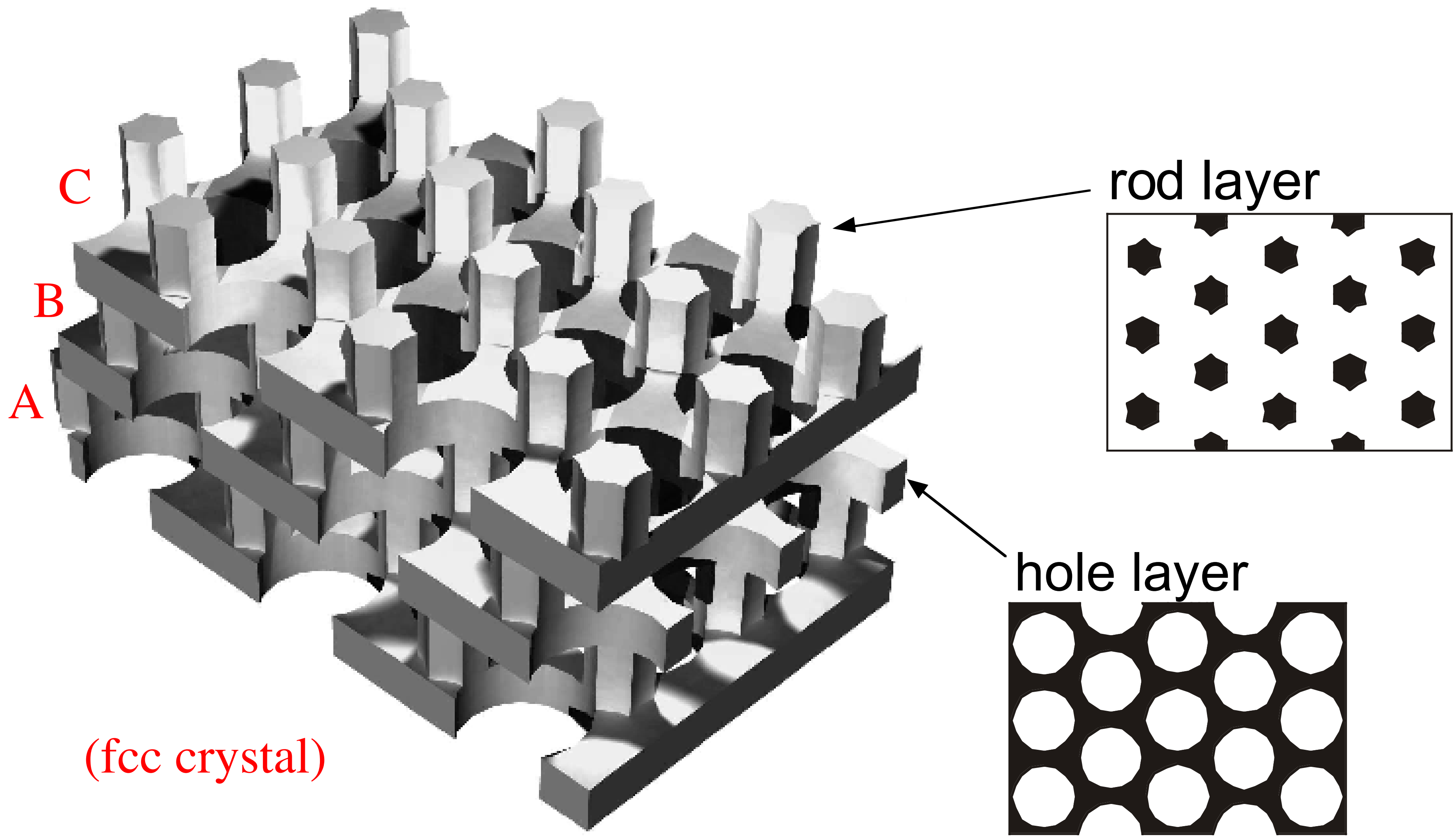


Q ~ 30,000

Power threshold ~ 40 μW

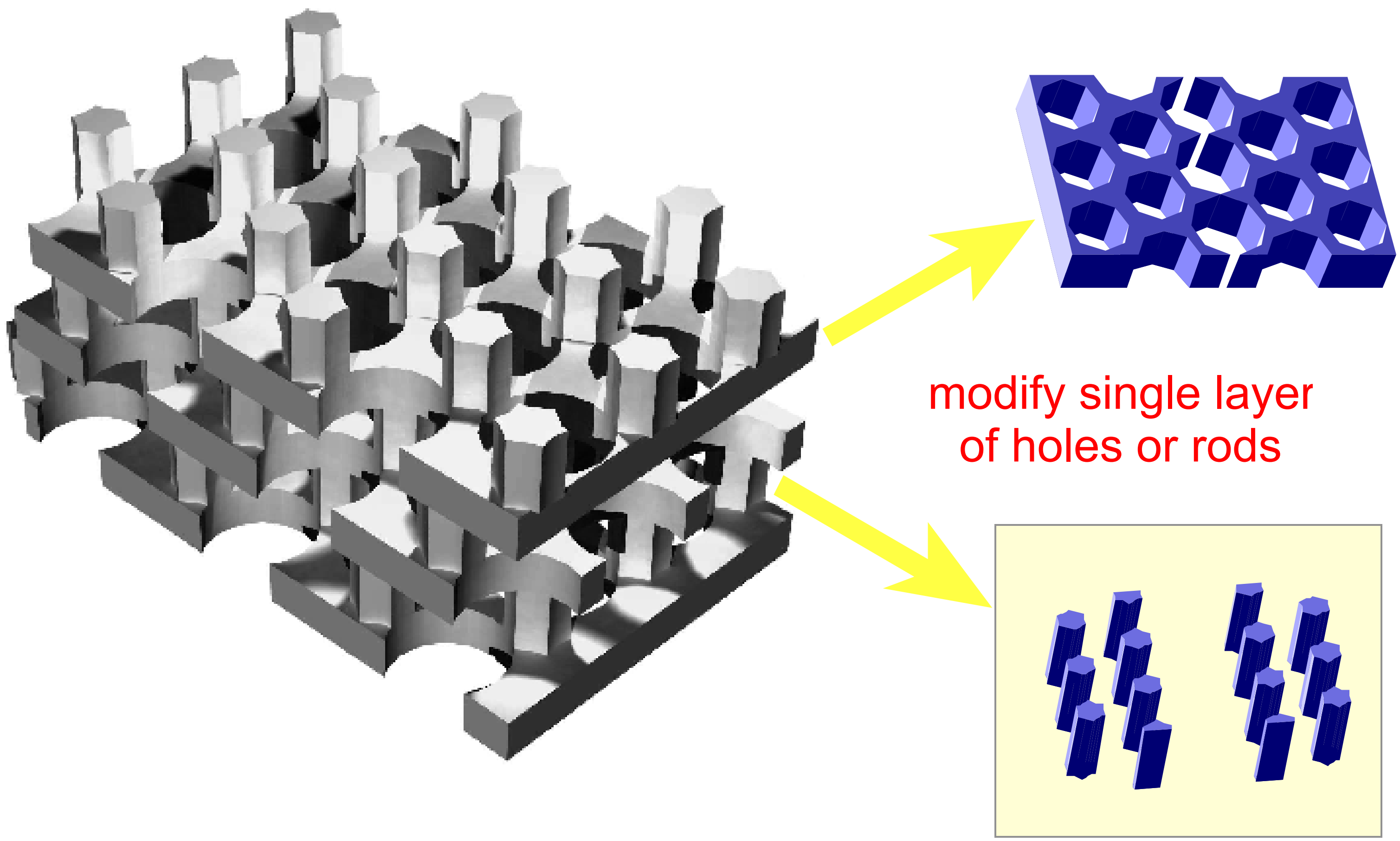
Switching energy ~ 4 pJ

Same principles apply in 3d…

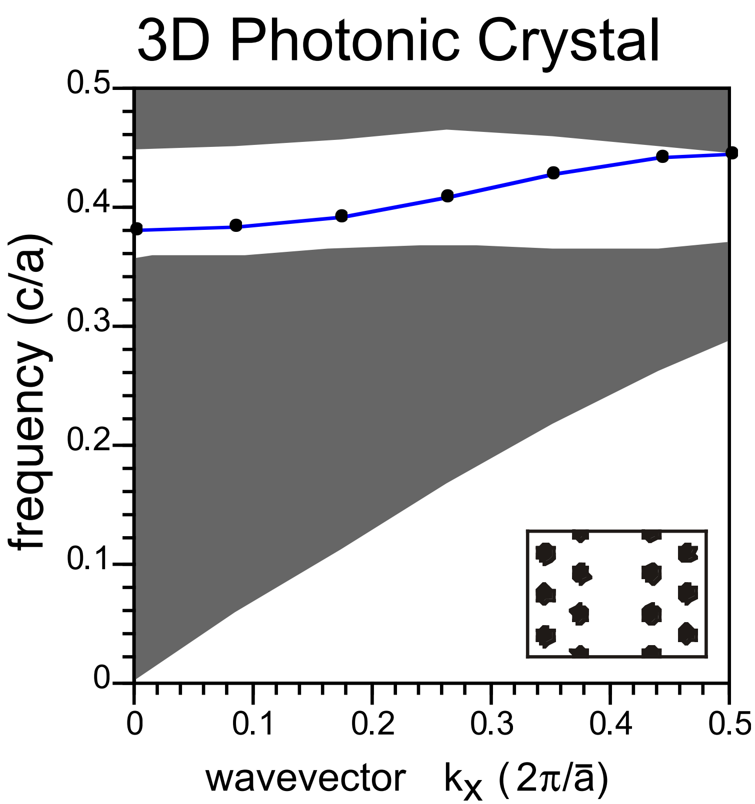


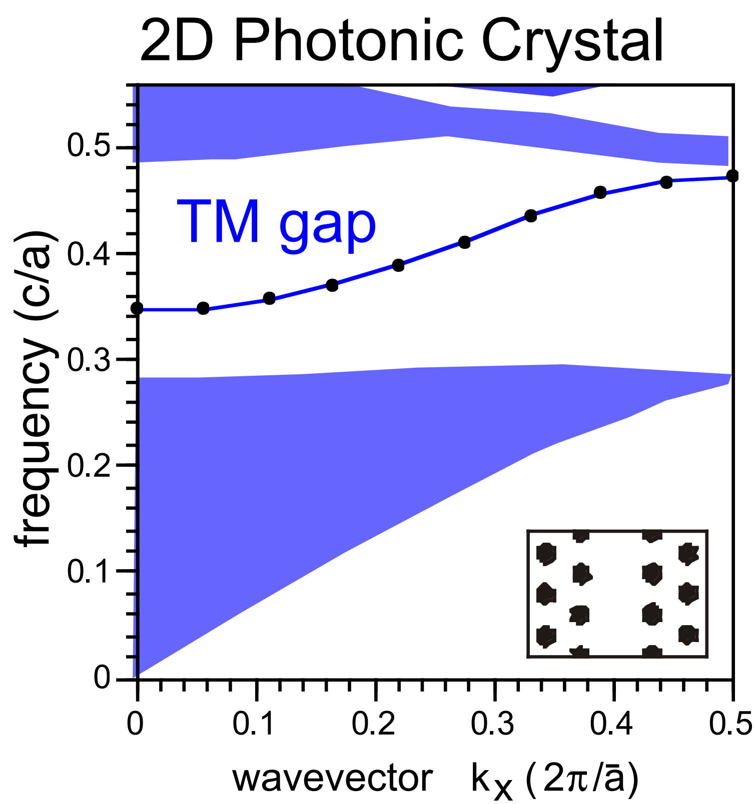
2d-like defects in 3d

[M. L. Povinelli *et al.*, *Phys. Rev. B* **64**, 075313 (2001)]

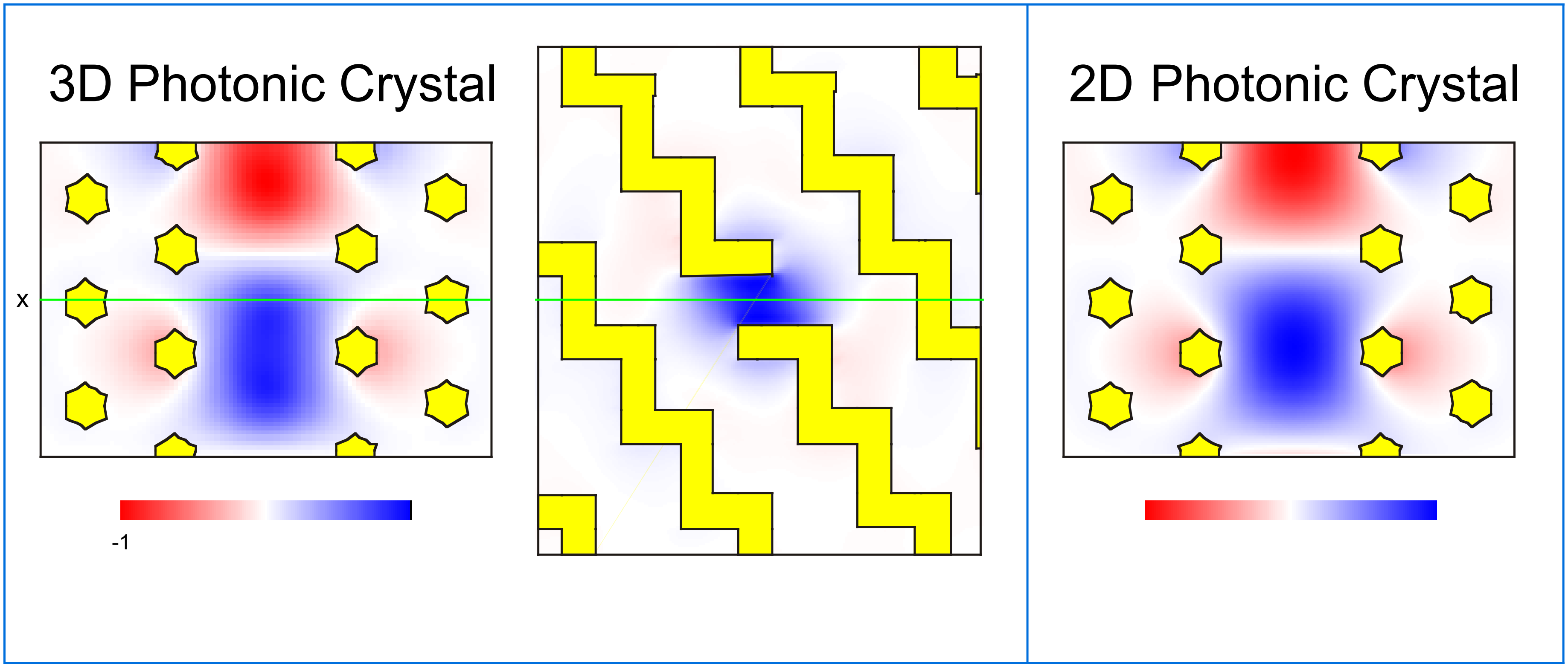


3d projected band diagram

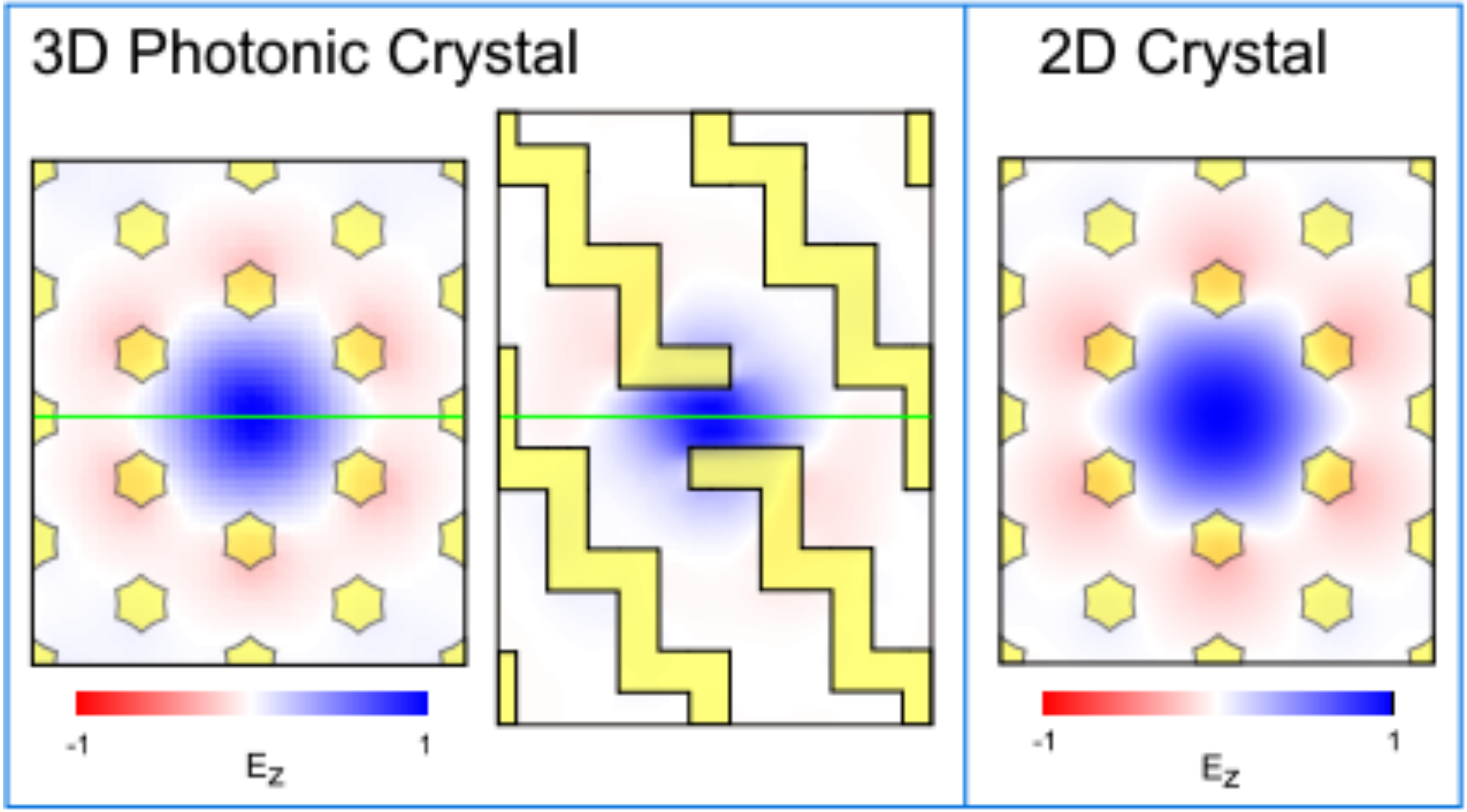




2d-like waveguide mode



2d-like cavity mode



The Upshot

To design an interesting device, you need only:

symmetry

+ single-mode (usually)

+ resonance

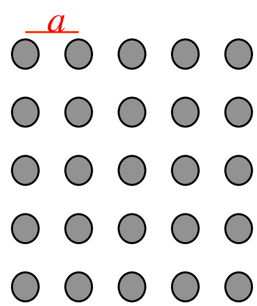
+ (ideally) a band gap to forbid losses

Oh, and a full Maxwell simulator to get Q parameters, *etcetera*.

Outline

- Preliminaries: waves in periodic media
- Photonic crystals in theory and practice
- Bulk crystal properties
- Intentional defects and devices
- Index-guiding and incomplete gaps
- Photonic-crystal fibers
- Perturbations, tuning, and disorder

Review: Bloch Basics



Waves in periodic media can have:

- propagation with no scattering (conserved **k**)
- photonic band gaps (with proper ε function)

Eigenproblem gives simple insight:

Bloch form: $$\vec{H} = e^{i(\vec{k}\cdot\vec{x} - \omega t)} \vec{H}_{\vec{k}}(\vec{x})$$

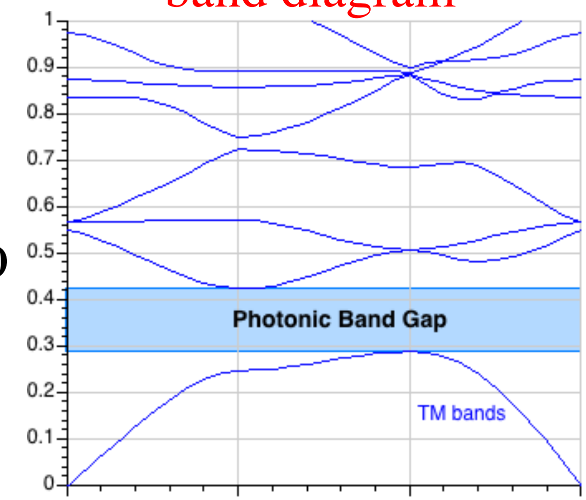
$$\underline{\left[(\vec{\nabla} + i\vec{k}) \times \frac{1}{\varepsilon} (\vec{\nabla} + i\vec{k}) \times \right]} \vec{H}_{\vec{k}} = \left(\frac{\omega_n(\vec{k})}{c} \right)^2 \vec{H}_{\vec{k}}$$

$\hat{\Theta}_{\vec{k}}$

Hermitian –> complete, orthogonal, variational theorem, *etc*.

band diagram

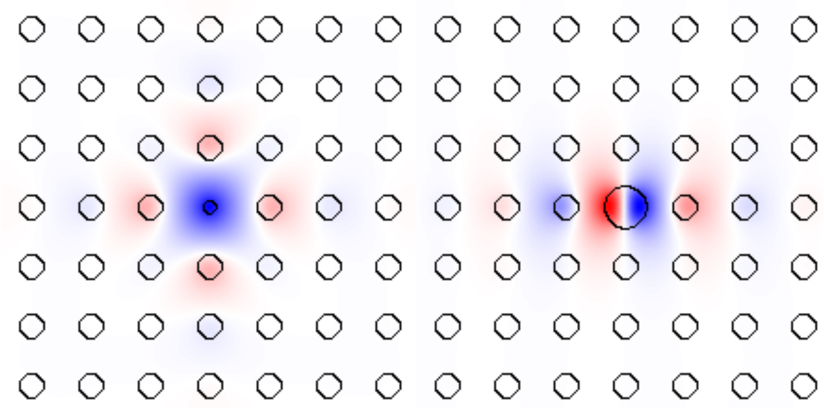
ω



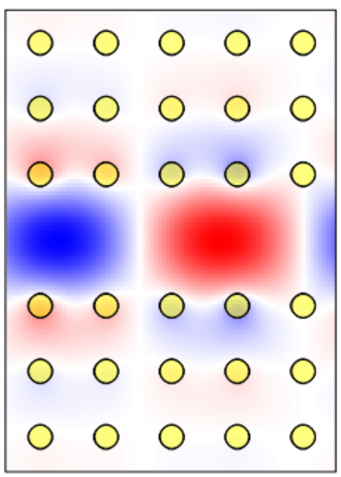
k

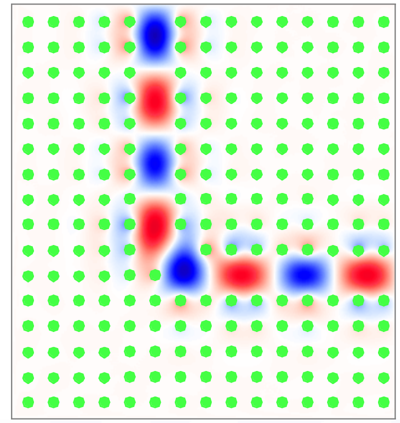
Review: Defects and Devices

Point defects = Cavities

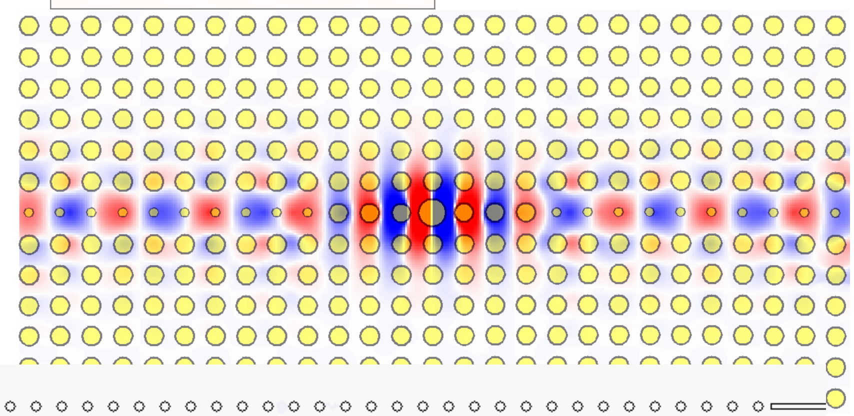


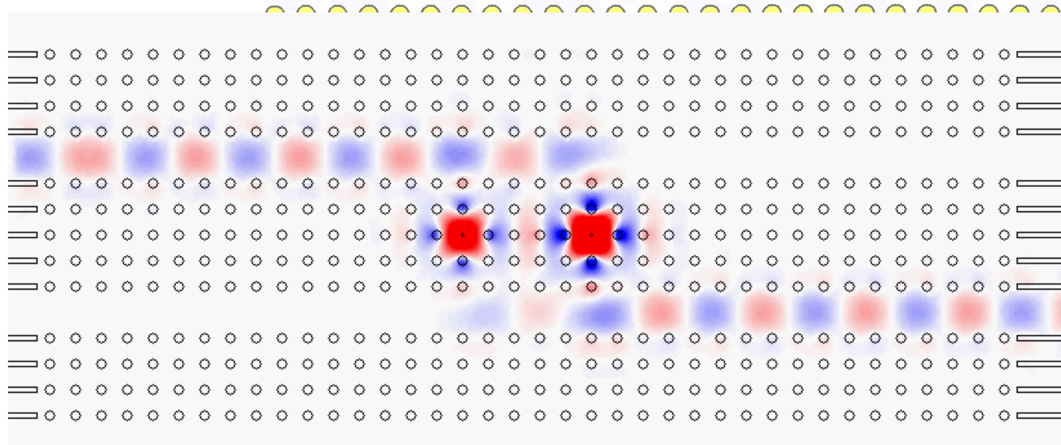
Line defects = Waveguides



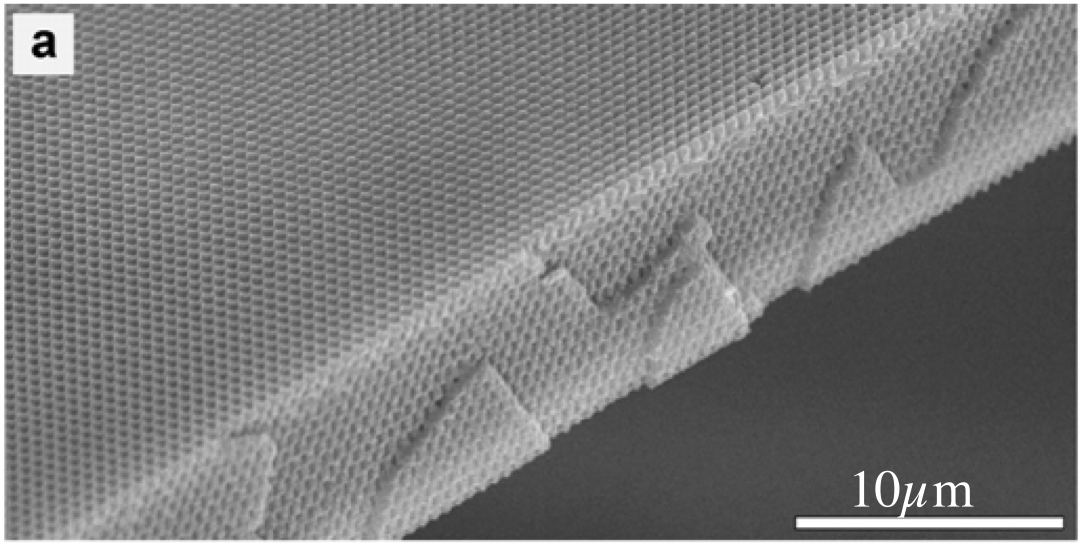


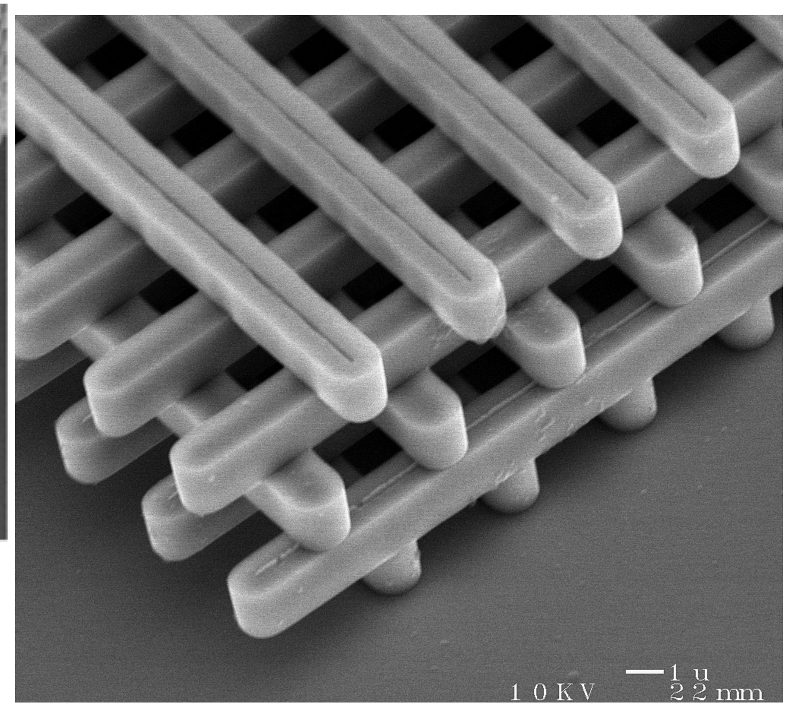
Waveguides
+
Resonant
Cavities

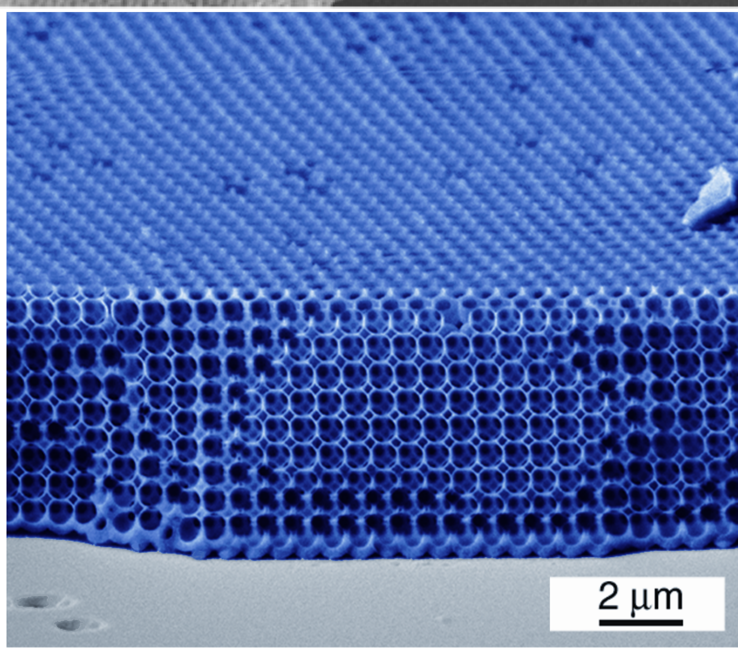




Review: 3d Crystals and Fabrication



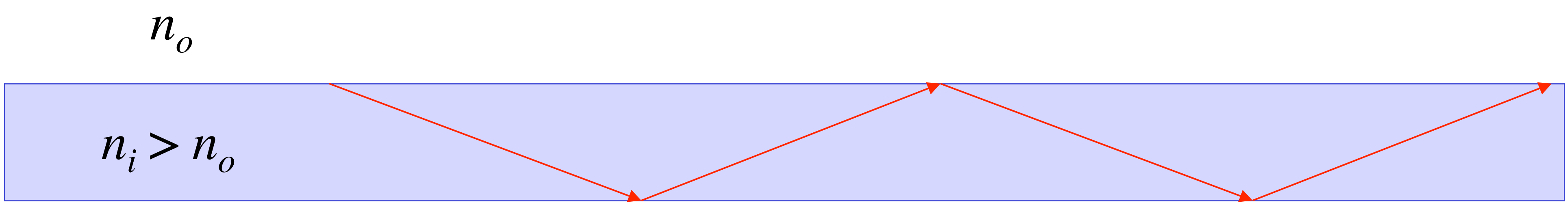




Much progress
in making complex structures
…
incorporation of defects & devices
still in early stages

How *else* can we confine light?

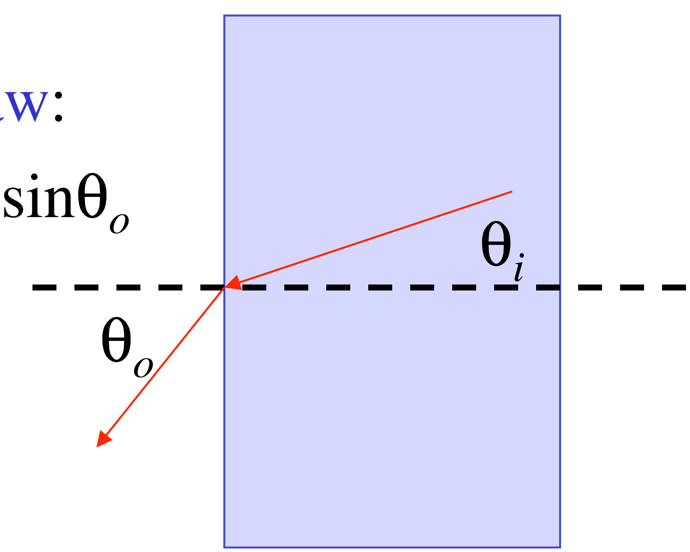
Total Internal Reflection



rays at shallow angles $> \theta_c$
are totally reflected

Snell's Law:

$n_i \sin\theta_i = n_o \sin\theta_o$

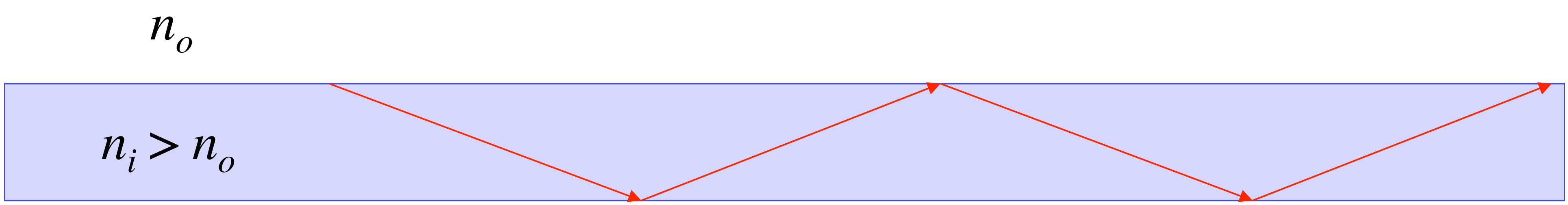


$\sin\theta_c = n_o / n_i$

< 1, so θ_c is real

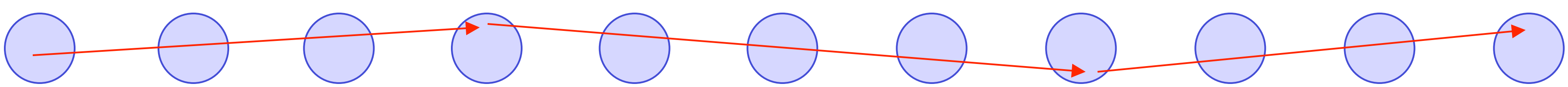
i.e. TIR can only guide
within higher index
unlike a band gap

Total Internal Reflection?



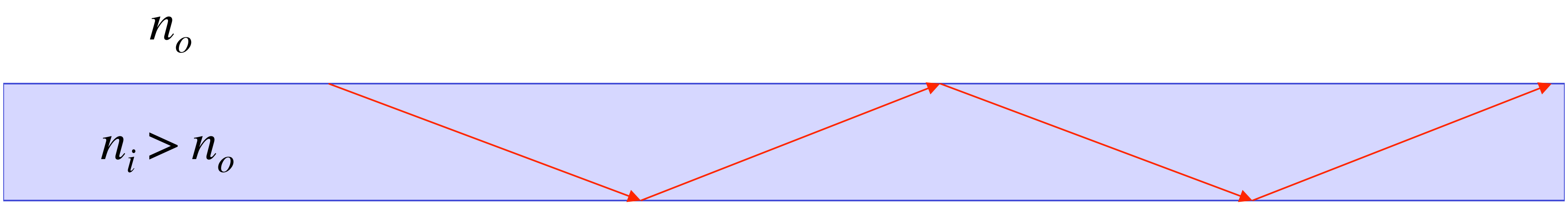
rays at shallow angles $> \theta_c$
are totally reflected

So, for example,
a discontiguous structure can't possibly guide by TIR…



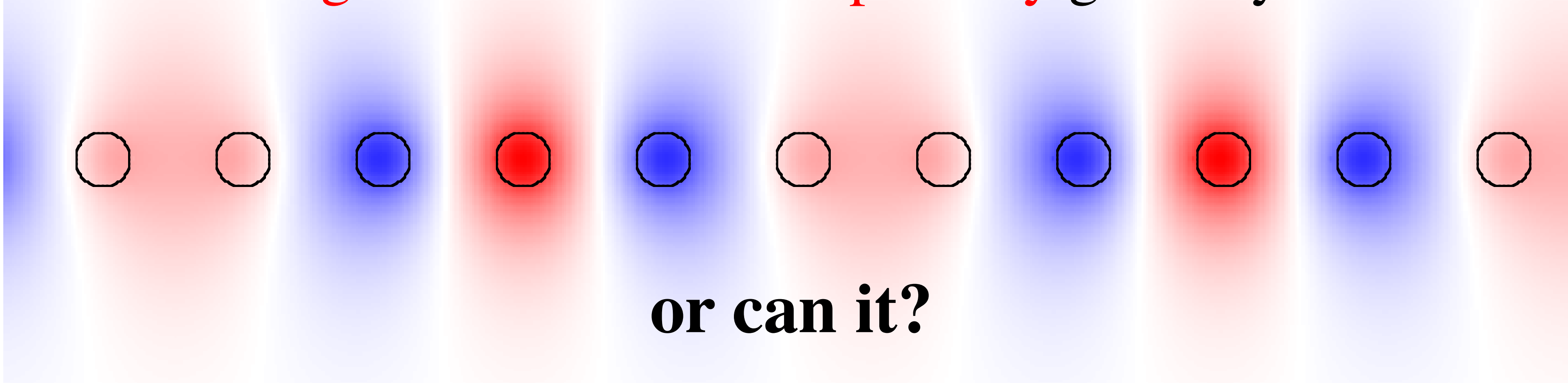
the rays can't stay inside!

Total Internal Reflection?



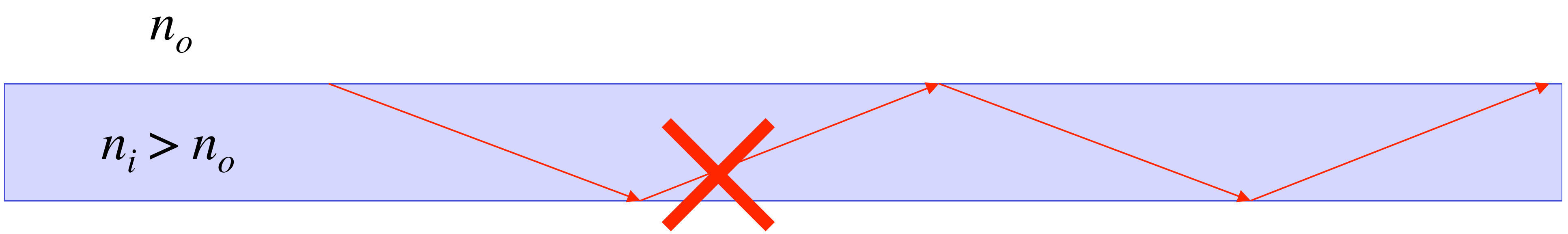
rays at shallow angles $> \theta_c$
are totally reflected

So, for example,
a discontiguous structure can't possibly guide by TIR…

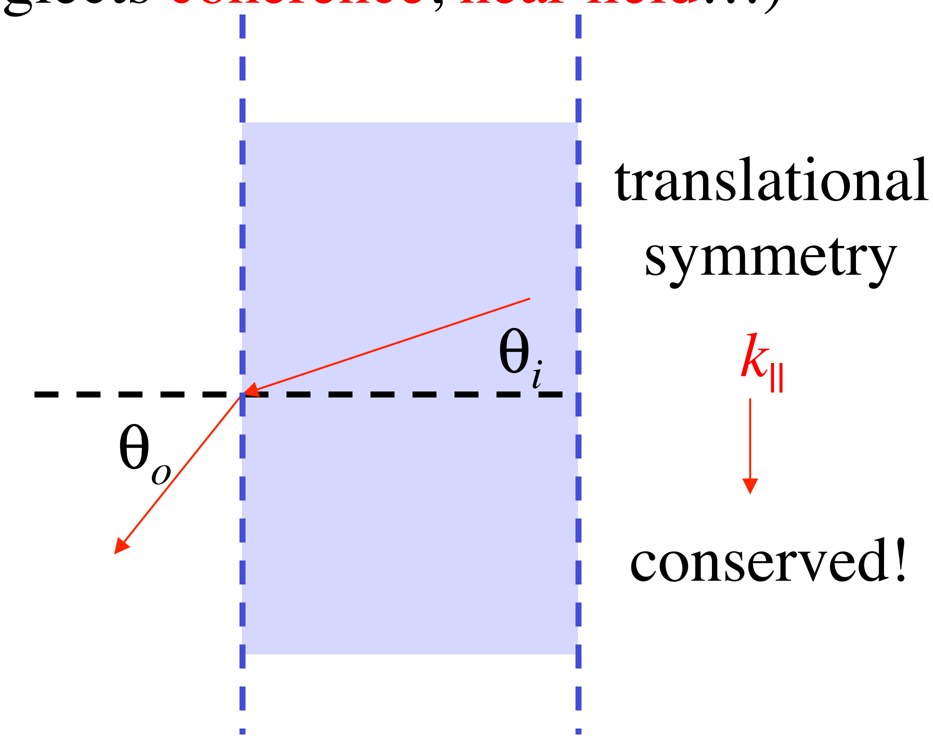


or can it?

Total Internal Reflection Redux



ray-optics picture is invalid on λ scale
(neglects coherence, near field…)



Snell's Law is really conservation of $k_{||}$ and ω:

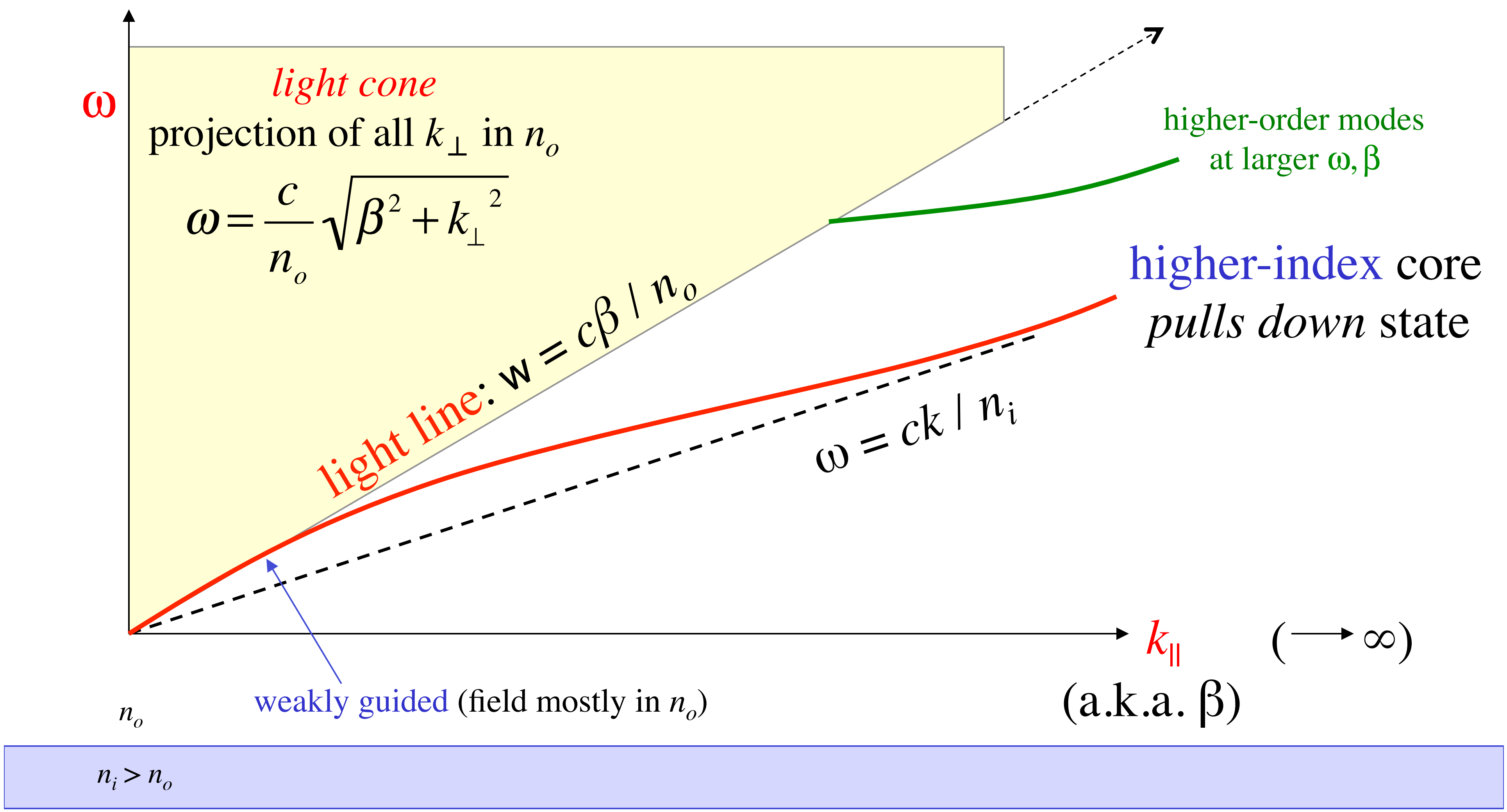
$$|k_i| \sin\theta_i = |k_o| \sin\theta_o$$

$$|k| = n\omega/c$$

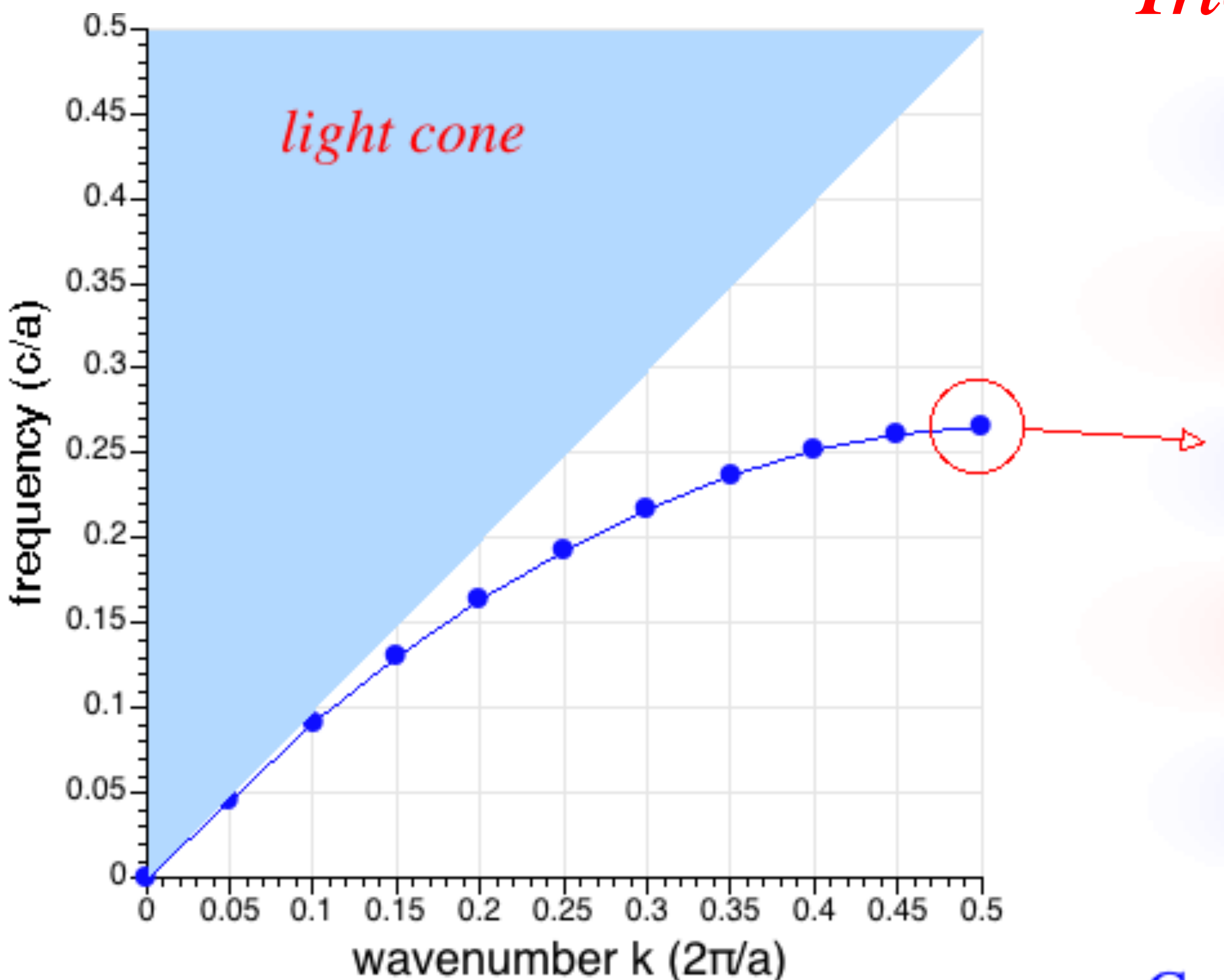
(wavevector) (frequency)

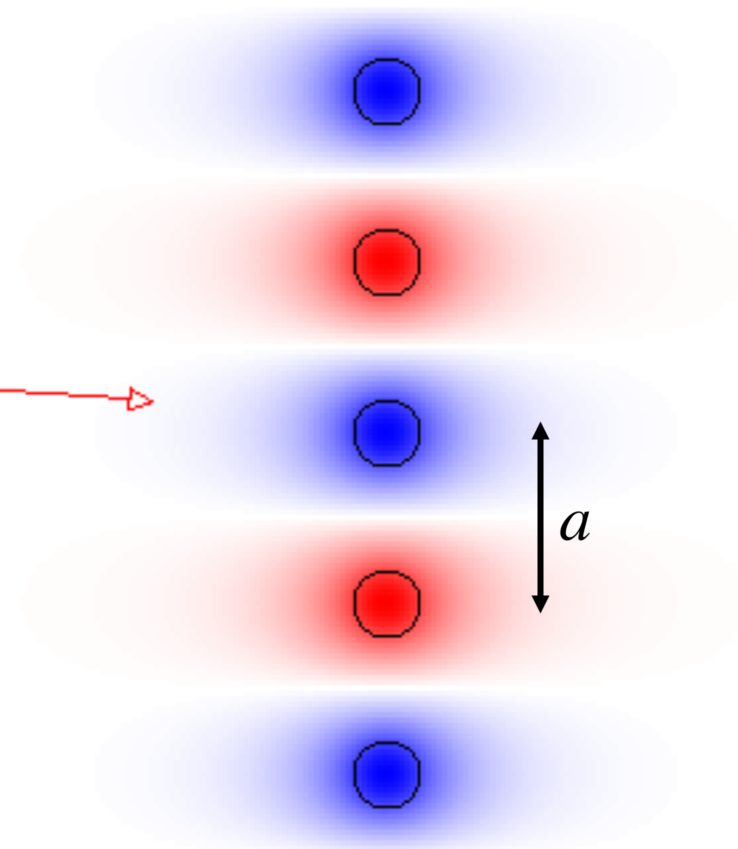
Waveguide Dispersion Relations

i.e. projected band diagrams



Strange ~~Total Internal Reflection~~ *Index Guiding*

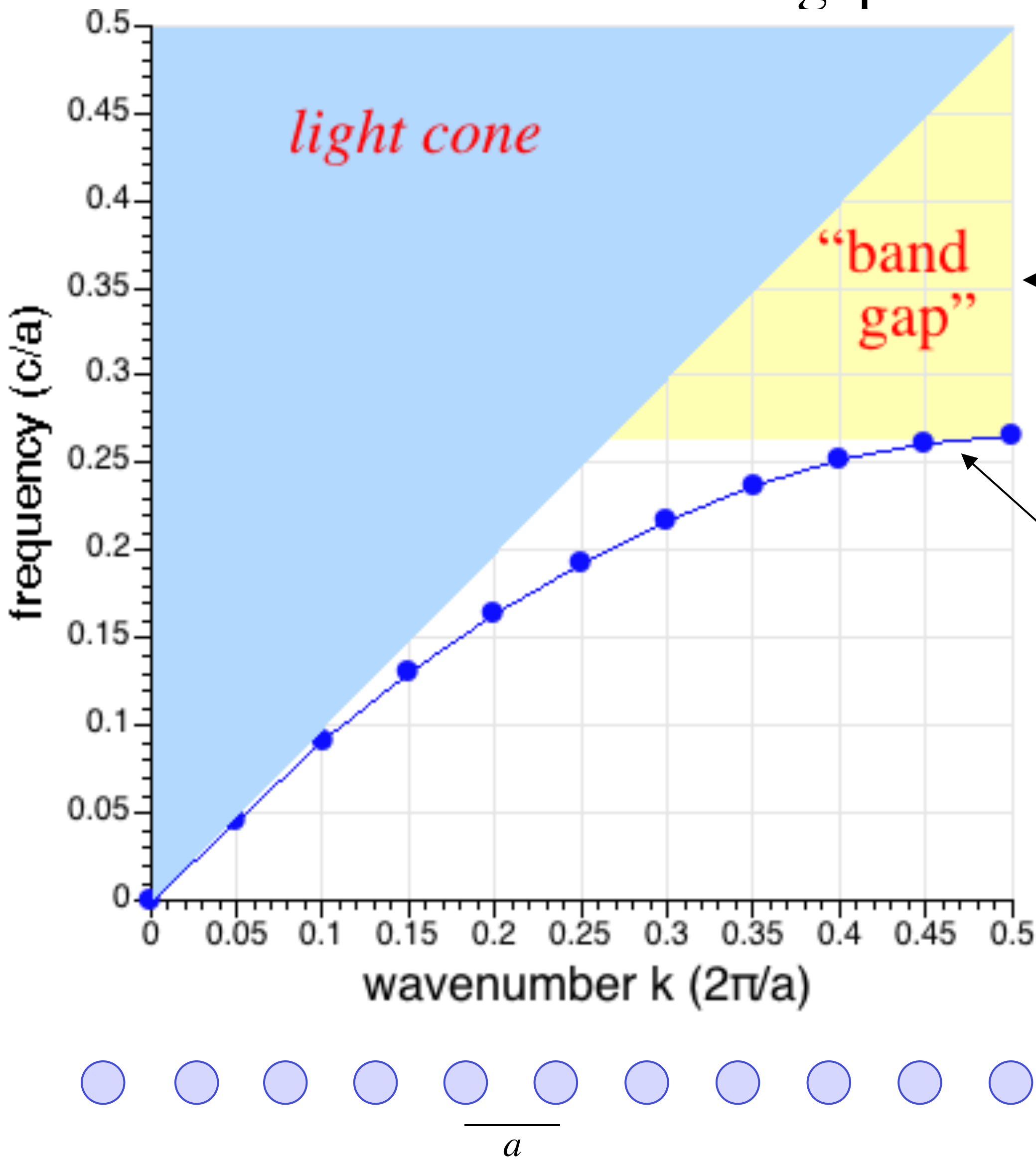




Conserved k and ω
+ higher index to pull down state
= localized/guided mode.

A Hybrid Photonic Crystal:

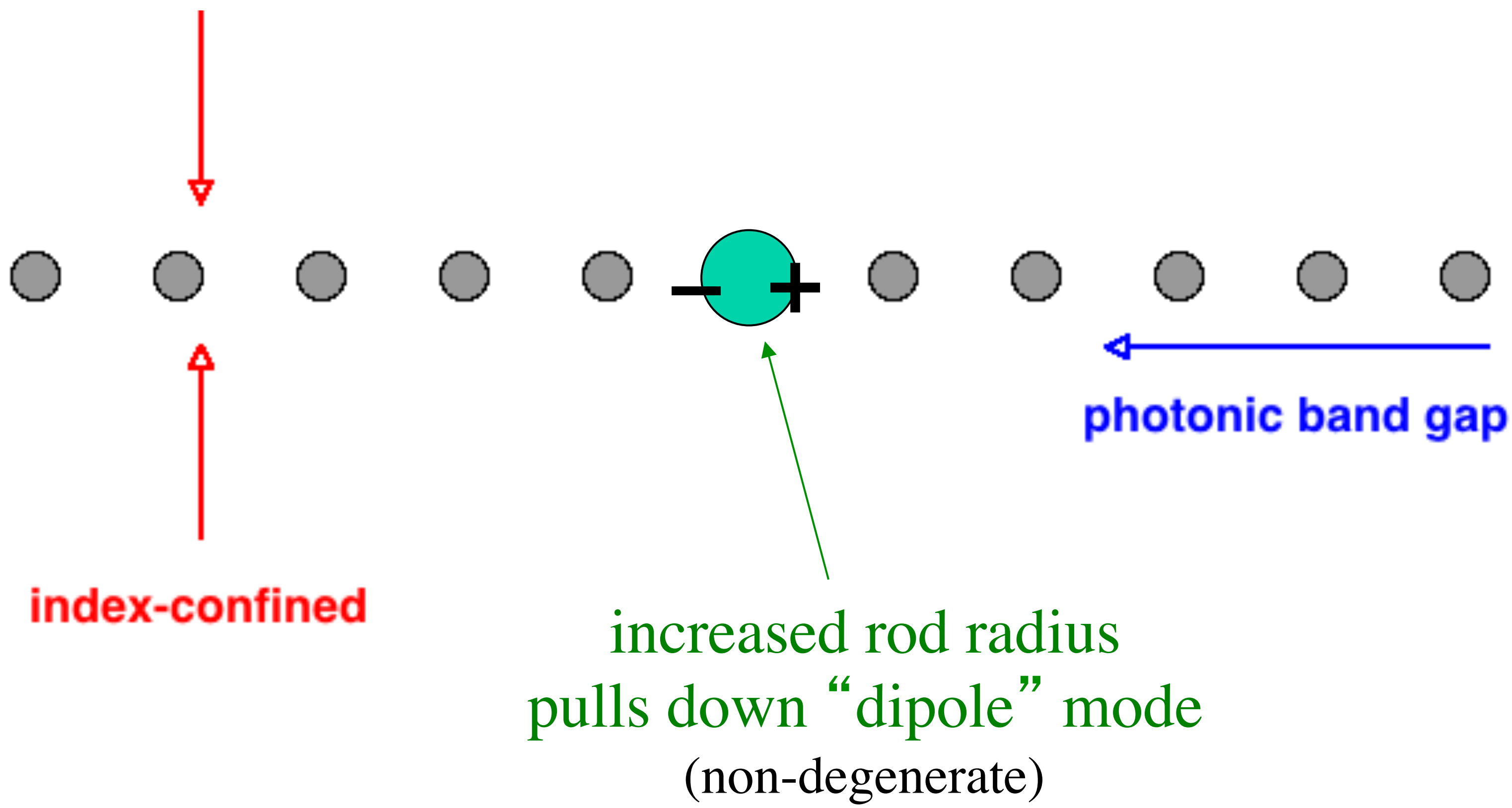
1d band gap + index guiding



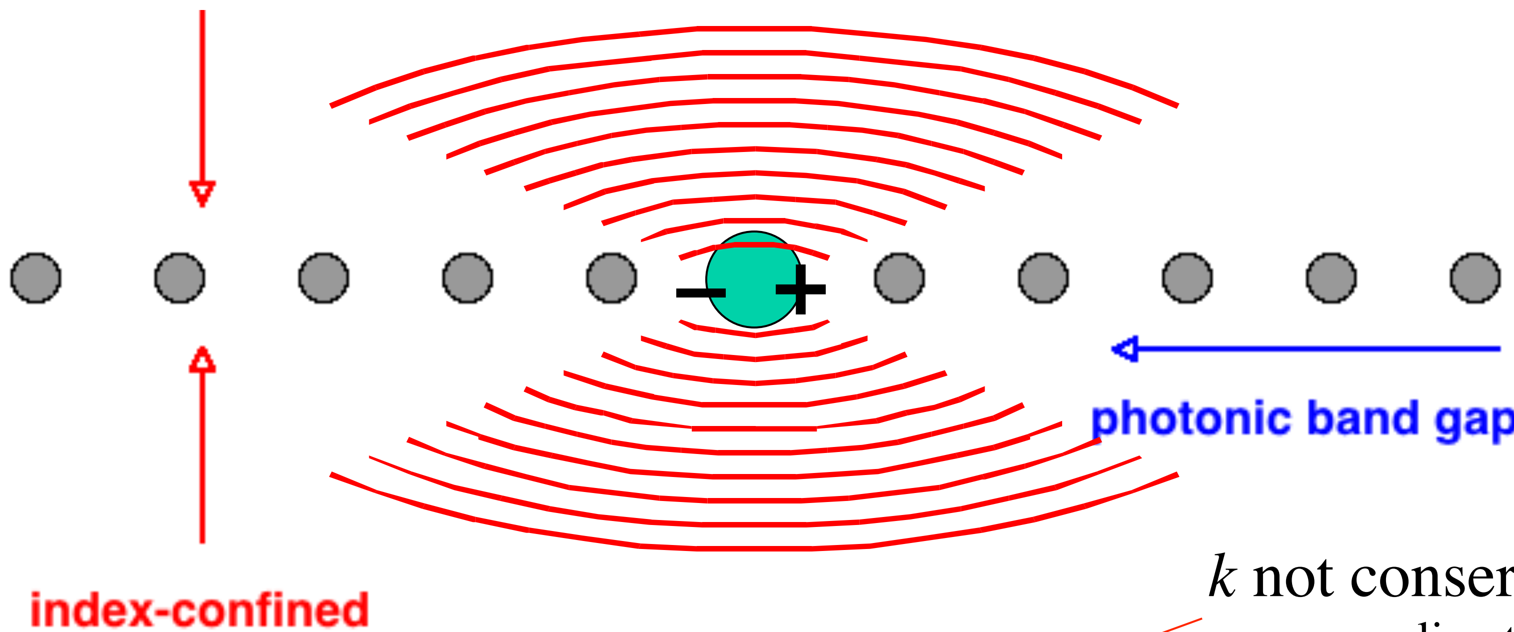
range of frequencies in which there are no guided modes

slow-light band edge

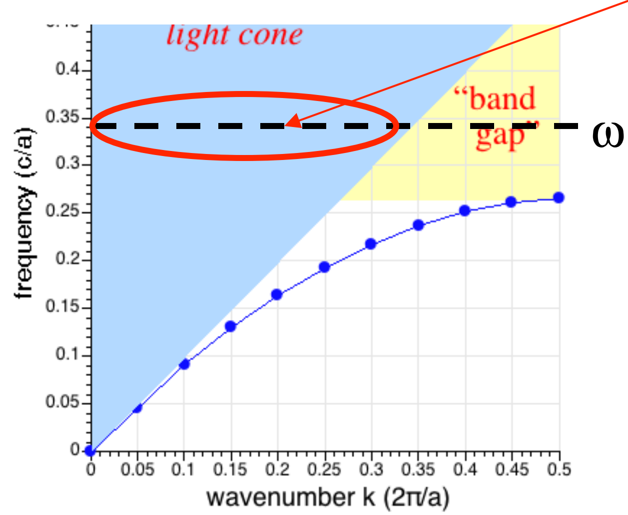
A Resonant Cavity



A Resonant Cavity



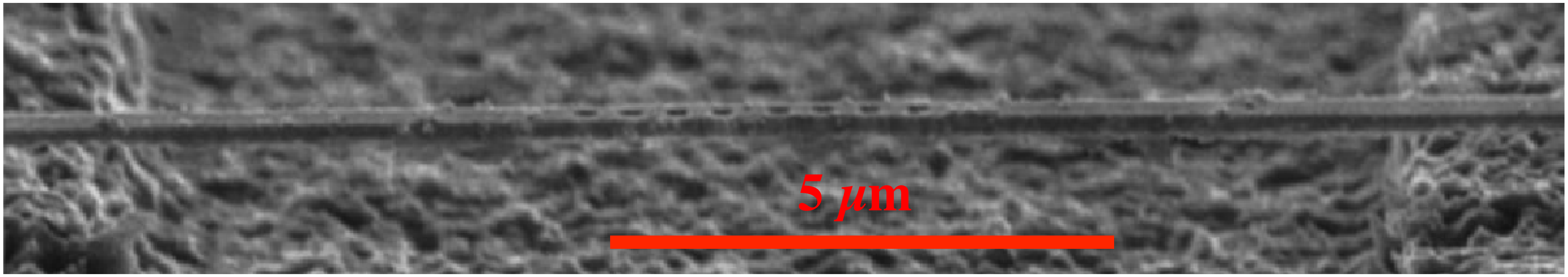
k not conserved
so coupling to
light cone:
radiation

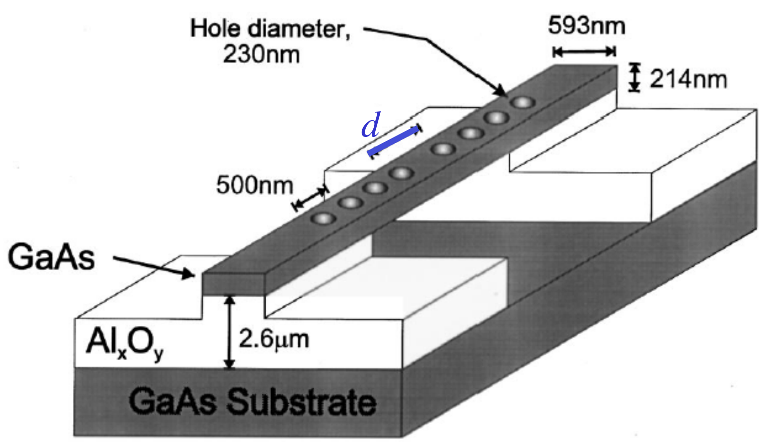


The trick is to keep the radiation small…
(more on this later)

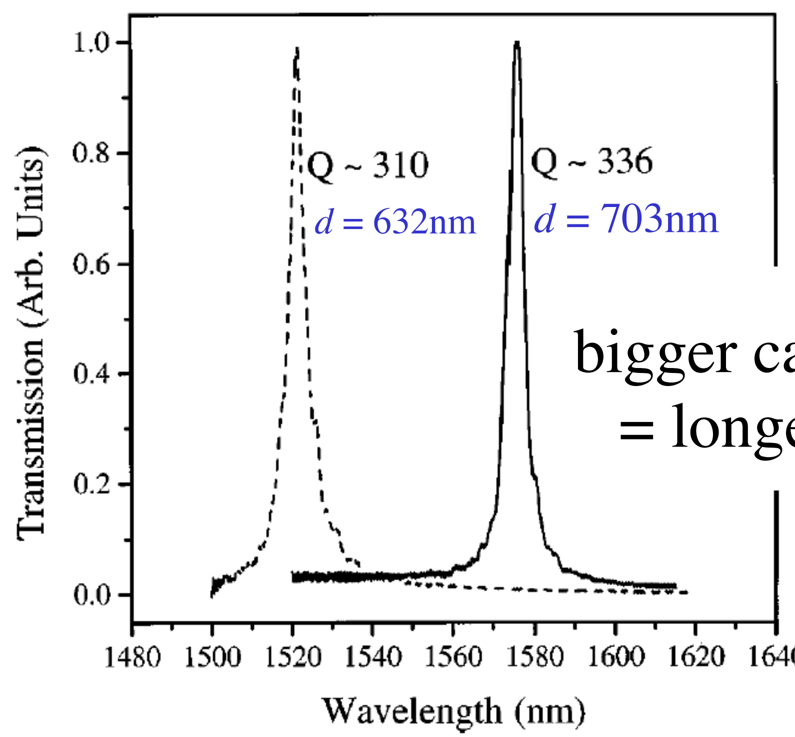
Meanwhile, back in reality…

Air-bridge Resonator: 1d gap + 2d index guiding





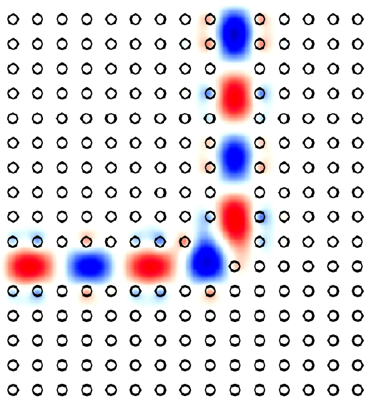
[D. J. Ripin *et al*., *J. Appl. Phys*. **87**, 1578 (2000)]

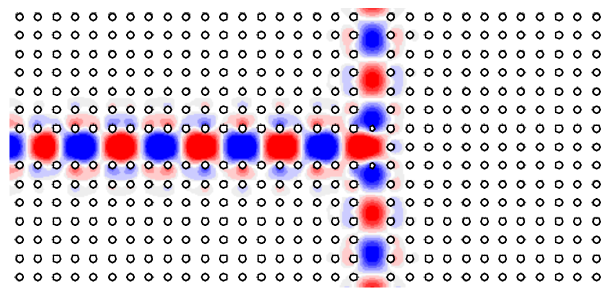


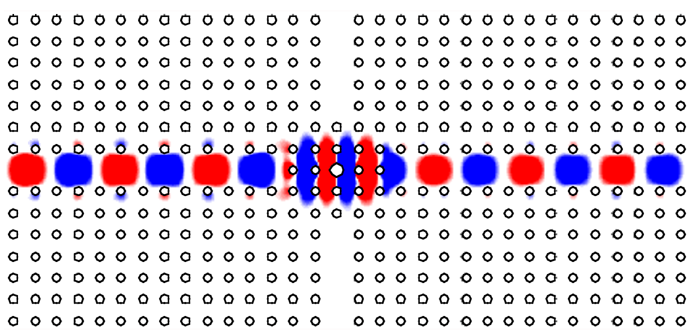
bigger cavity
= longer λ

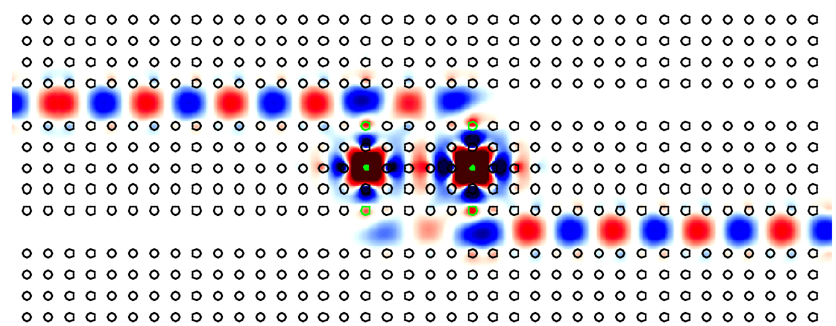
Time for Two Dimensions…

2d is all we really need for many interesting devices
…darn z direction!

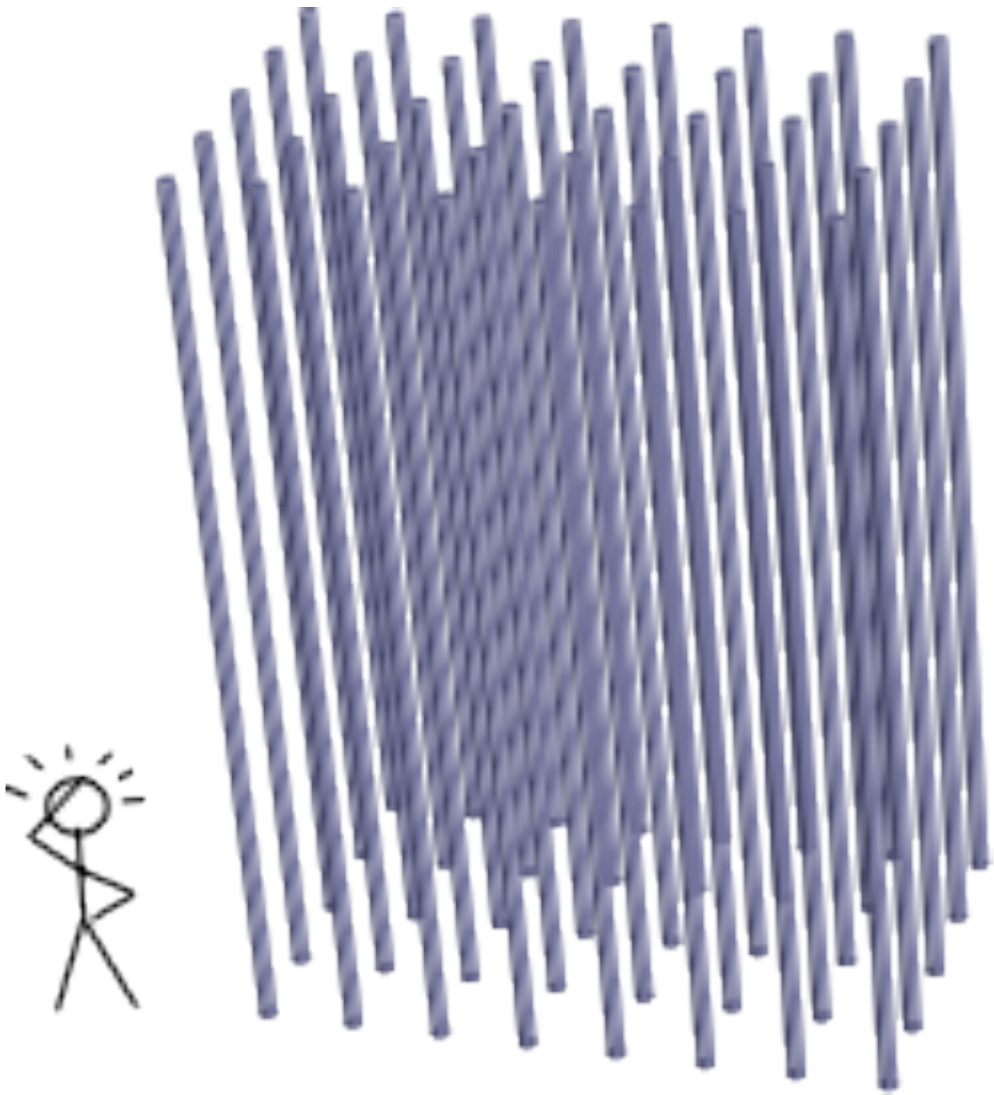








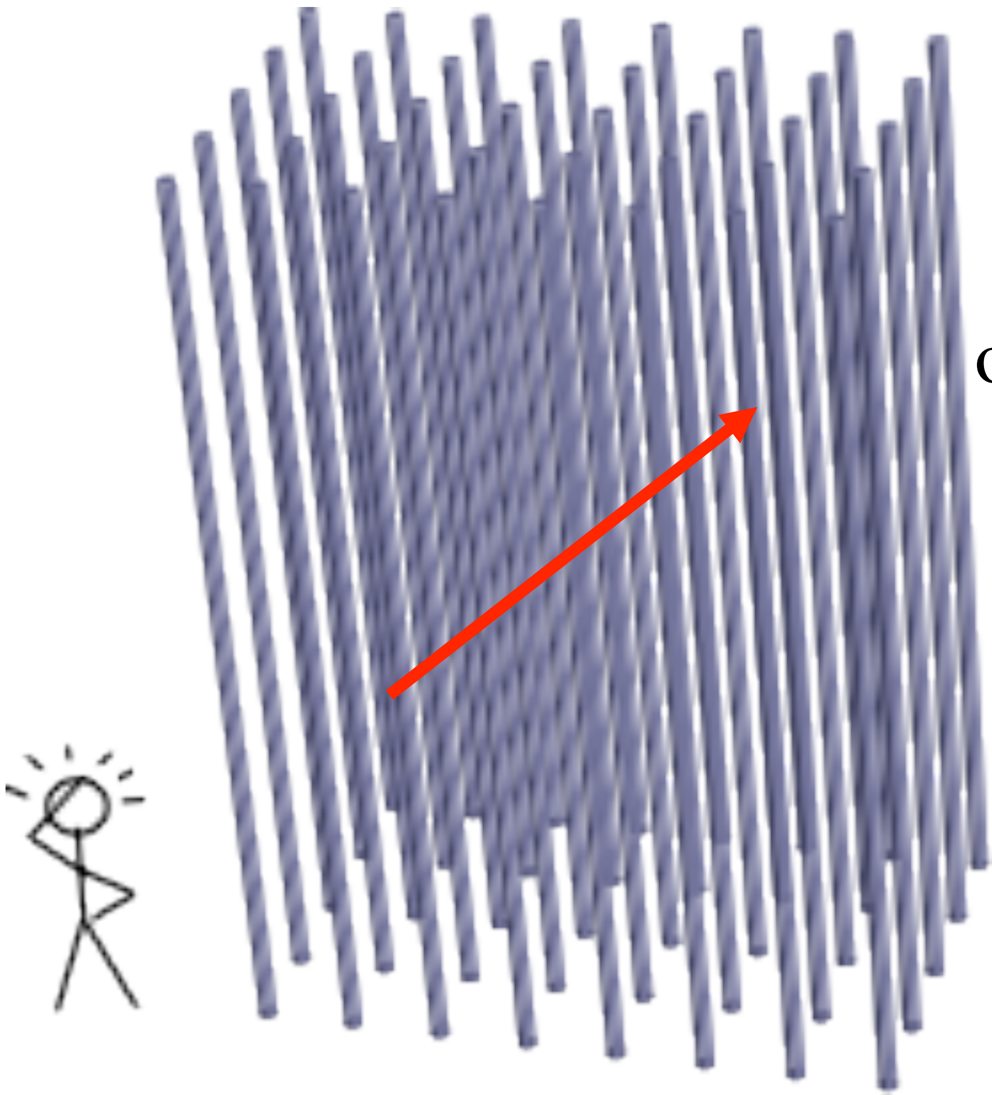
How do we make a 2d bandgap?



Most obvious solution?

make
2d pattern
really tall

How do we make a 2d bandgap?



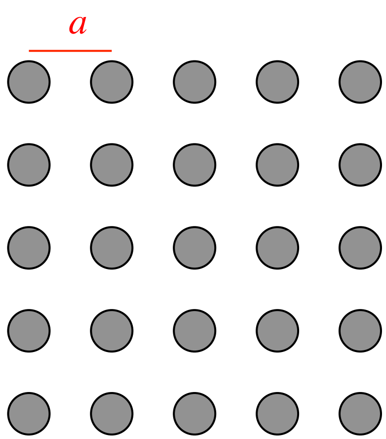
If height is finite,
we must couple to
out-of-plane wavevectors…

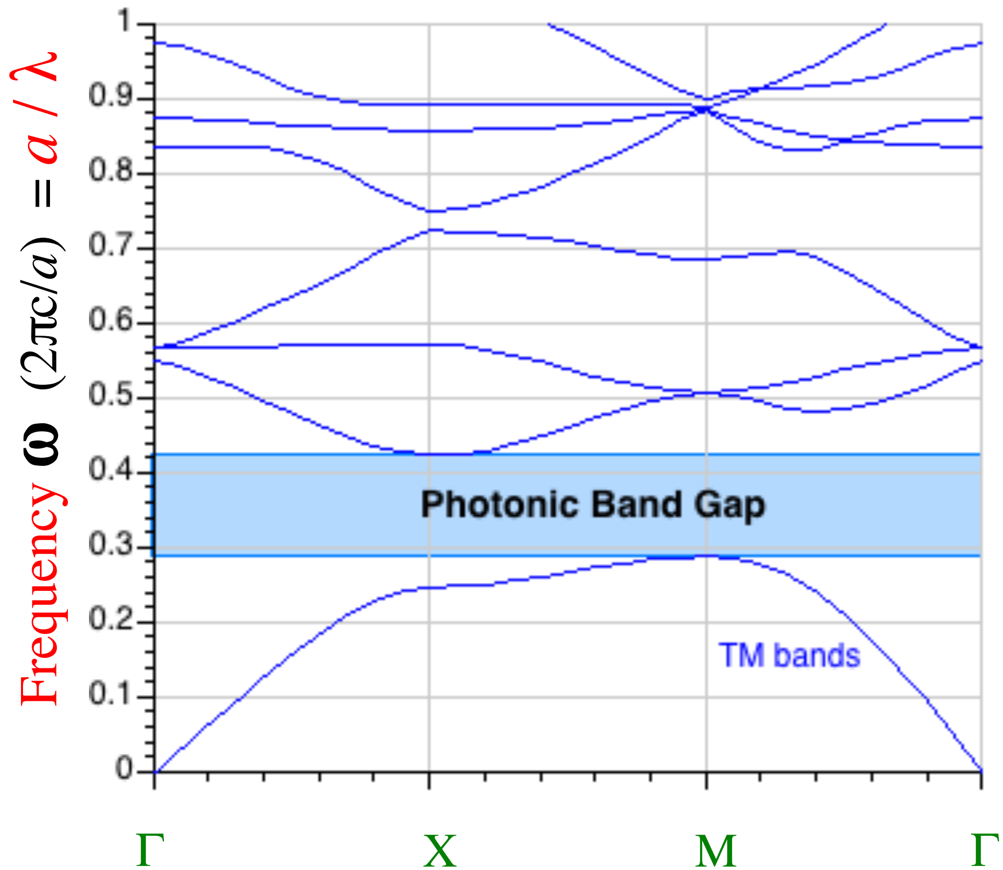
k_z not conserved

A 2d band diagram in 3d?

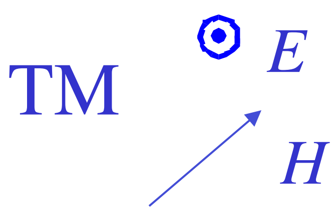
Recall the 2d band diagram:

…*what happens in 3d?*



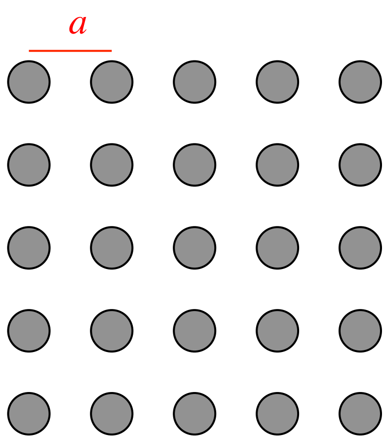


& what about polarization?

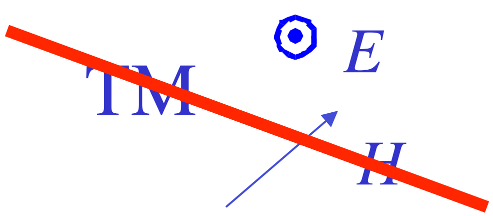


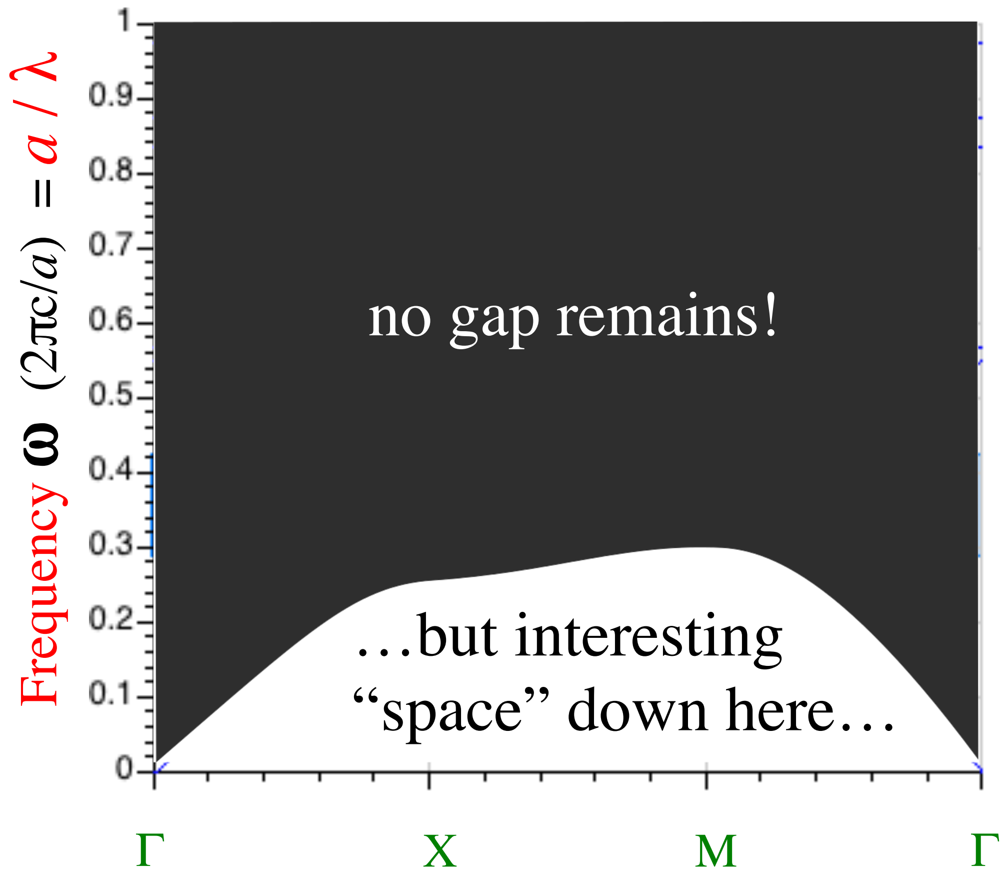
A 2d band diagram in 3d

In 3d, continuum of k_z
fills upwards from 1st band:

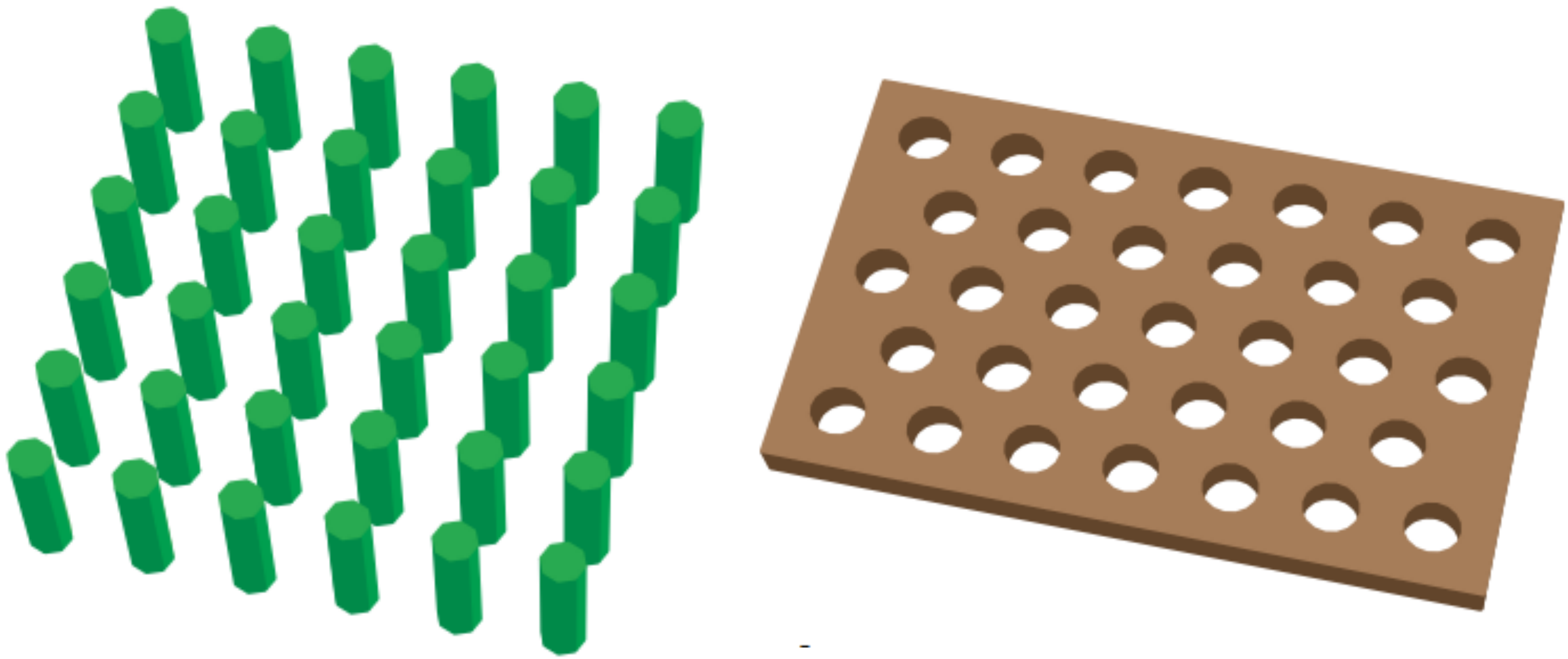


& pure polarizations disappear





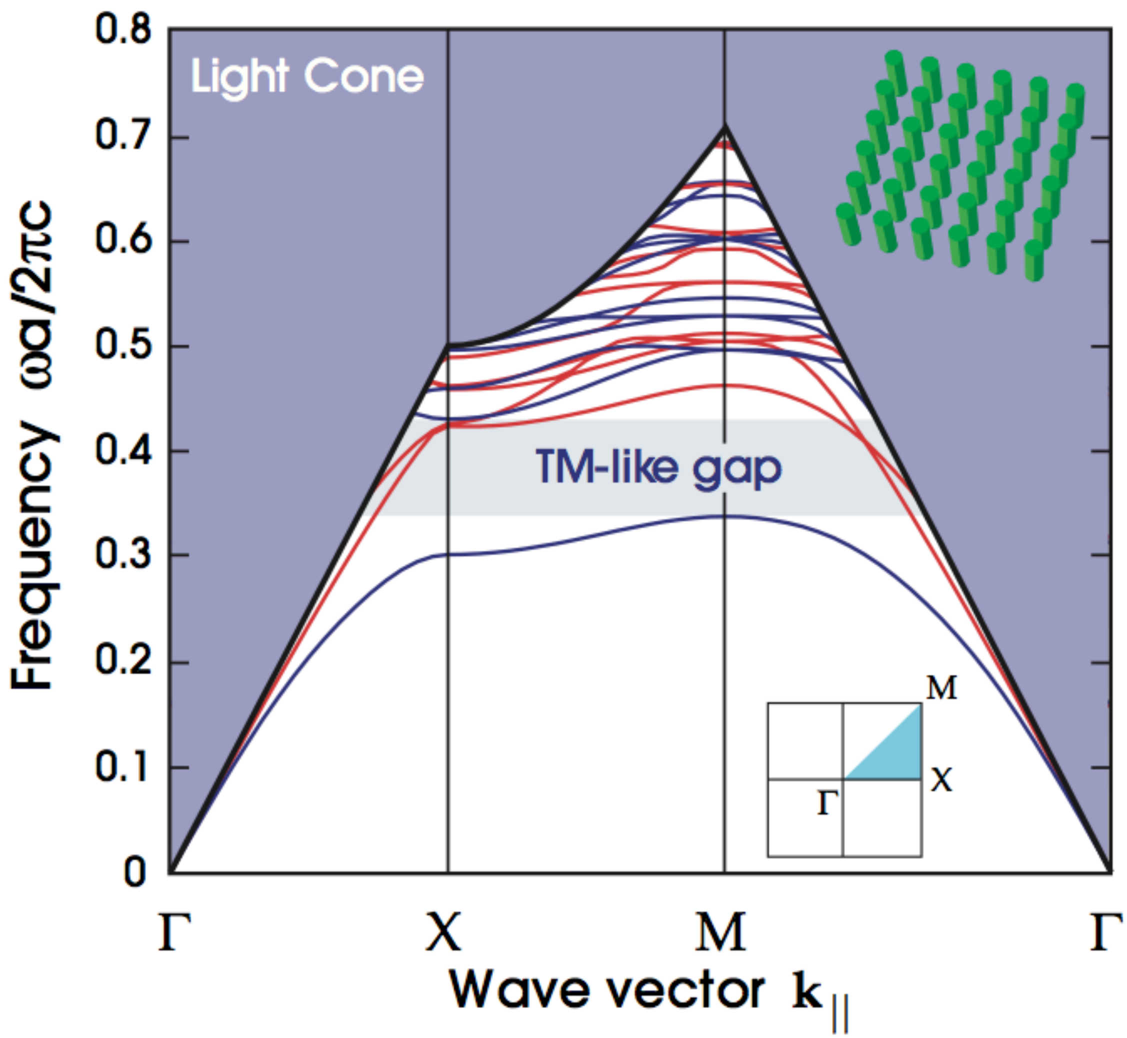
Photonic-Crystal Slabs



2d photonic bandgap + vertical index guiding

[J. D. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade, *Photonic Crystals: Molding the Flow of Light*, 2nd edition, chapter 8]

Rod-Slab Projected Band Diagram



Light cone = all solutions in medium above/below slab

Guided modes below light cone = no radiation

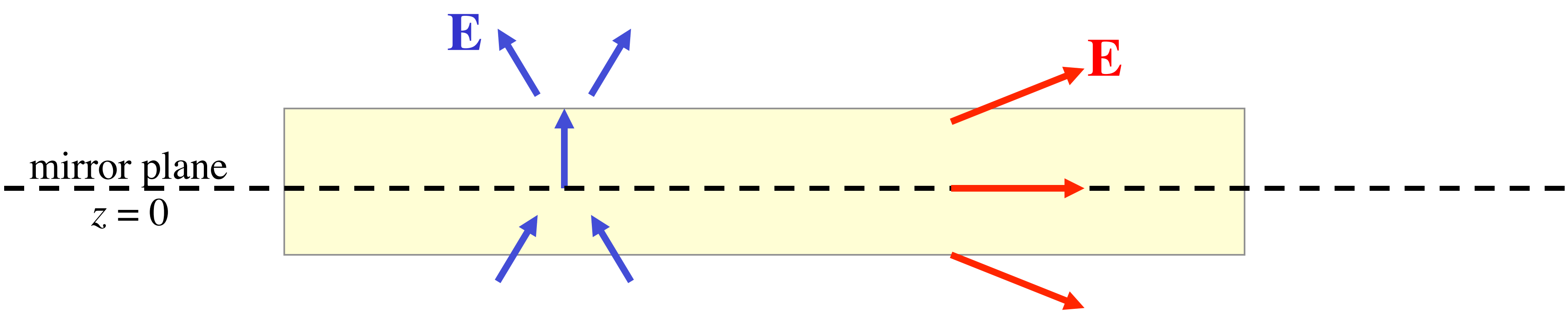
Two "polarizations:" TM-like & TE-like

"Gap" in guided modes … *not* a complete gap

Slab thickness is crucial to obtain gap…

Slab symmetry & "polarization"

2d: TM and TE modes



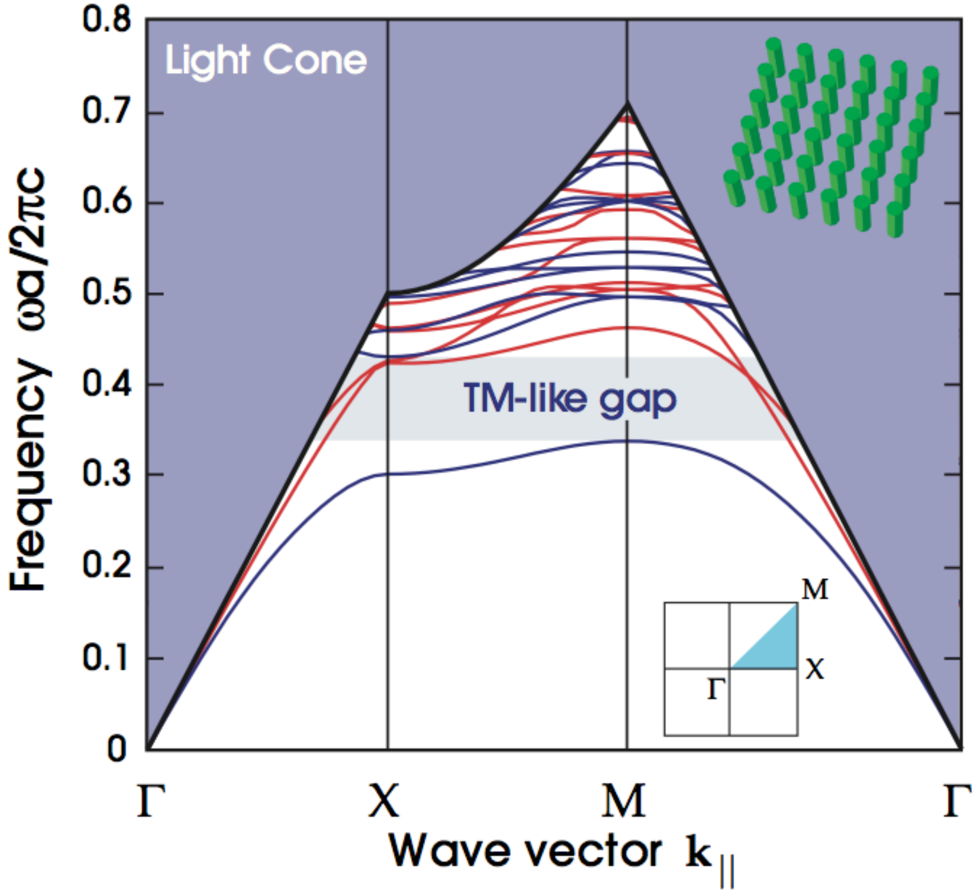
slab: odd (TM-like) and even (TE-like) modes

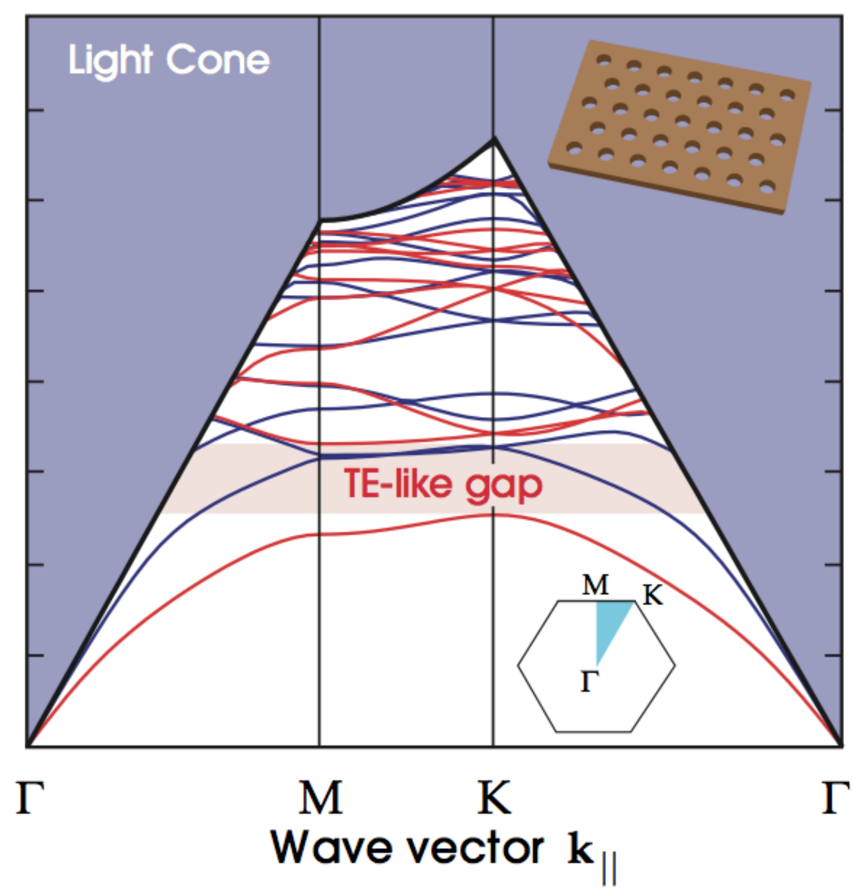
Like in 2d, there may only be a band gap in one symmetry/polarization

Slab Gaps

Rod slab

Hole slab





TM-like gap

TE-like gap

Substrates, for the Gravity-Impaired

(rods or holes)

superstrate restores symmetry

substrate breaks symmetry:
some even/odd mixing "kills" gap

BUT

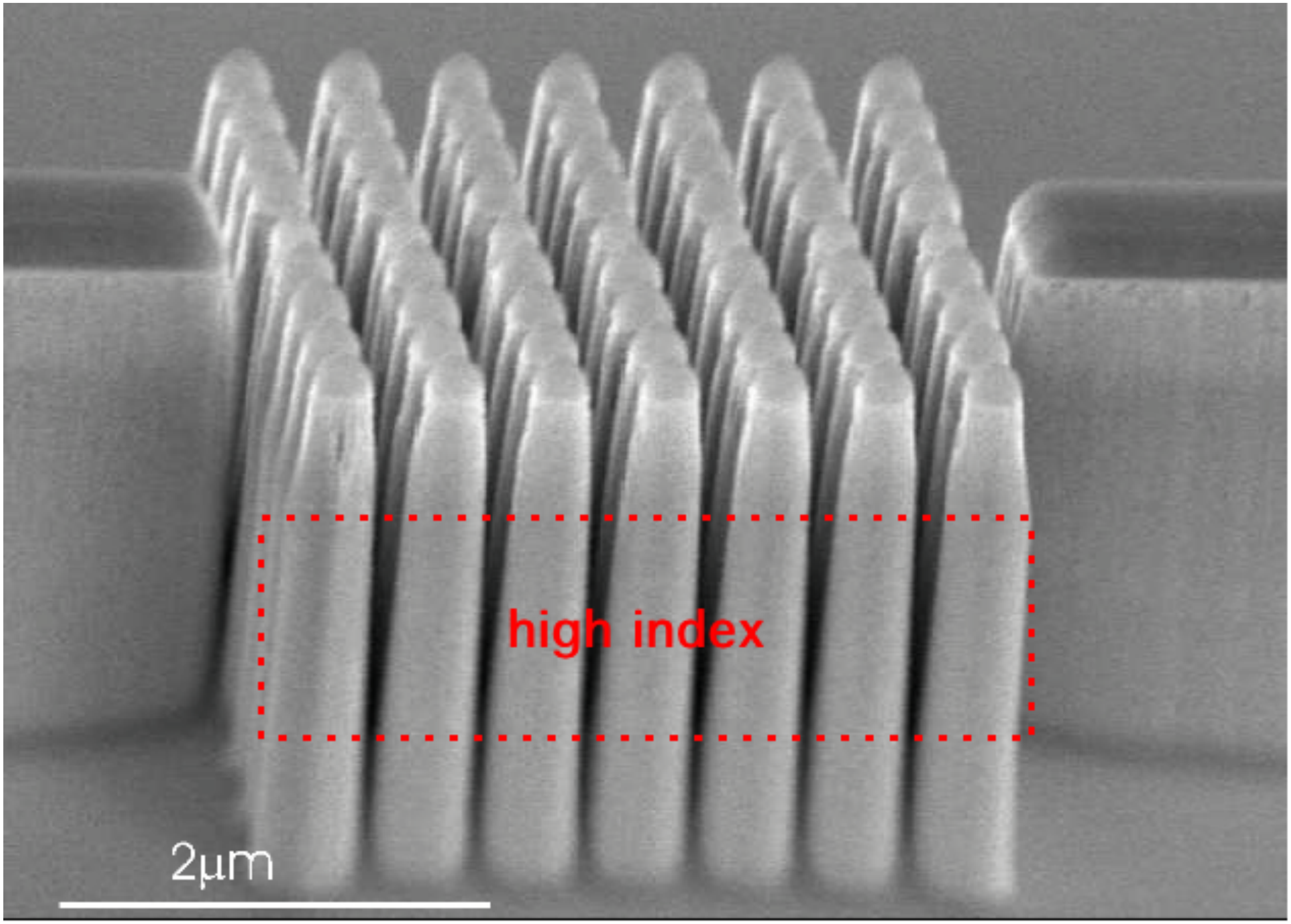
with strong confinement
(high index contrast)

mixing can be weak

"extruded" substrate
= stronger confinement

(less mixing even
without superstrate

Extruded Rod Substrate



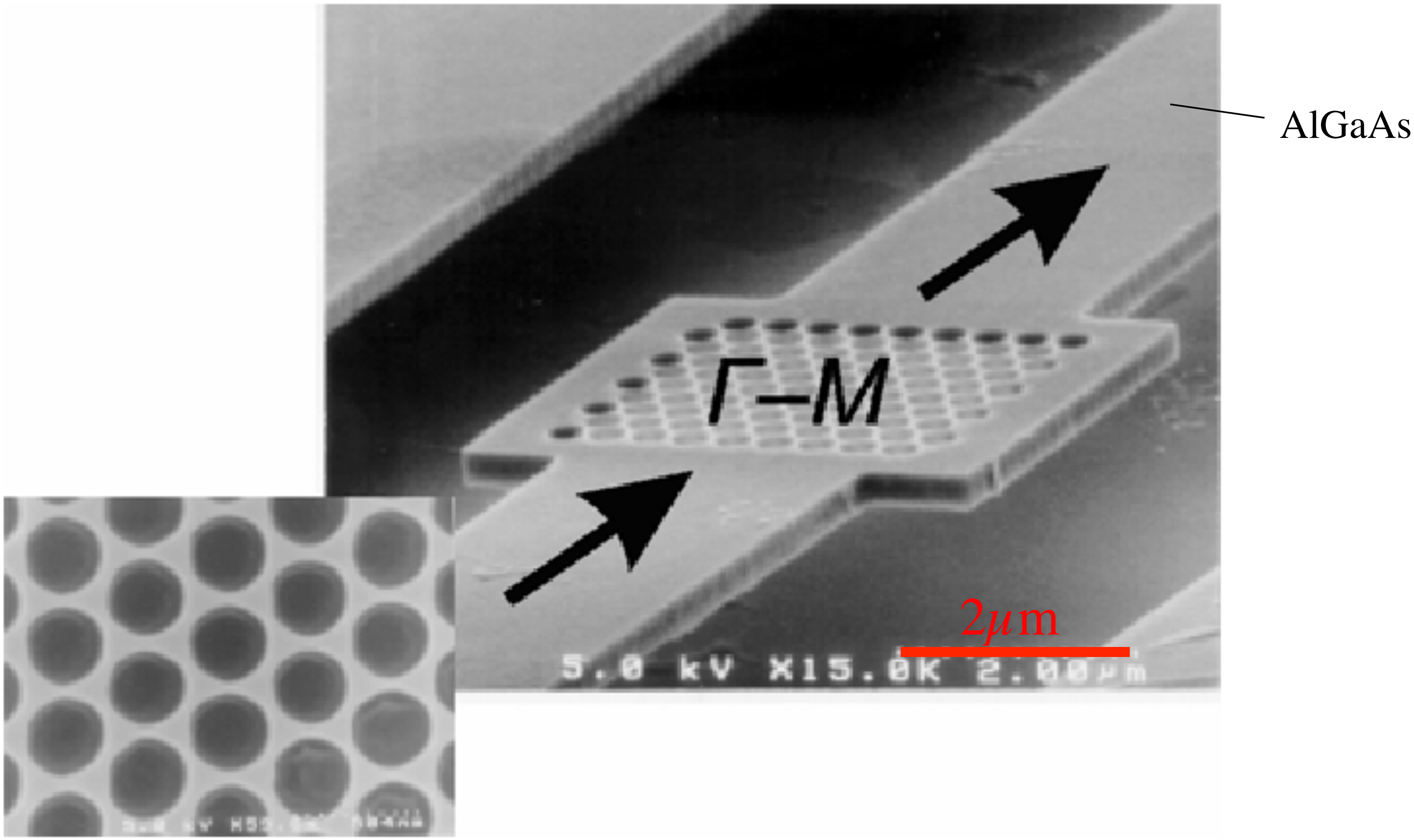
S. Assefa, L. A. Kolodziejski

(GaAs on AlO_x)

[S. Assefa *et al*., *APL* **85**, 6110 (2004).]

Air-membrane Slabs

who needs a substrate?

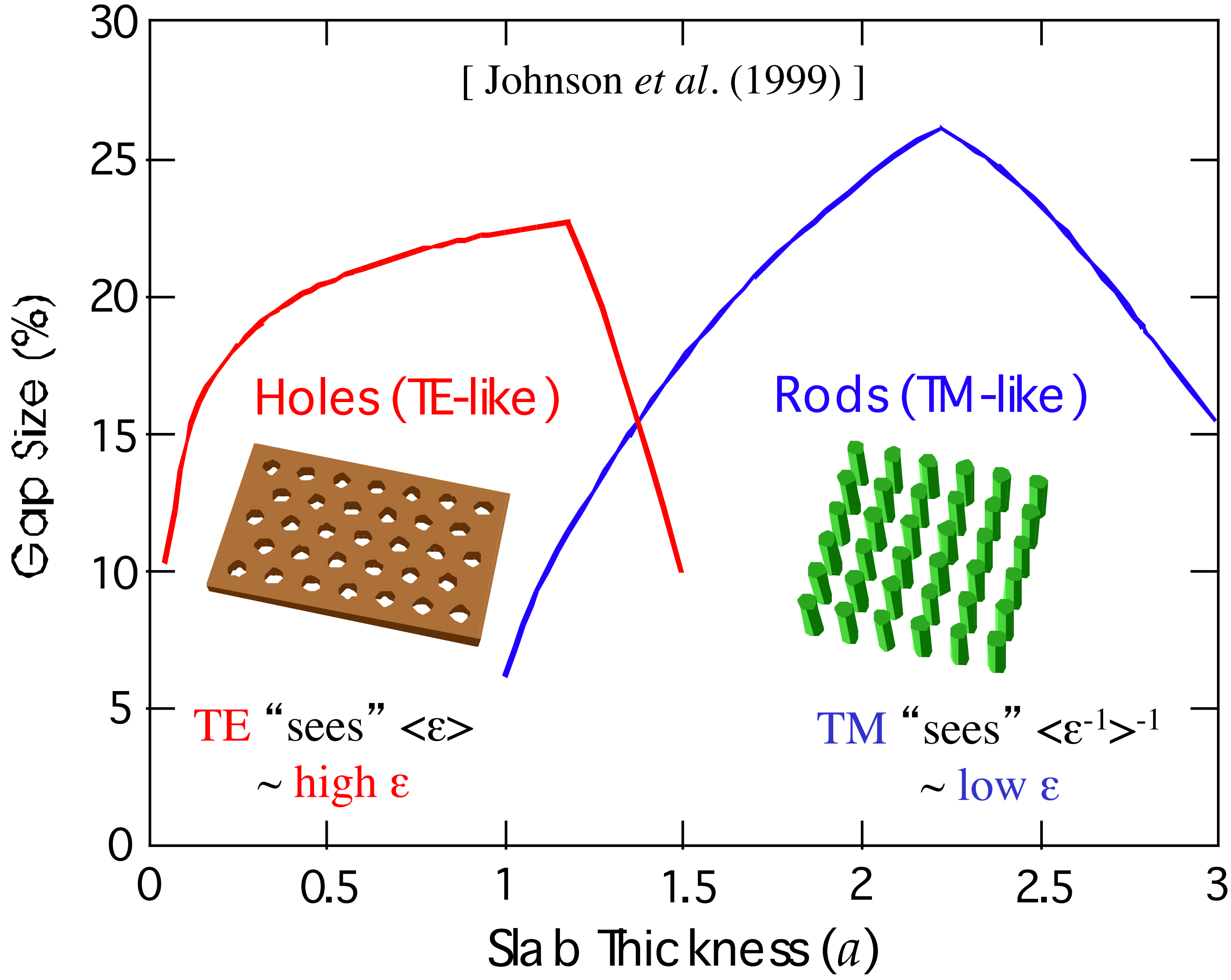


[N. Carlsson *et al*., *Opt*. *Quantum Elec*. **34**, 123 (2002)]

Optimal Slab Thickness

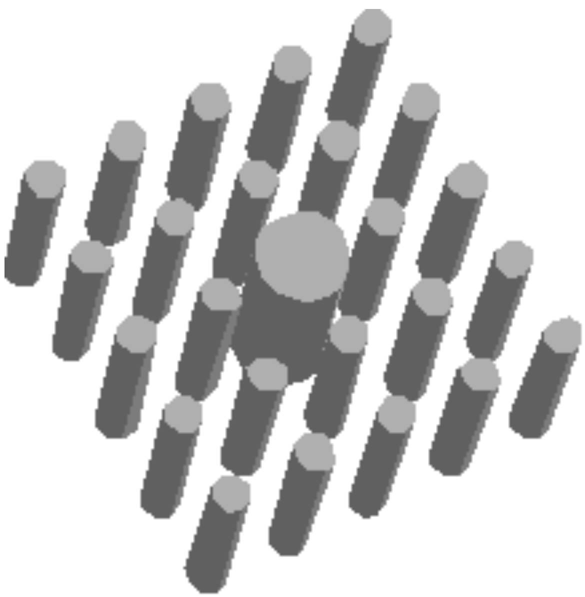
~ λ/2, but λ/2 in what material?

effective medium theory: effective ε depends on polarization

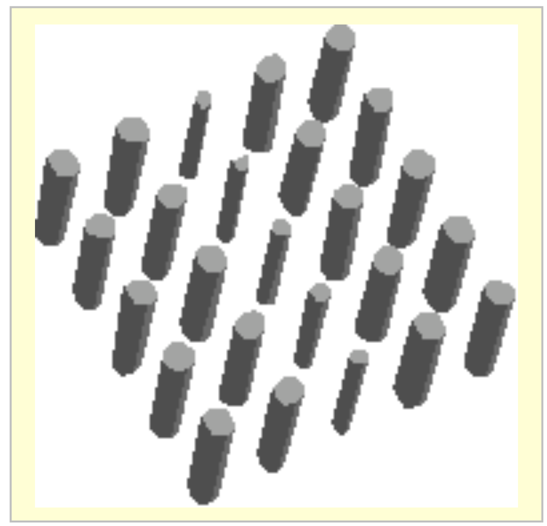


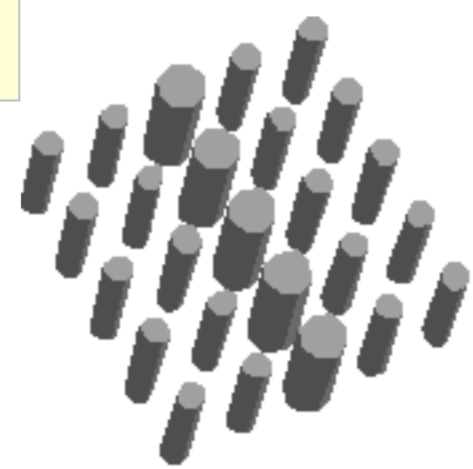
Photonic-Crystal Building Blocks

point defects
(cavities)

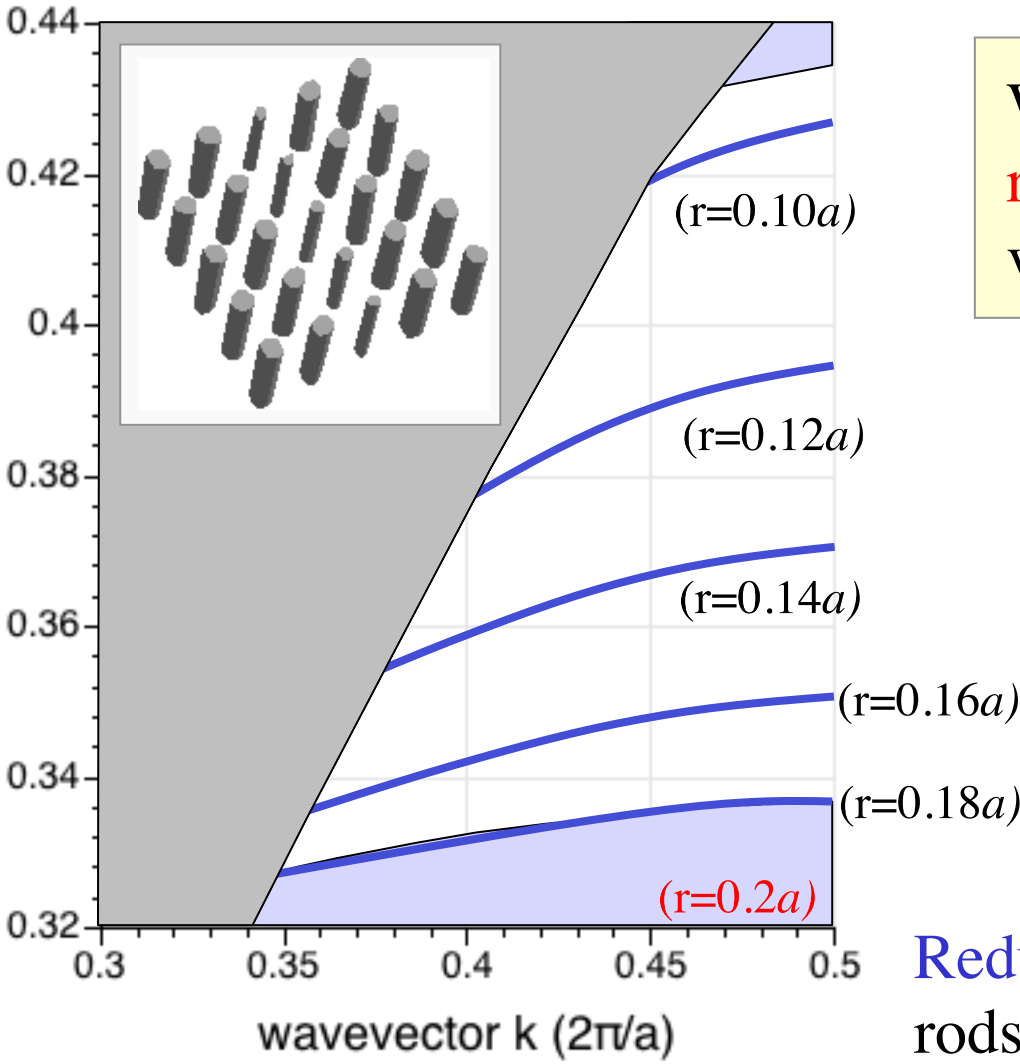


line defects
(waveguides)





A Reduced-Index Waveguide

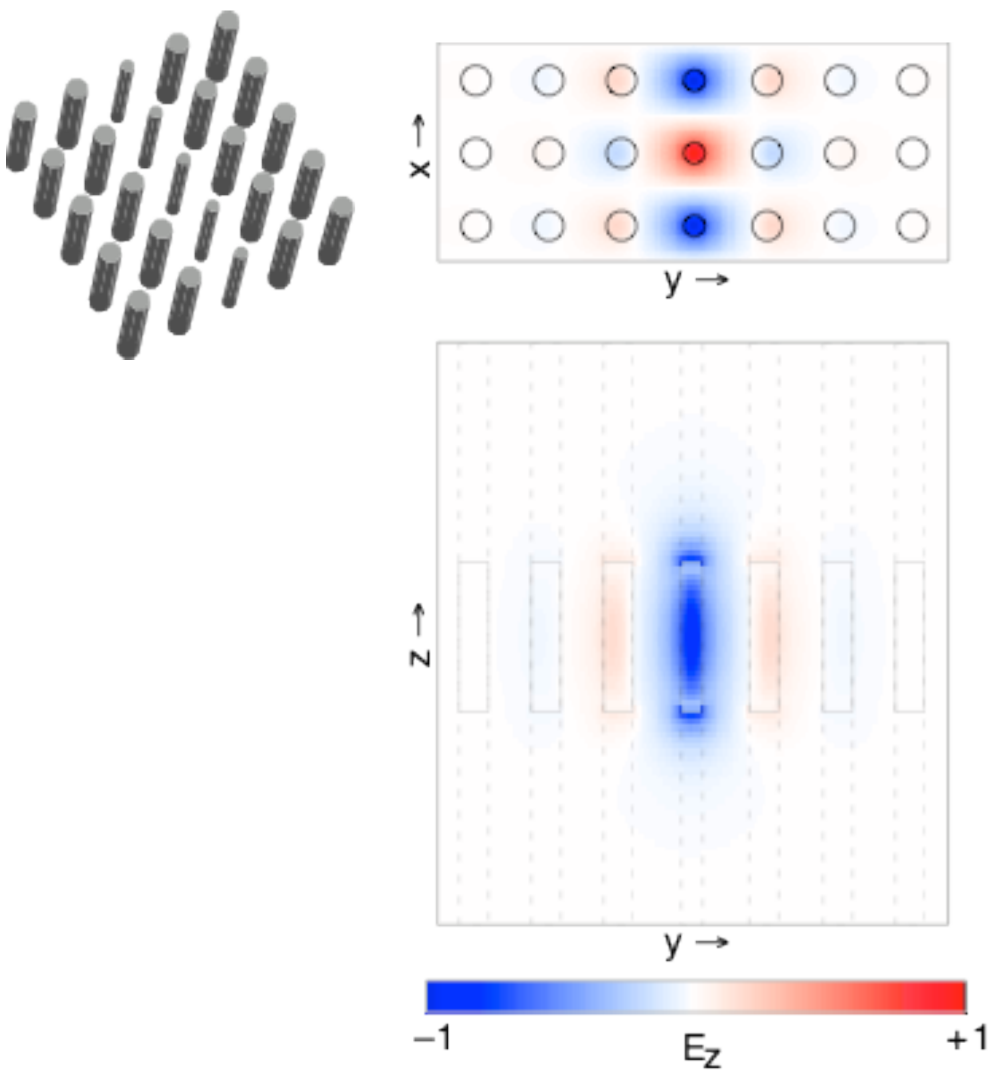


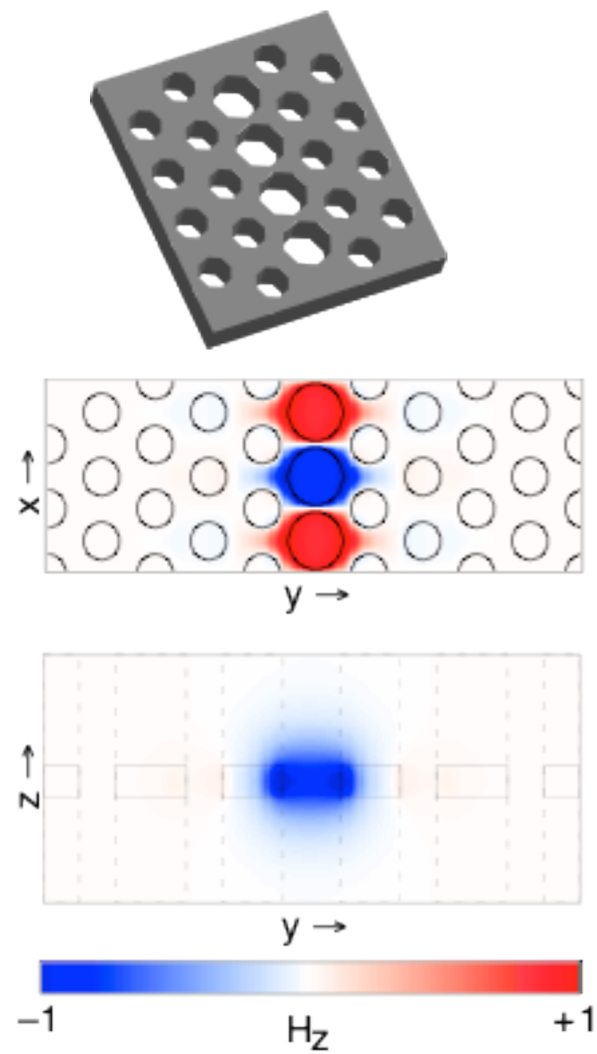
We *cannot* completely remove the rods—no vertical confinement!

Still have conserved wavevector—under the light cone, no radiation

Reduce the radius of a row of rods to "trap" a waveguide mode in the gap.

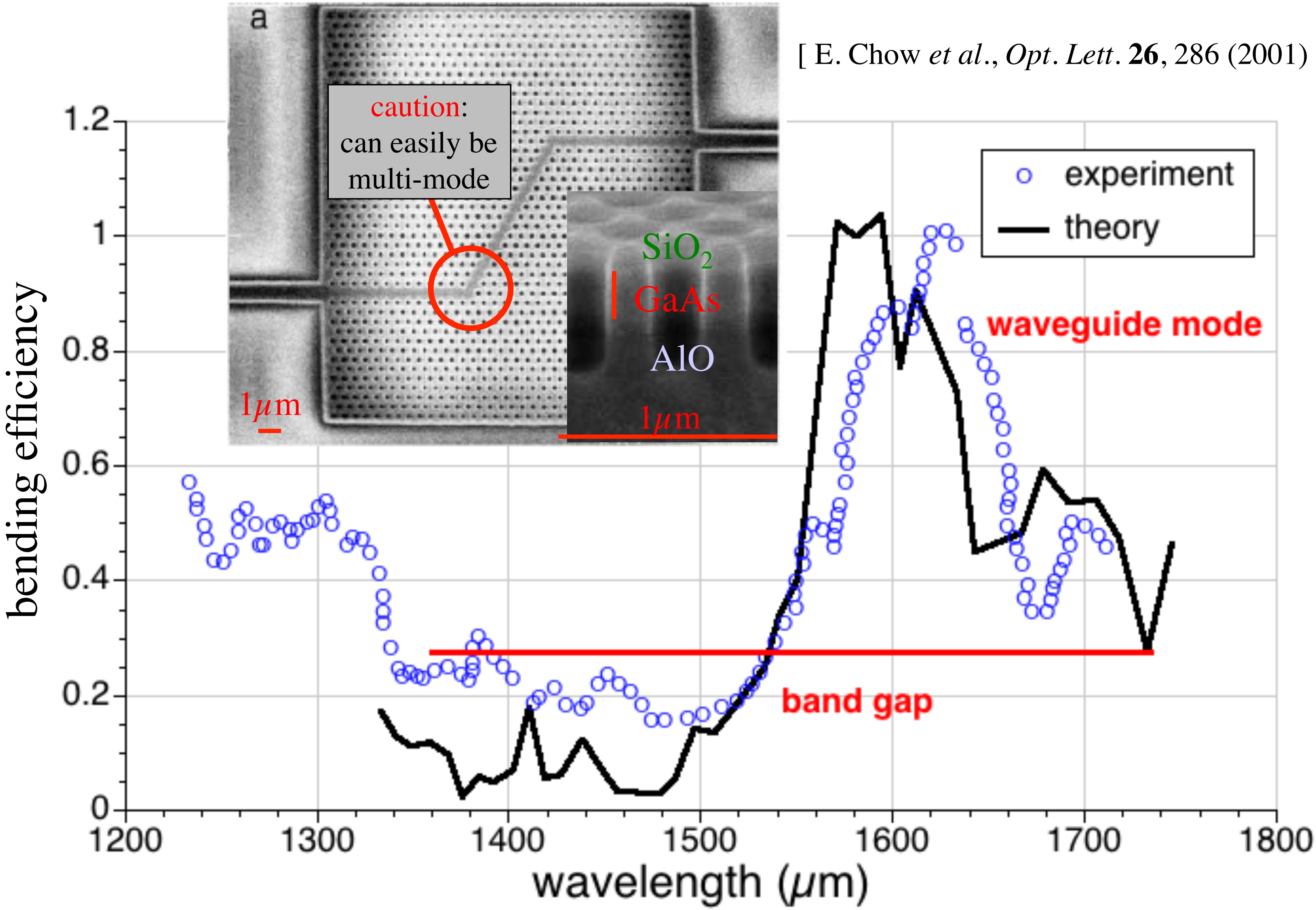
Reduced-Index Waveguide Modes





Experimental Waveguide & Bend

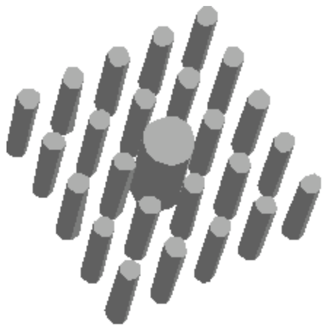
[E. Chow *et al*., *Opt. Lett*. **26**, 286 (2001)]

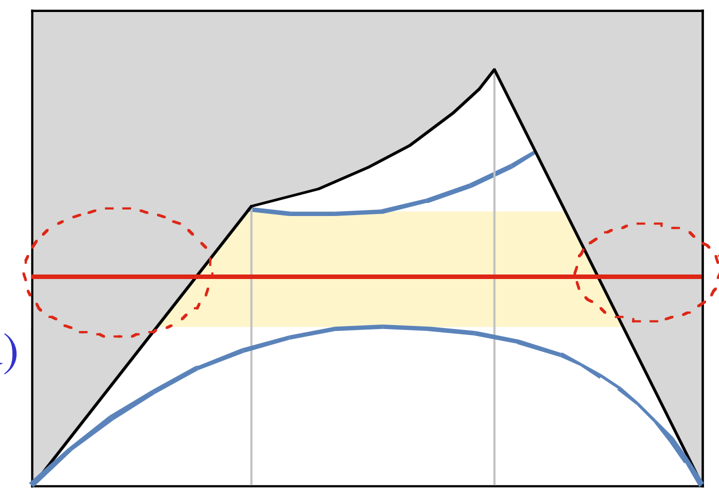


Inevitable Radiation Losses

whenever translational symmetry is broken

e.g. at cavities, waveguide bends, disorder…





w (conserved)

coupling to light cone = radiation losses

k is *no longer* conserved!

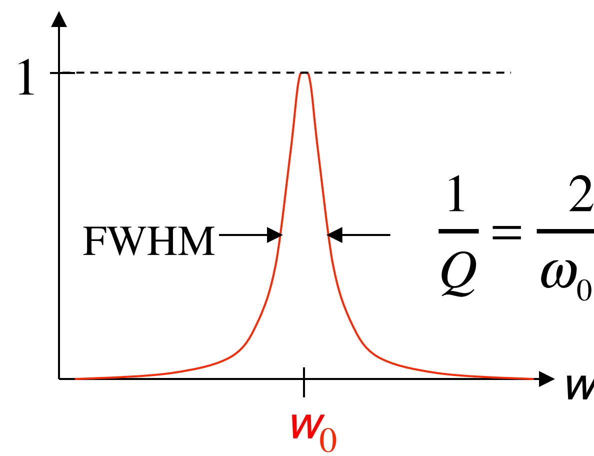
Dimensionless Losses: Q

quality factor Q = # optical periods for energy to decay by exp(–2π)

energy ~ $\exp(-\omega t/Q)$

in frequency domain: $1/Q$ = bandwidth

from last time:
(coupling-of-modes-in-time)



$$\frac{1}{Q} = \frac{2}{\omega_0 \tau}$$

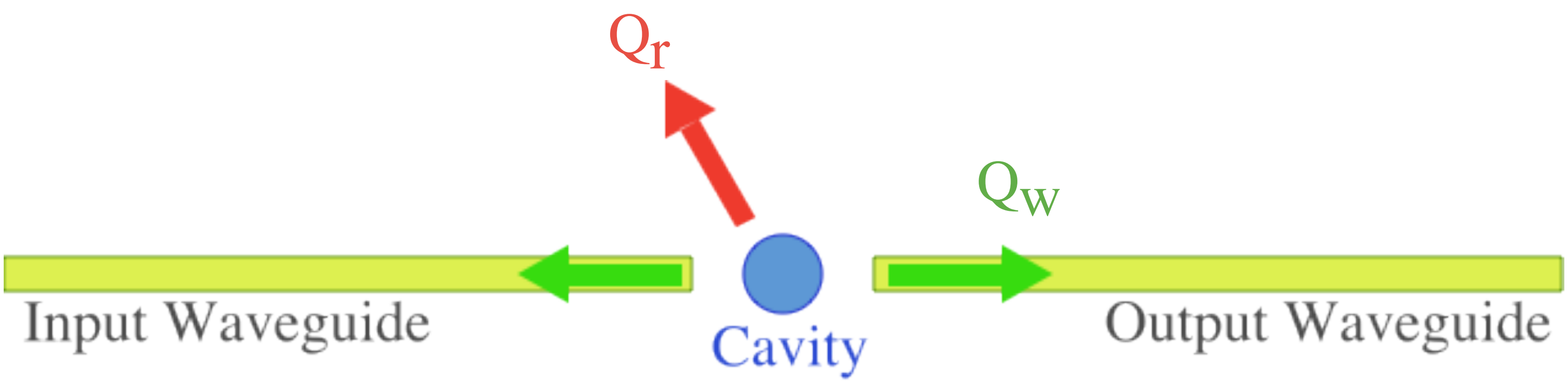
T = Lorentzian filter

$$= \frac{\frac{4}{\tau^2}}{(\omega - \omega_0)^2 + \frac{4}{\tau^2}}$$

…quality factor Q

All Is Not Lost

A simple model device (filters, bends, …):



$$\frac{1}{Q} = \frac{1}{Q_r} + \frac{1}{Q_w}$$

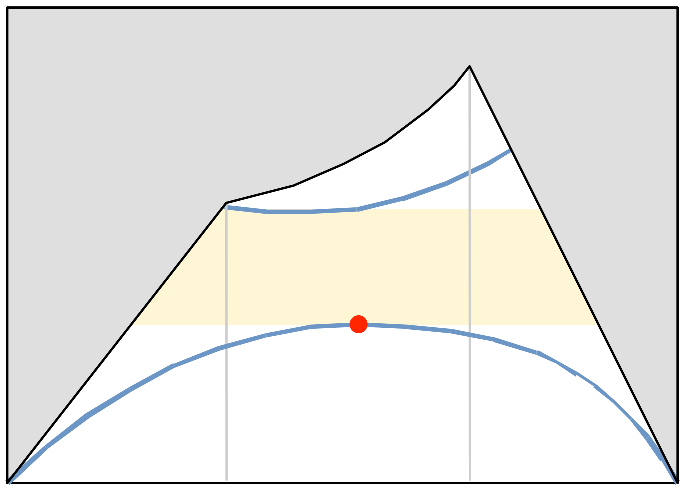
Q = lifetime/period
= frequency/bandwidth

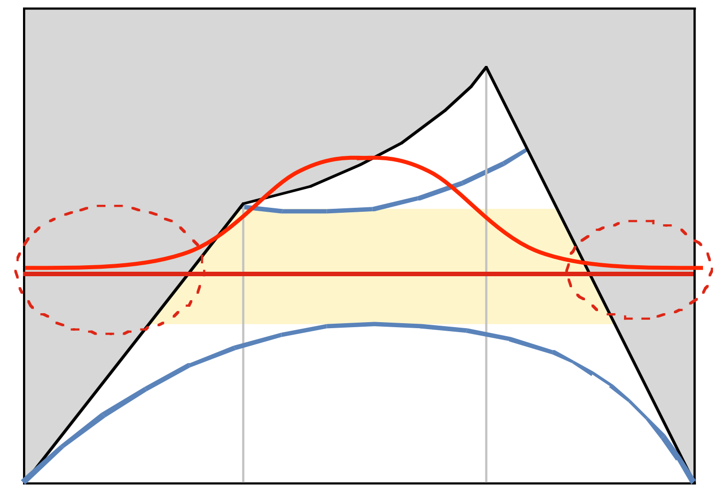
We want: $Q_r >> Q_w$

$1 - \text{transmission} \sim 2Q / Q_r$

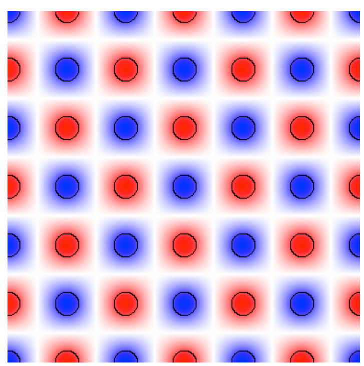
worst case: high-Q (narrow-band) cavities

Radiation loss: A Fourier picture

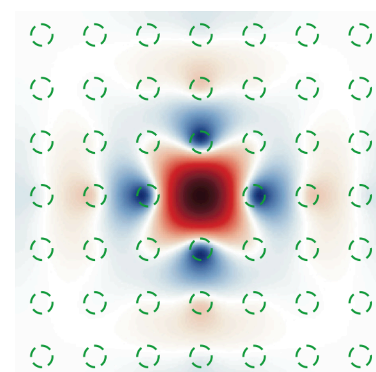




spatial:

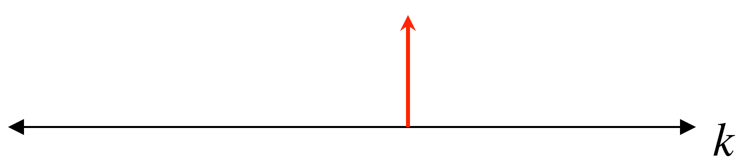


infinitely extended in space



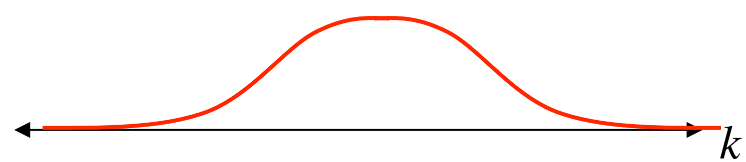
localized in space

Fourier:



delta function(s) [Fourier series]

below light cone = no radiation



delocalized in Fourier

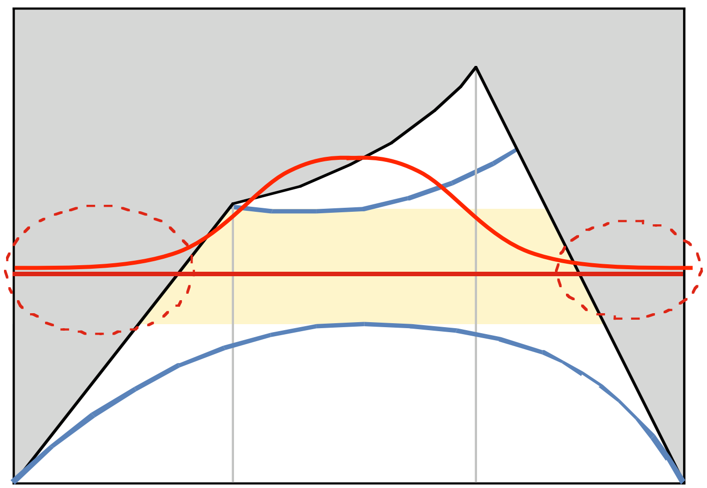
tails in light cone = radiation

A tradeoff: Localization vs. Loss

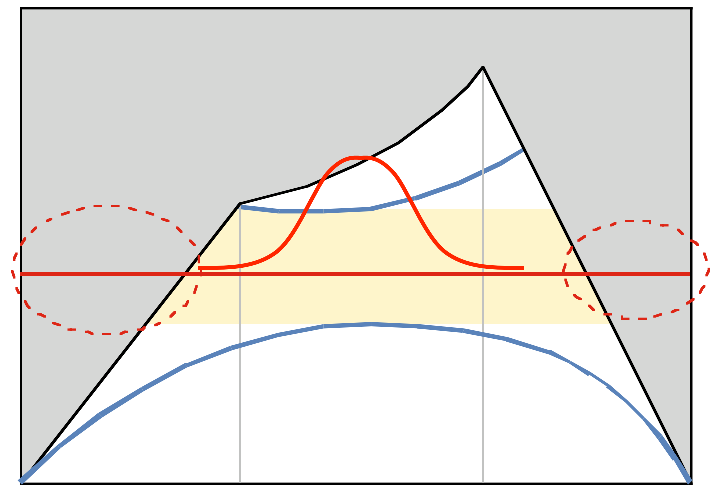
"Uncertainty principle:"

less spatial localization = *more Fourier* localization

= less radiation loss

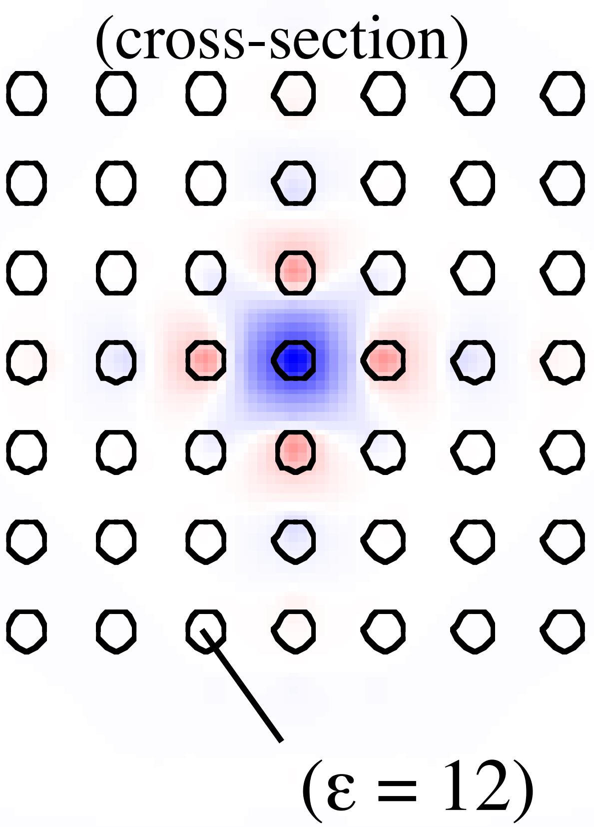


stronger spatial localization

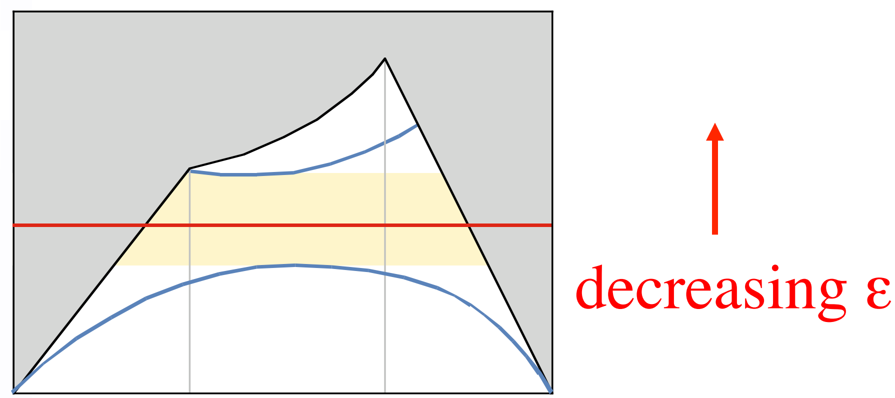


weaker spatial localization

Monopole Cavity in a Slab

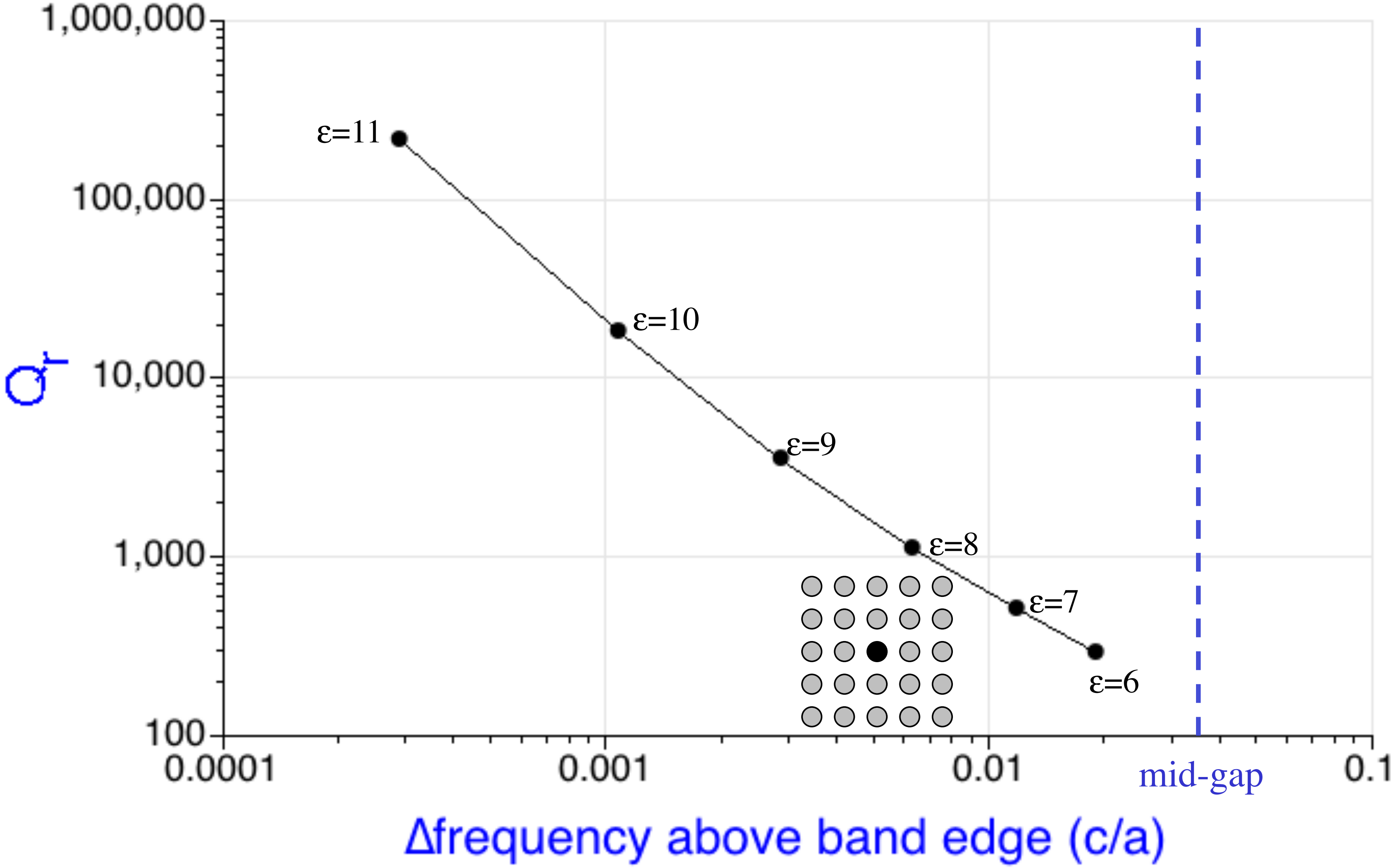


Lower the ε of a single rod: push up a monopole (singlet) state.



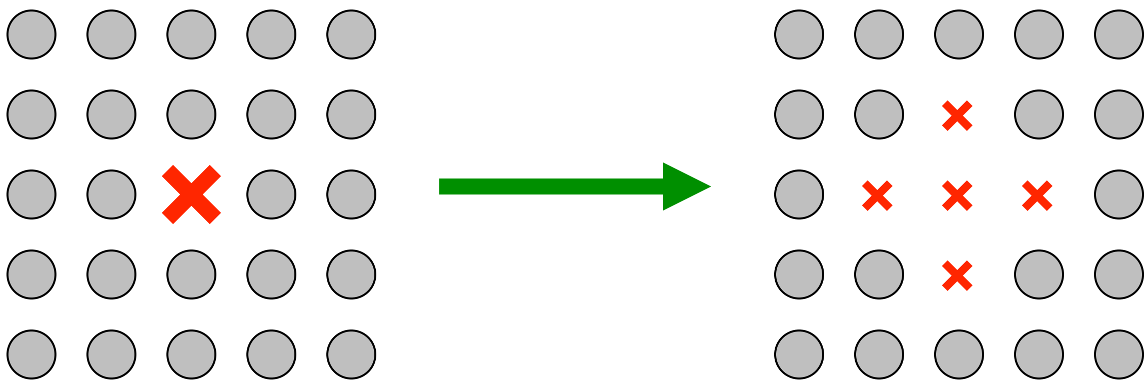
Use small Δε: delocalized in-plane, & high-Q (we hope)

Delocalized Monopole Q



[S. G. Johnson *et al*., *Computing in Sci. and Eng*. **3**, 38 (2001).]

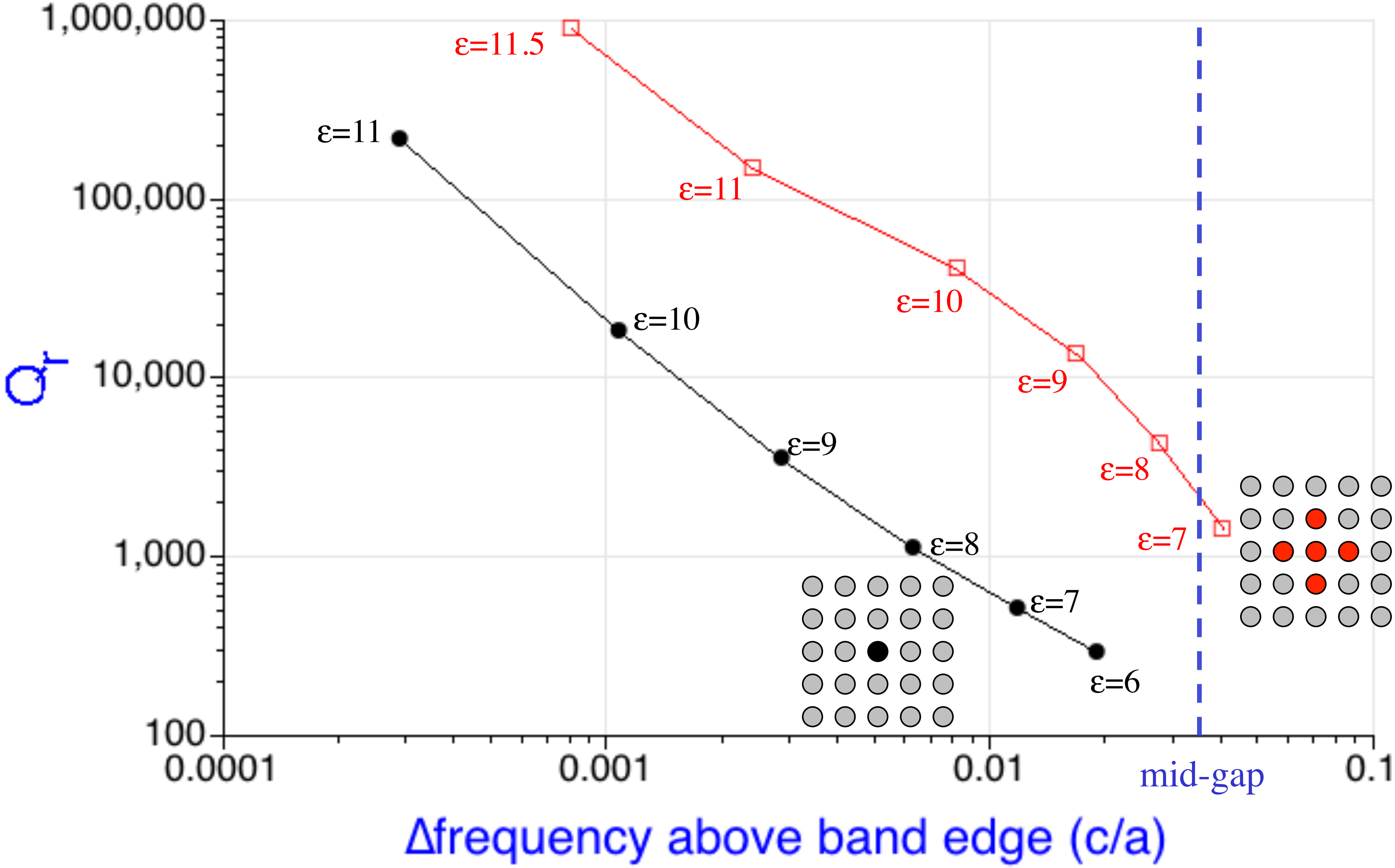
Super-defects



Weaker defect with more unit cells.

More delocalized
at the same point in the gap
(*i.e.* at same bulk decay rate)

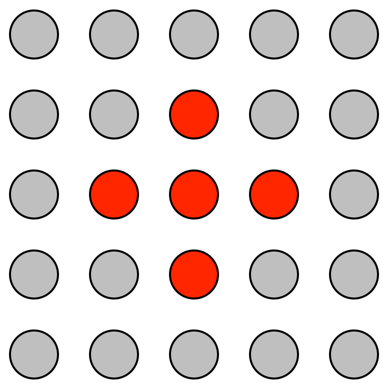
Super-Defect vs. Single-Defect Q



[S. G. Johnson *et al*., *Computing in Sci. and Eng*. **3**, 38 (2001).]

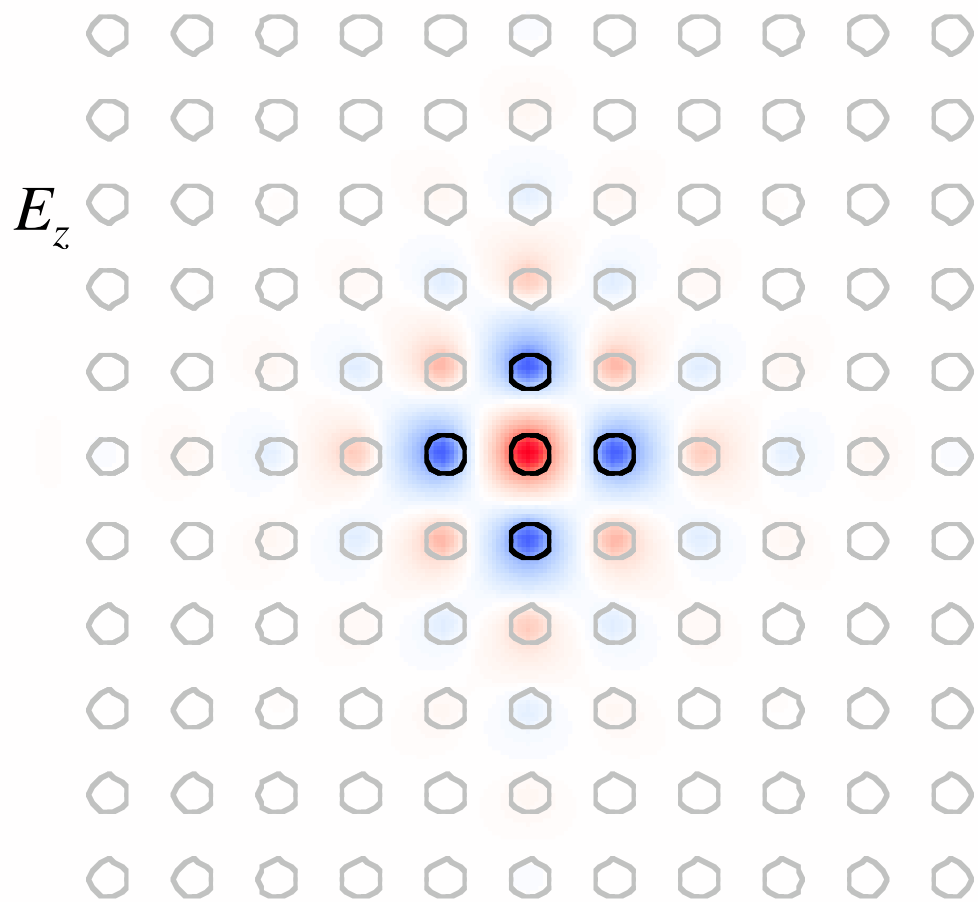
Super-Defect State

(cross-section)



(super defect)

$\Delta\varepsilon = -3$, $Q_{rad} = 13{,}000$

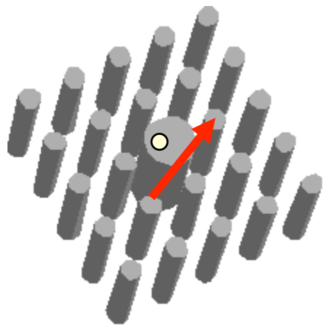


still ~localized: *In-plane* $Q_{||}$ is > 50,000 for only 4 bulk periods

How do we compute Q?

(via 3d FDTD [finite-difference time-domain] simulation)

1



excite cavity with dipole source

(broad bandwidth, *e.g*. Gaussian pulse)

… monitor field at some point ◦

…extract frequencies, decay rates via fancy signal processing (not just FFT/fit)

[V. A. Mandelshtam, *J. Chem. Phys*. **107**, 6756 (1997)]

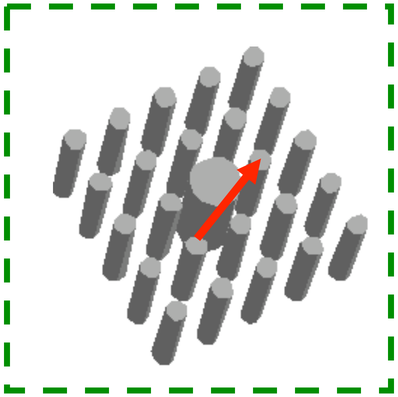
Pro: no *a priori* knowledge, get all ω' s and Q's at once

Con: no separate Q_w/Q_r,
mixed-up field pattern if multiple resonances

How do we compute Q?

(via 3d FDTD [finite-difference time-domain] simulation)

2



excite cavity with
narrow-band dipole source
(*e.g.* temporally broad Gaussian pulse)

— source is at ω_0 resonance,
which must already be known (via 1)

…measure outgoing power P and energy U

$$\boxed{Q = \omega_0 \, U / P}$$

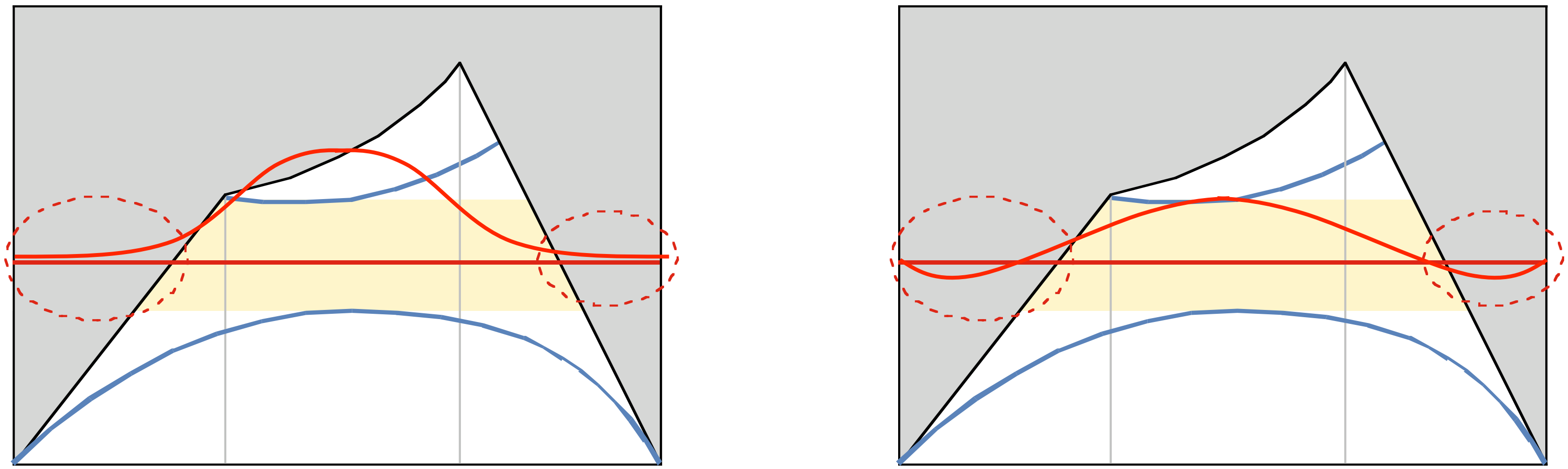
Pro: separate Q_w/Q_r, also get field pattern when multimode

Con: requires separate run 1 to get ω_0,
long-time source for closely-spaced resonances

Can we increase Q

without delocalizing (much)?

Cancellations?



Maybe we can make the Fourier transform oscillate through zero at some important k in the light cone?

But what k's are "important?"

Equivalently, some kind of destructive interference in the radiated field?

Need a more compact representation

Cannot cancel infinitely many $\mathbf{E}(x)$ integrals

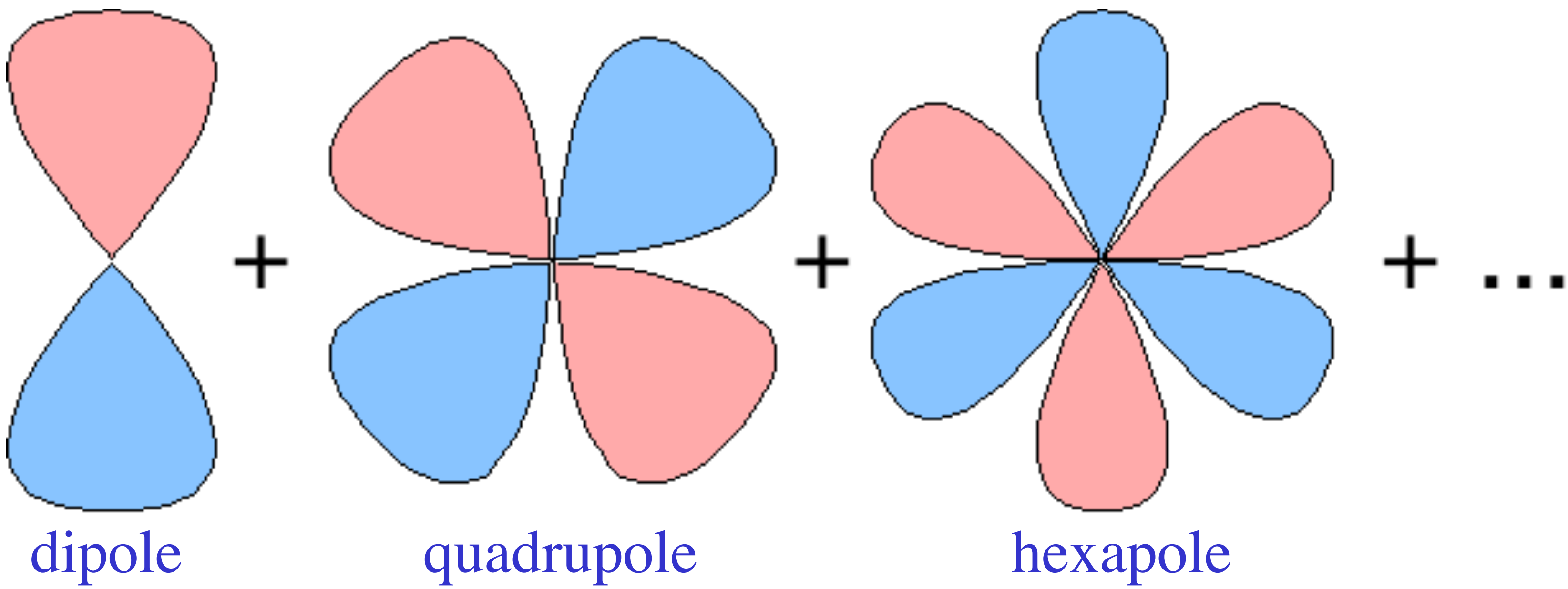
Radiation pattern from localized source…

— use multipole expansion
& cancel largest moment

Multipole Expansion

[Jackson, *Classical Electrodynamics*]

radiated field =



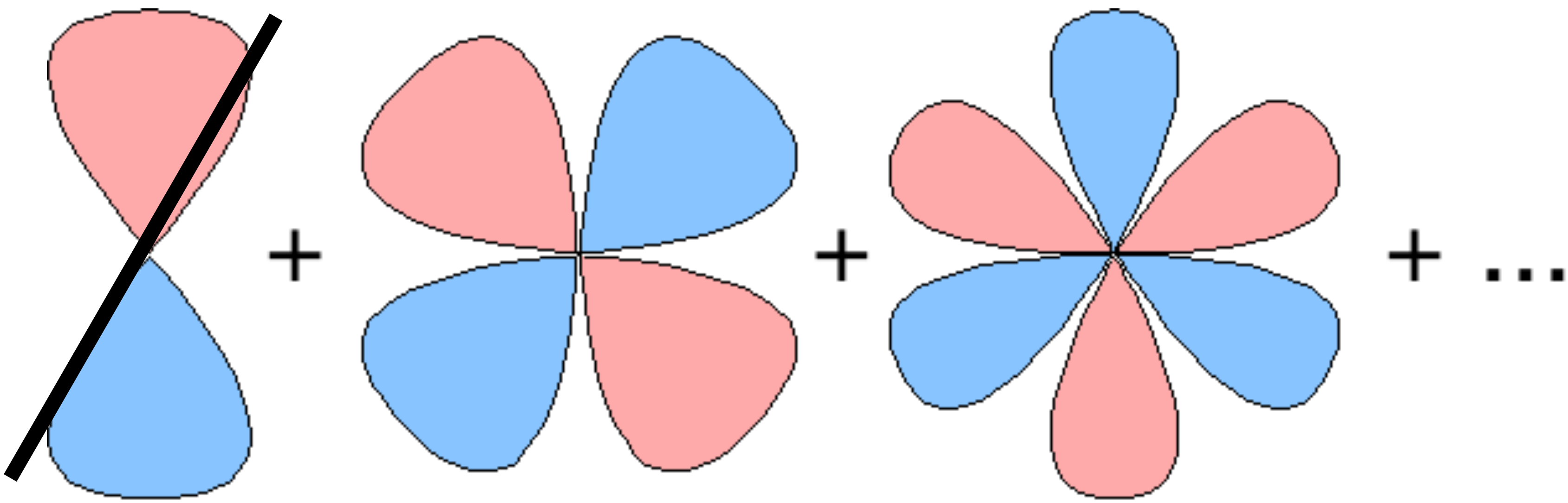
Each term's strength = single integral over near field

…one term is cancellable by tuning one defect parameter

Multipole Expansion

[Jackson, *Classical Electrodynamics*]

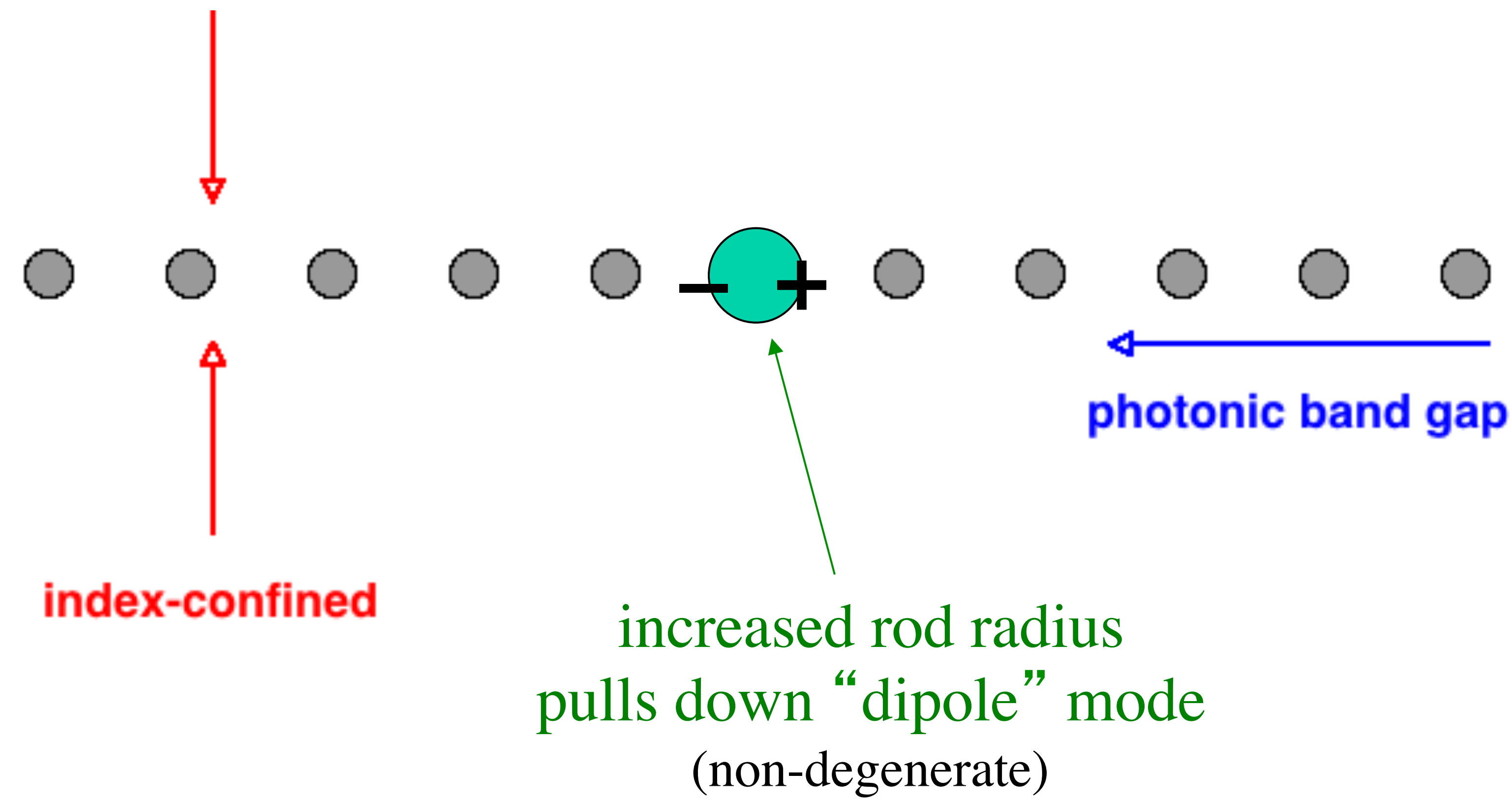
radiated field =



dipole quadrupole hexapole

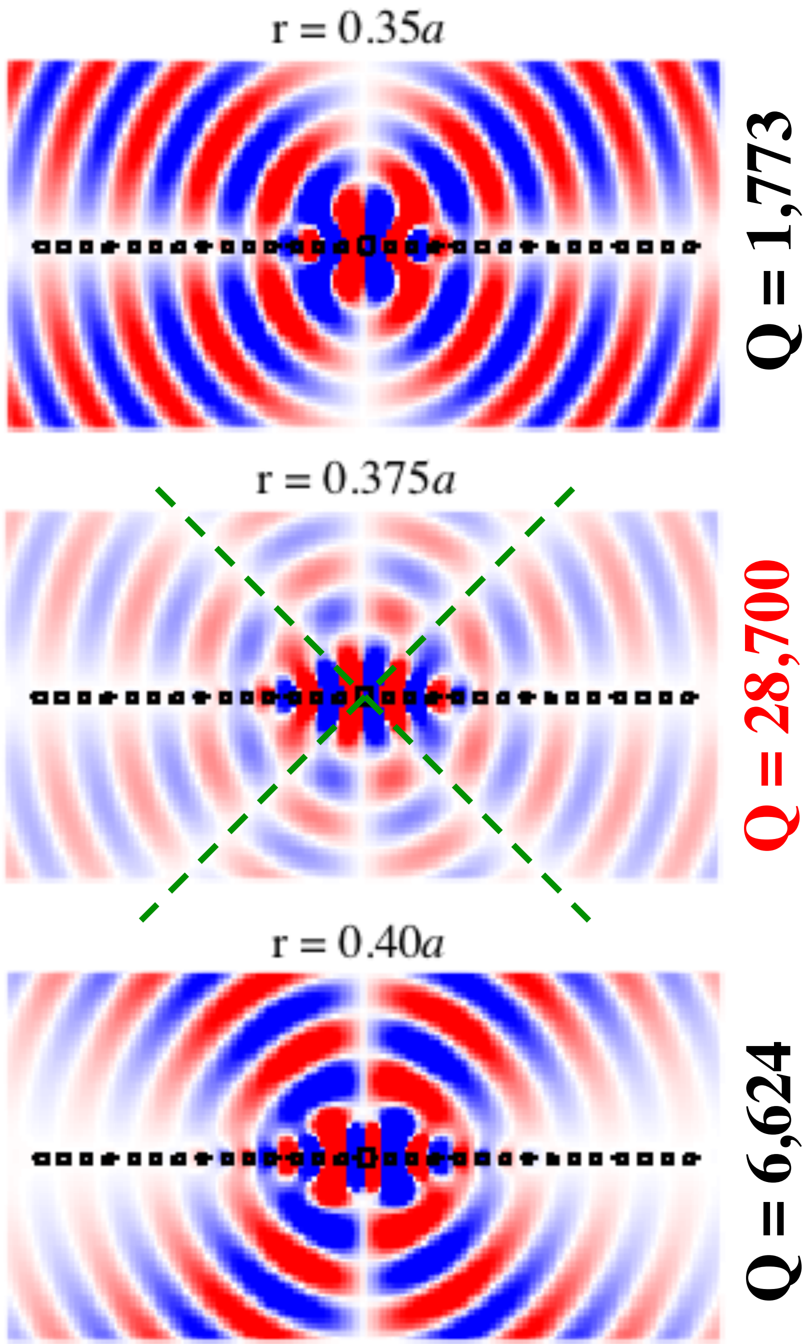
peak Q (cancellation) = transition to higher-order radiation

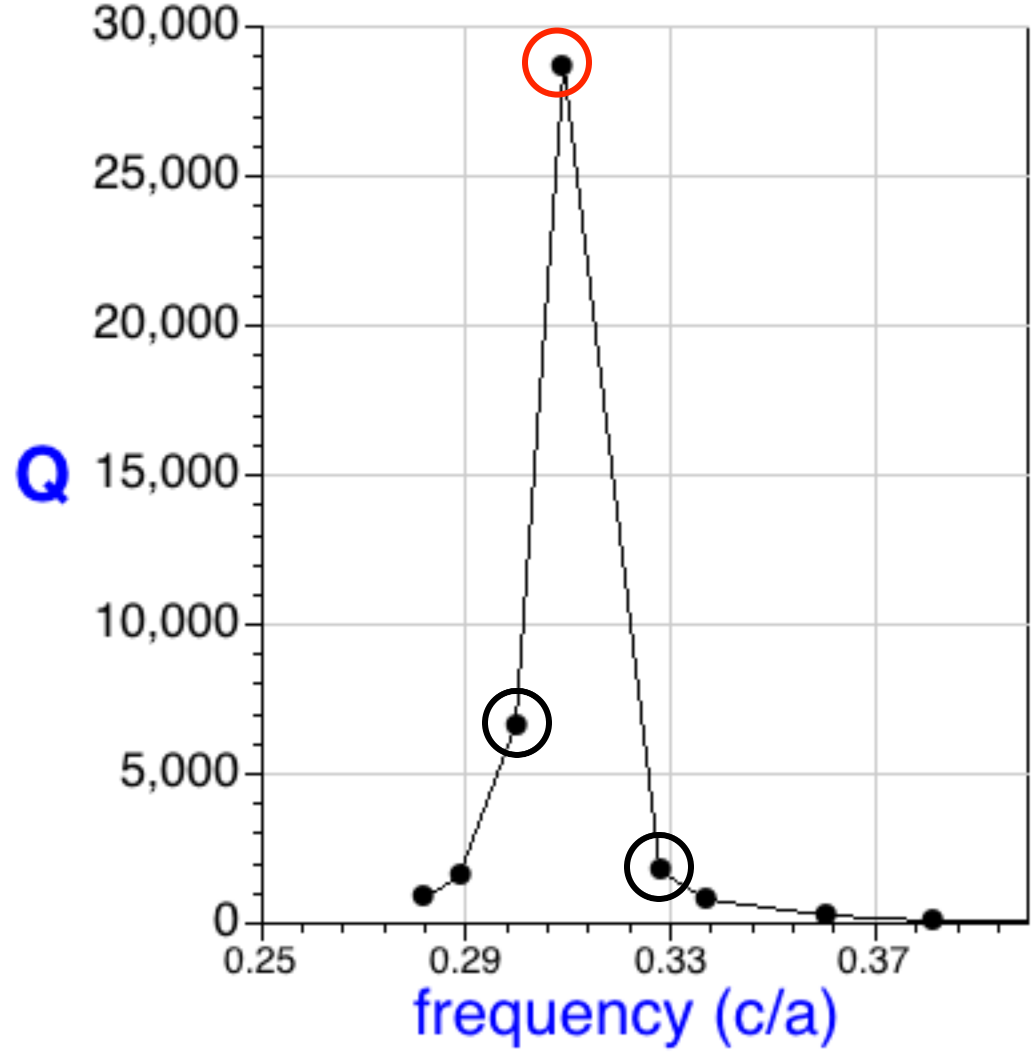
Multipoles in a 2d example



as we change the radius, ω sweeps across the gap

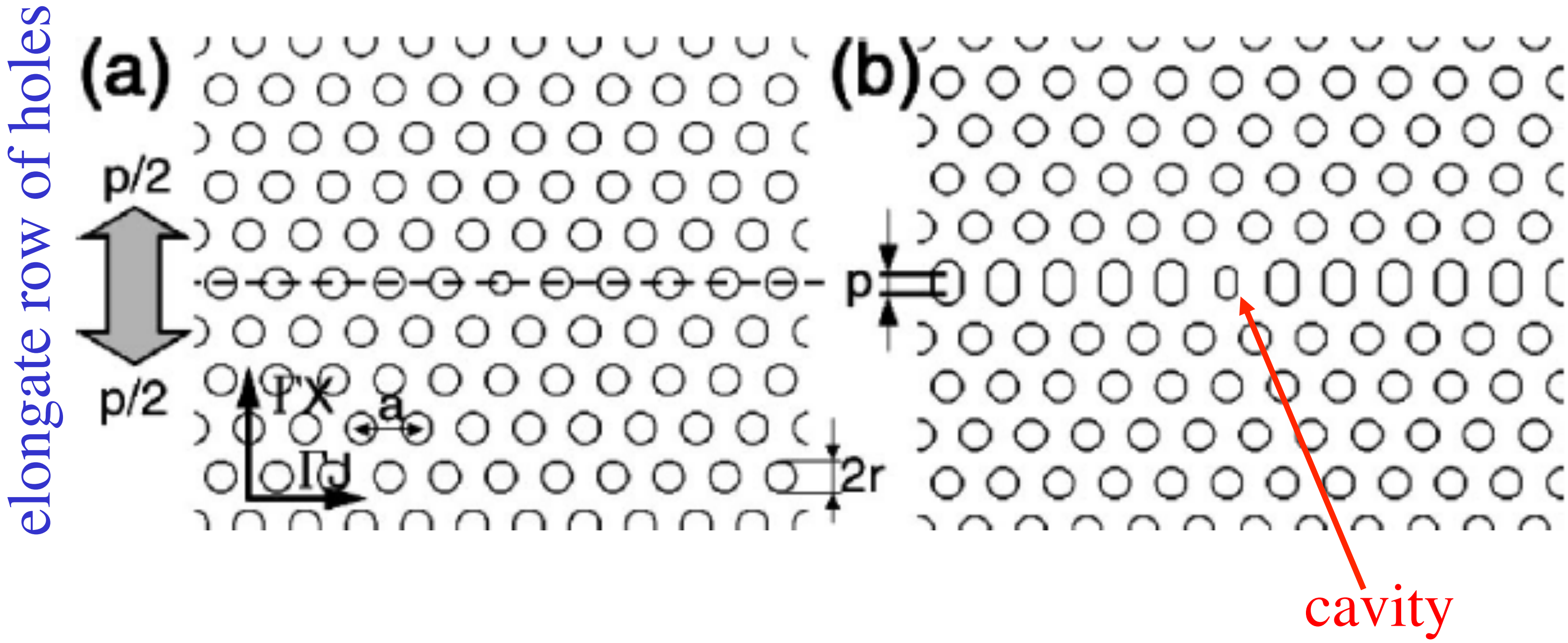
2d multipole cancellation





An Experimental (Laser) Cavity

[M. Loncar *et al*., *Appl*. *Phys*. *Lett*. **81**, 2680 (2002)]



Elongation p is a tuning parameter for the cavity…

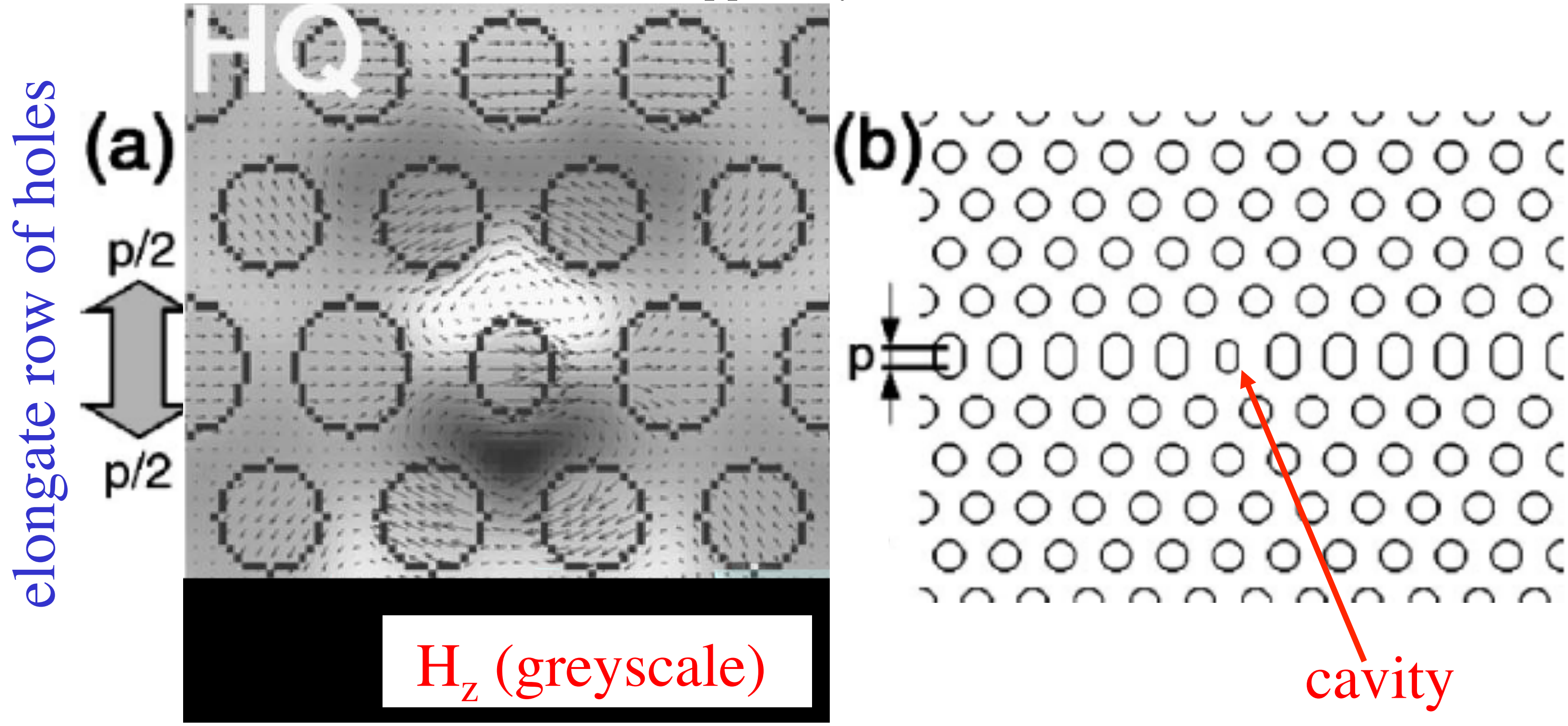
…in simulations, Q peaks sharply to ~10000 for $p = 0.1a$

(likely to be a multipole-cancellation effect)

* actually, there are two cavity modes; p breaks degeneracy

An Experimental (Laser) Cavity

[M. Loncar *et al*., *Appl*. *Phys*. *Lett*. **81**, 2680 (2002)]



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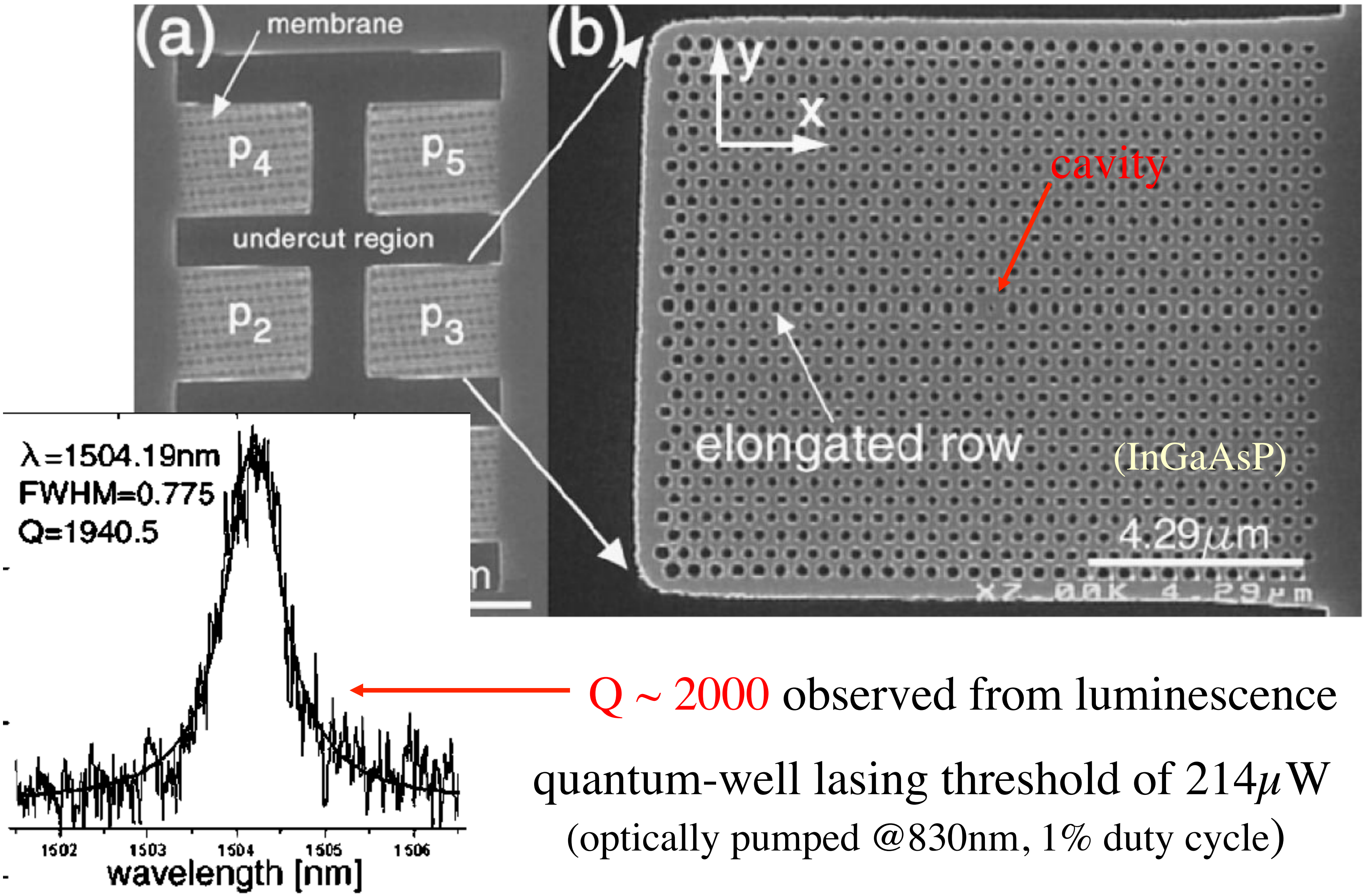


Q ~ 2000 observed from luminescence

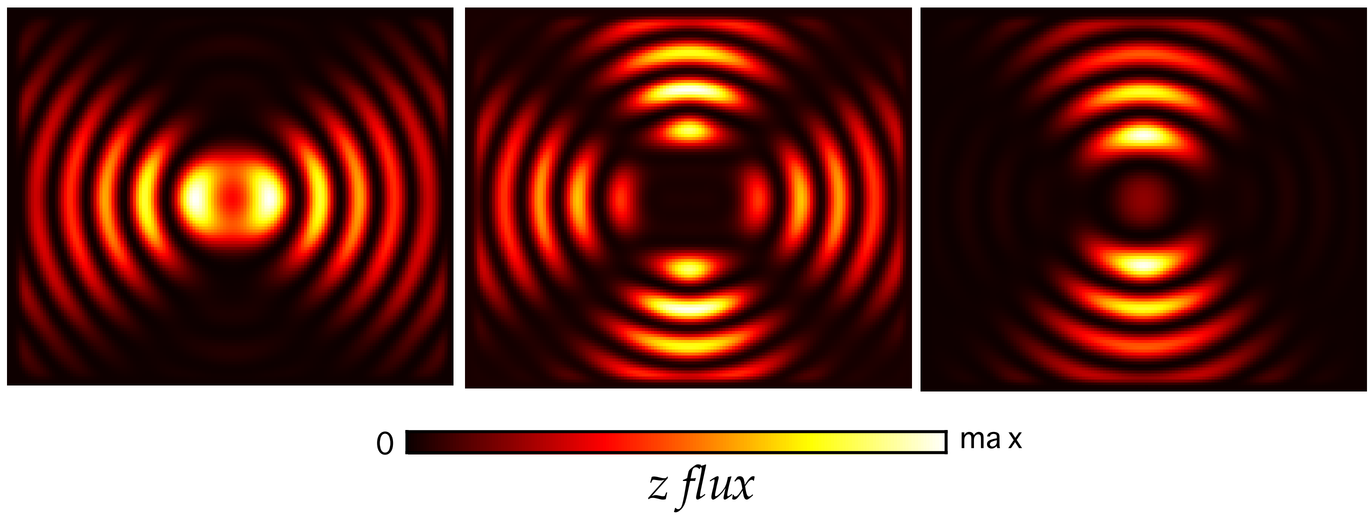
quantum-well lasing threshold of 214μW

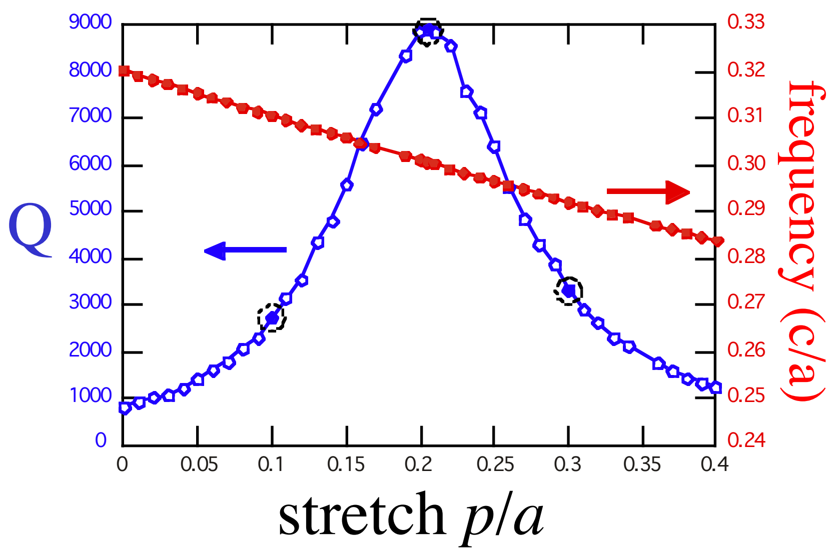
(optically pumped @830nm, 1% duty cycle)

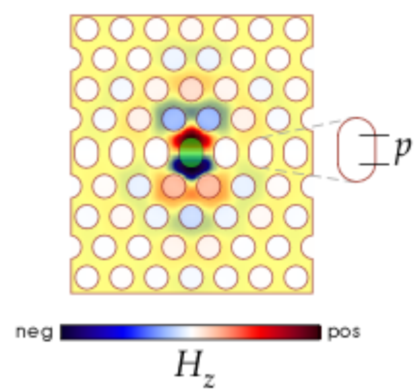
Multipole Cancellation in Stretched Cavity

[calculations courtesy A. Rodriguez, 2006]

$p = 0.1a$ $p = 0.205a$ $p = 0.3a$

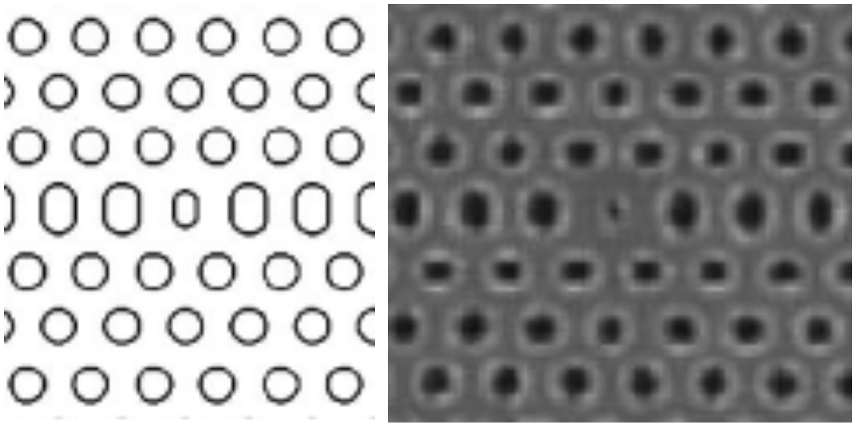






Slab Cavities in Practice: Q vs. V

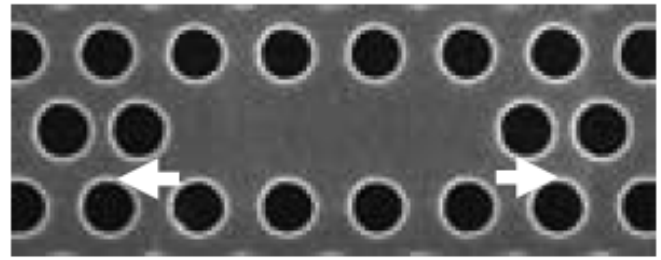
[Loncar, *APL* **81**, 2680 (2002)]



$Q \sim 10{,}000$ ($V \sim 4 \times$ optimum)

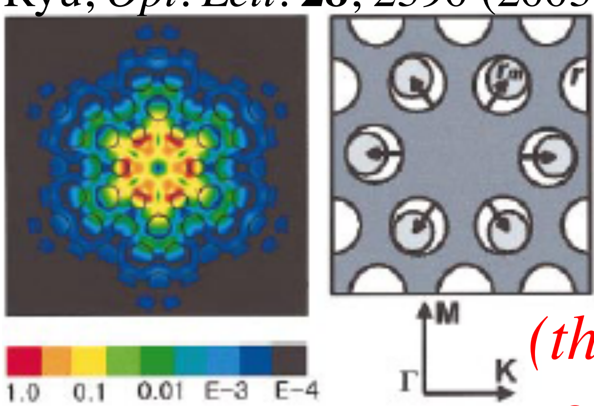
$= (\lambda/2n)^3$

[Akahane, *Nature* **425**, 944 (2003)]



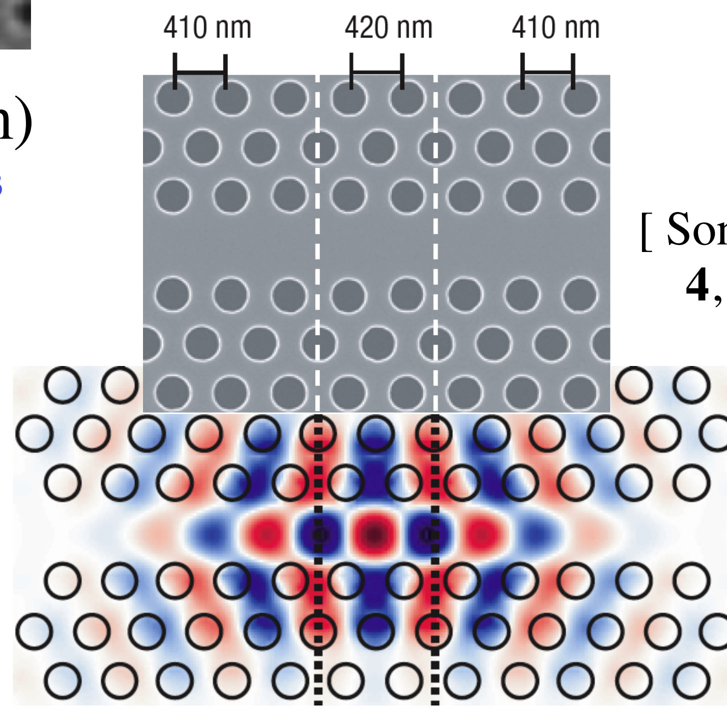
$Q \sim 45{,}000$ ($V \sim 6 \times$ optimum)

[Ryu, *Opt. Lett.* **28**, 2390 (2003)]



(theory only)

$Q \sim 10^6$ ($V \sim 11 \times$ optimum)



[Song, *Nature Mat.* **4**, 207 (2005)]

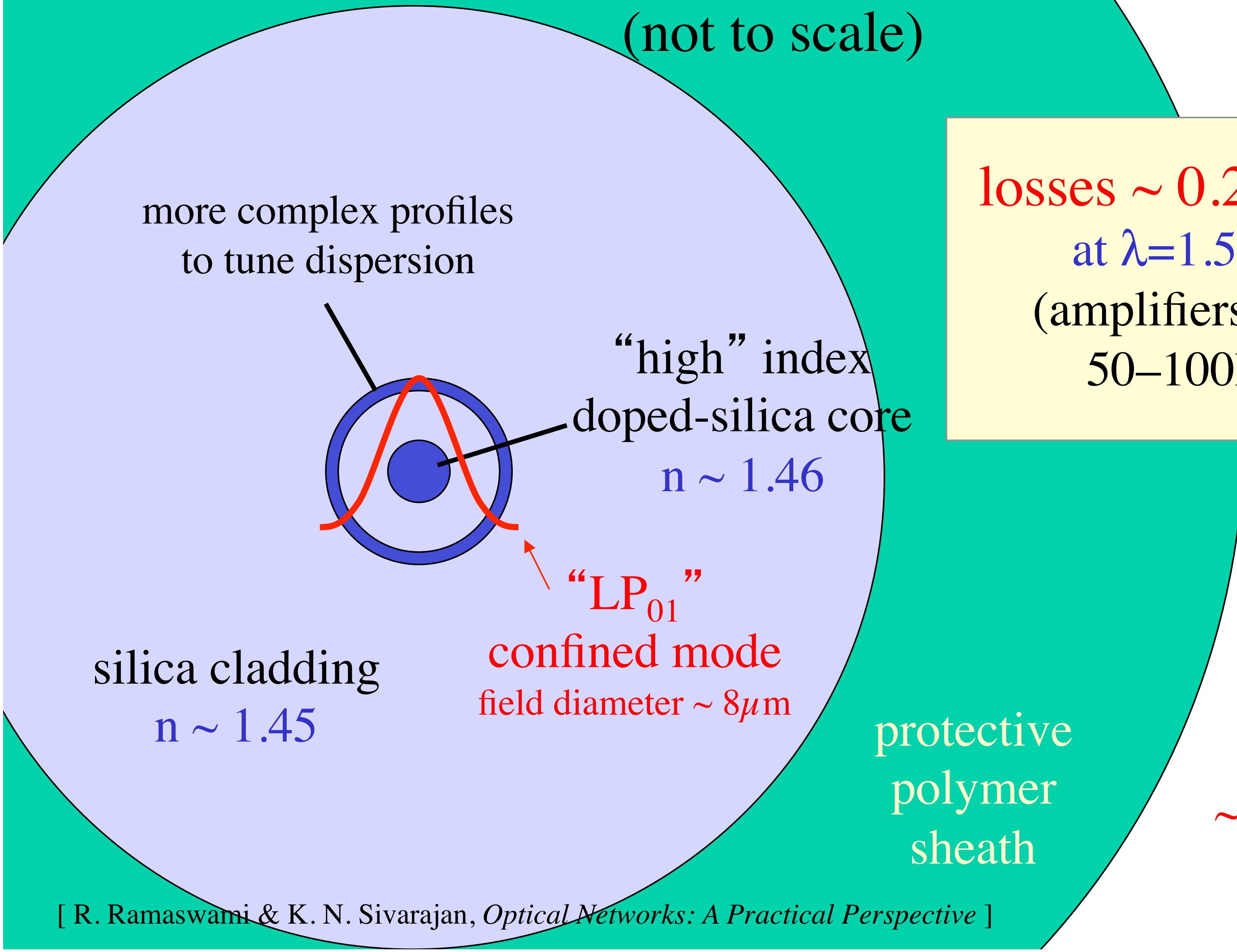
$Q \sim 600{,}000$ ($V \sim 10 \times$ optimum)

Outline

- Preliminaries: waves in periodic media
- Photonic crystals in theory and practice
- Bulk crystal properties
- Intentional defects and devices
- Index-guiding and incomplete gaps
- Photonic-crystal fibers
- Perturbations, tuning, and disorder

Optical Fibers Today

(not to scale)



losses ~ 0.2 dB/km
at λ=1.55μm
(amplifiers every
50–100km)

but this is
~ as good as
it gets…

[R. Ramaswami & K. N. Sivarajan, *Optical Networks: A Practical Perspective*]

The Glass Ceiling: *Limits of Silica*

Loss: amplifiers every 50–100km

…limited by Rayleigh scattering (molecular entropy)

…cannot use "exotic" wavelengths like 10.6μm

Nonlinearities: after ~100km, cause dispersion, crosstalk, power limits

(limited by mode area ~ single-mode, bending loss)

also cannot be made (very) **large** for compact nonlinear devices

Radical modifications to dispersion, polarization effects?

…tunability is limited by low index contrast

High Bit-Rates

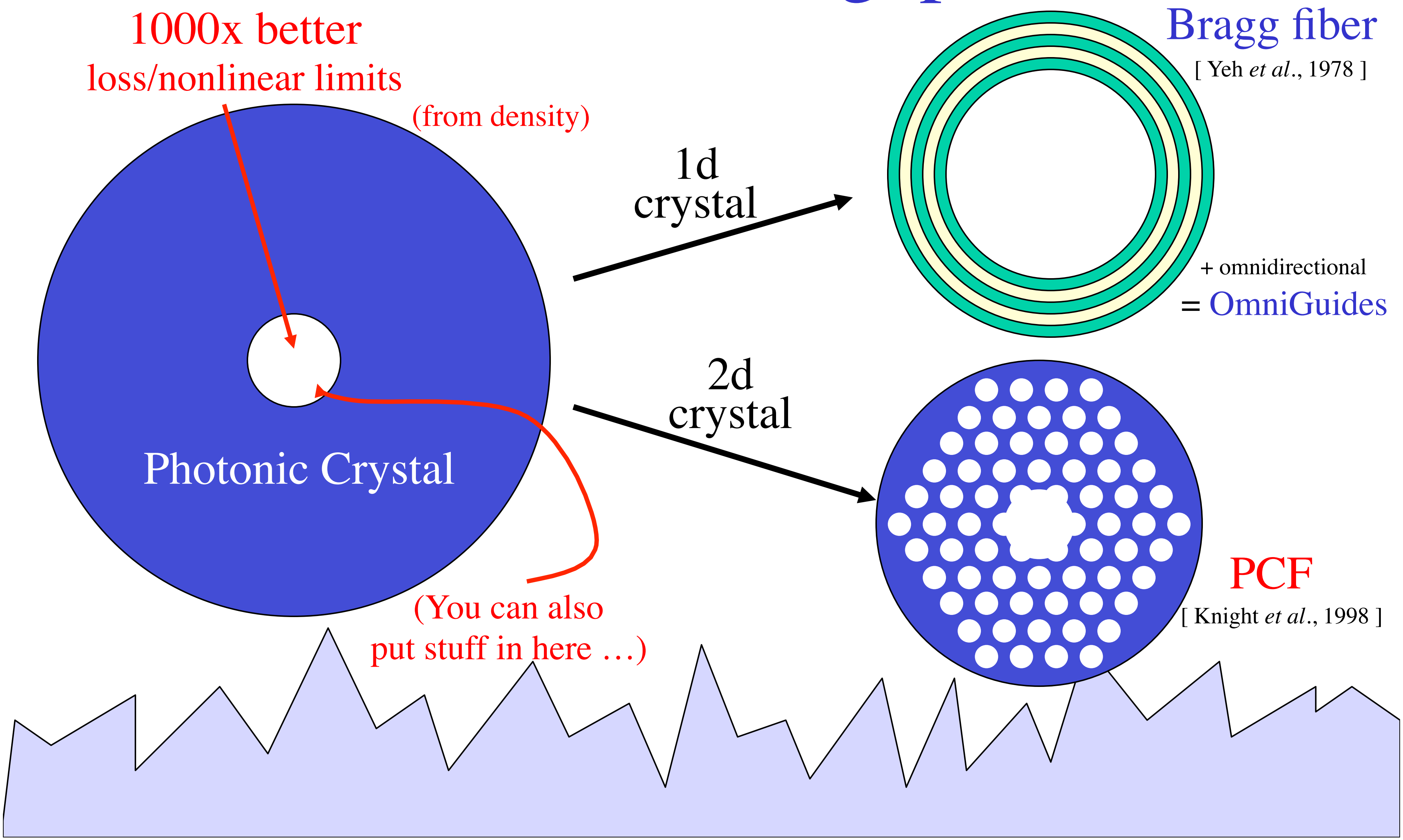
Compact Devices

Long Distances

Dense Wavelength Multiplexing (DWDM)

Breaking the Glass Ceiling:

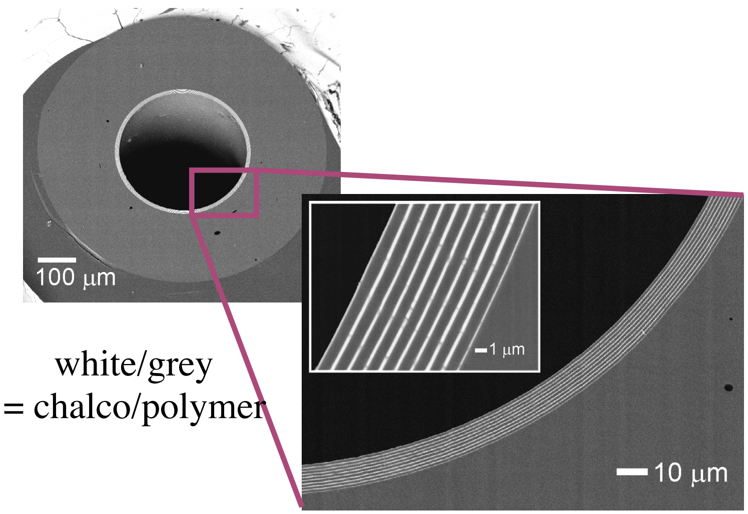
Hollow-core Bandgap Fibers



Breaking the Glass Ceiling:

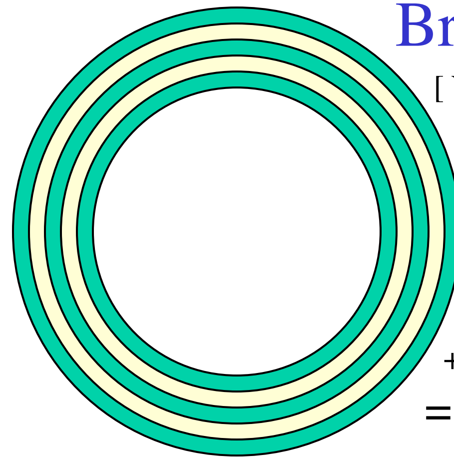
Hollow-core Bandgap Fibers

[figs courtesy
Y. Fink *et al*., MIT]



white/grey
= chalco/polymer

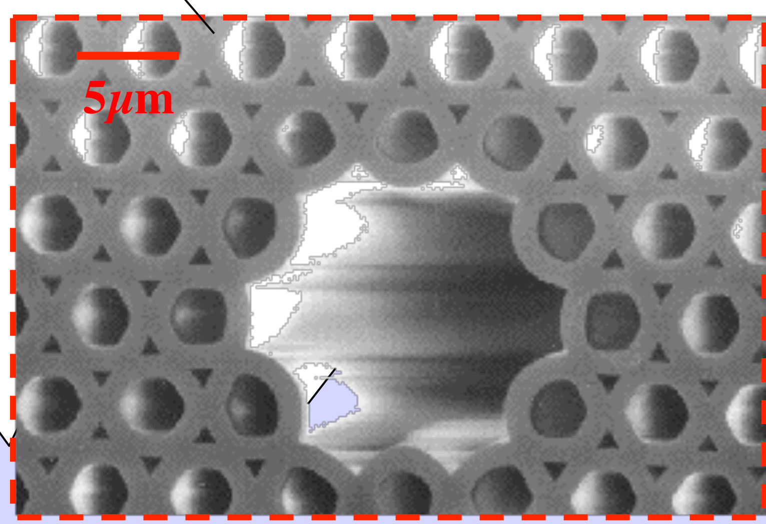
Bragg fiber
[Yeh *et al*., 1978]

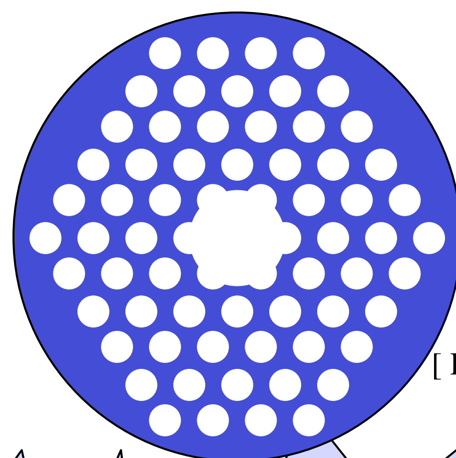


+ omnidirectional
= OmniGuide
fibers

silica

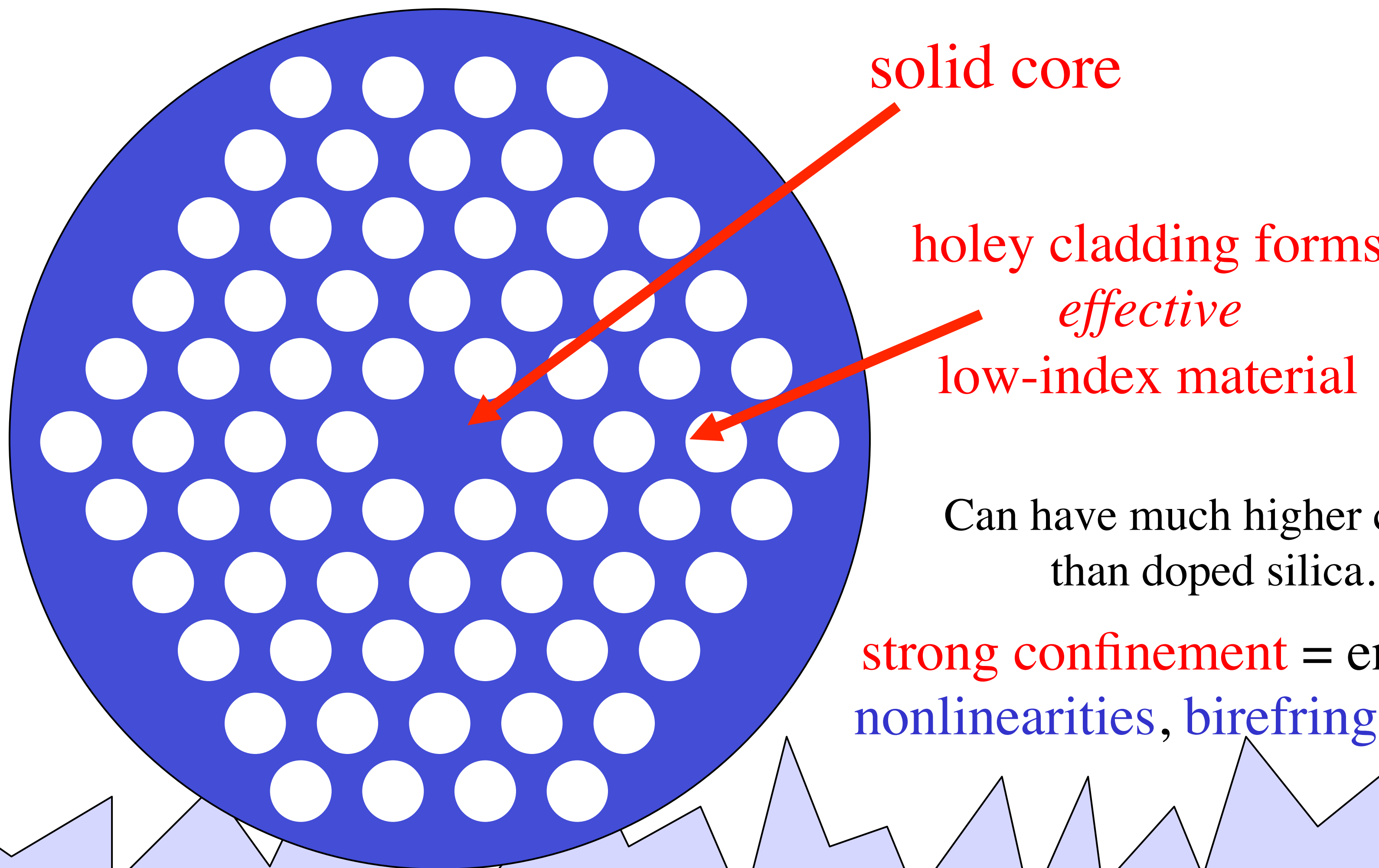
[R. F. Cregan
et al.,
Science **285**,
1537 (1999)]





PCF
[Knight *et al*., 1998]

Breaking the Glass Ceiling II:
Solid-core Holey Fibers



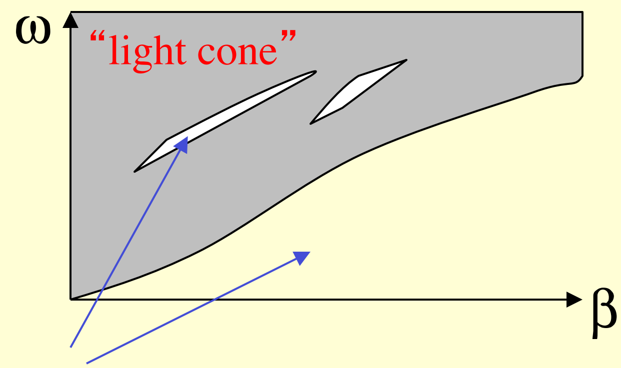
Can have much higher contrast
than doped silica…

strong confinement = enhanced
nonlinearities, birefringence, …

[J. C. Knight *et al*., *Opt*. *Lett*. **21**, 1547 (1996)]

Sequence of Analysis

1. Plot all solutions of infinite cladding as ω vs. β $(= k_z)$

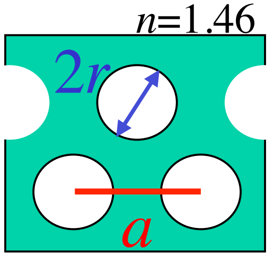


empty spaces (gaps): guiding possibilities

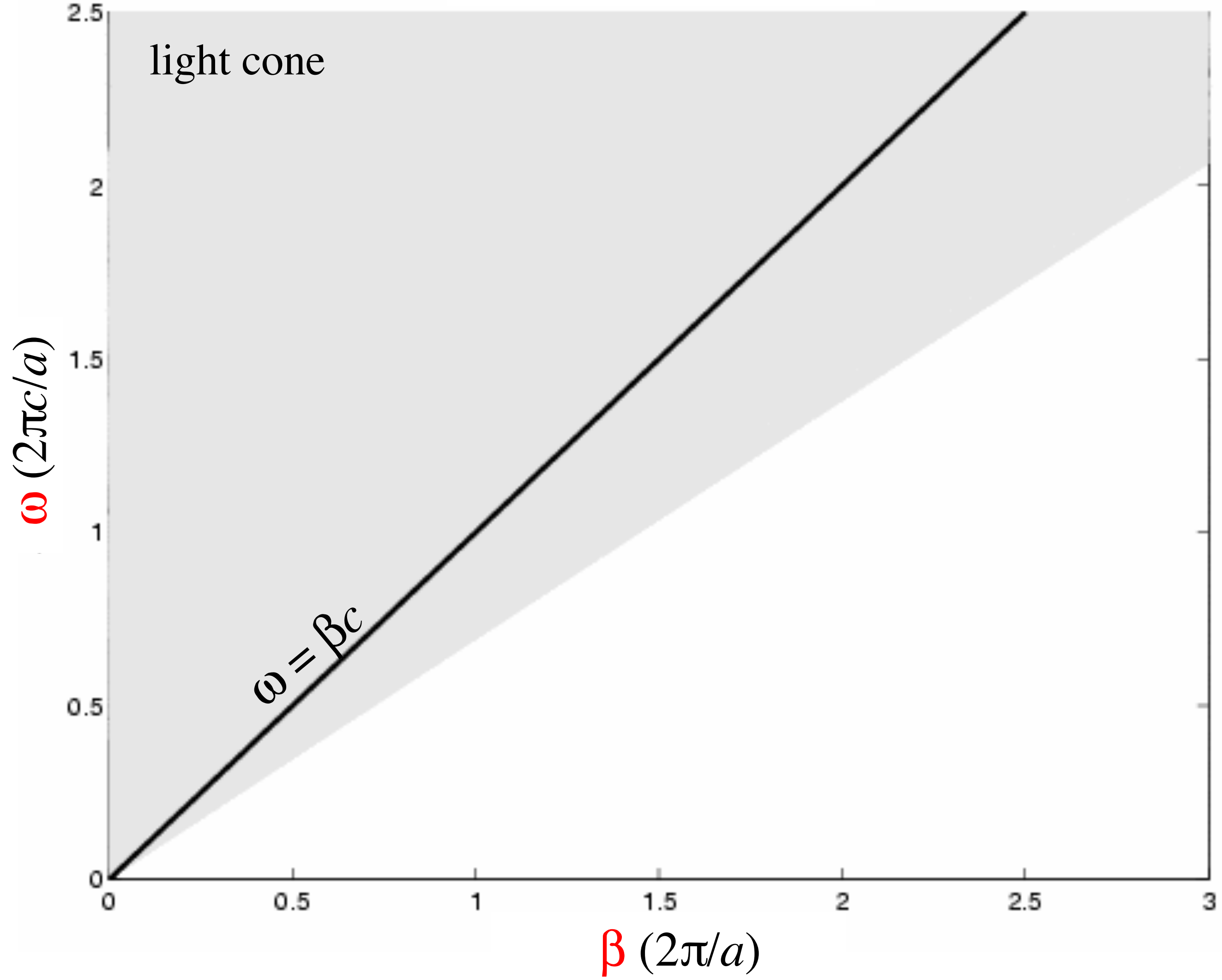
2. Core introduces new states in empty spaces — plot $\omega(\beta)$ dispersion relation

3. Compute other stuff…

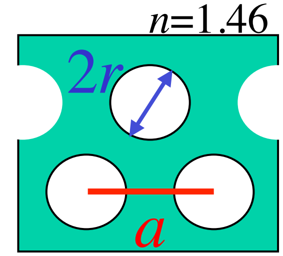
PCF: Holey Silica Cladding



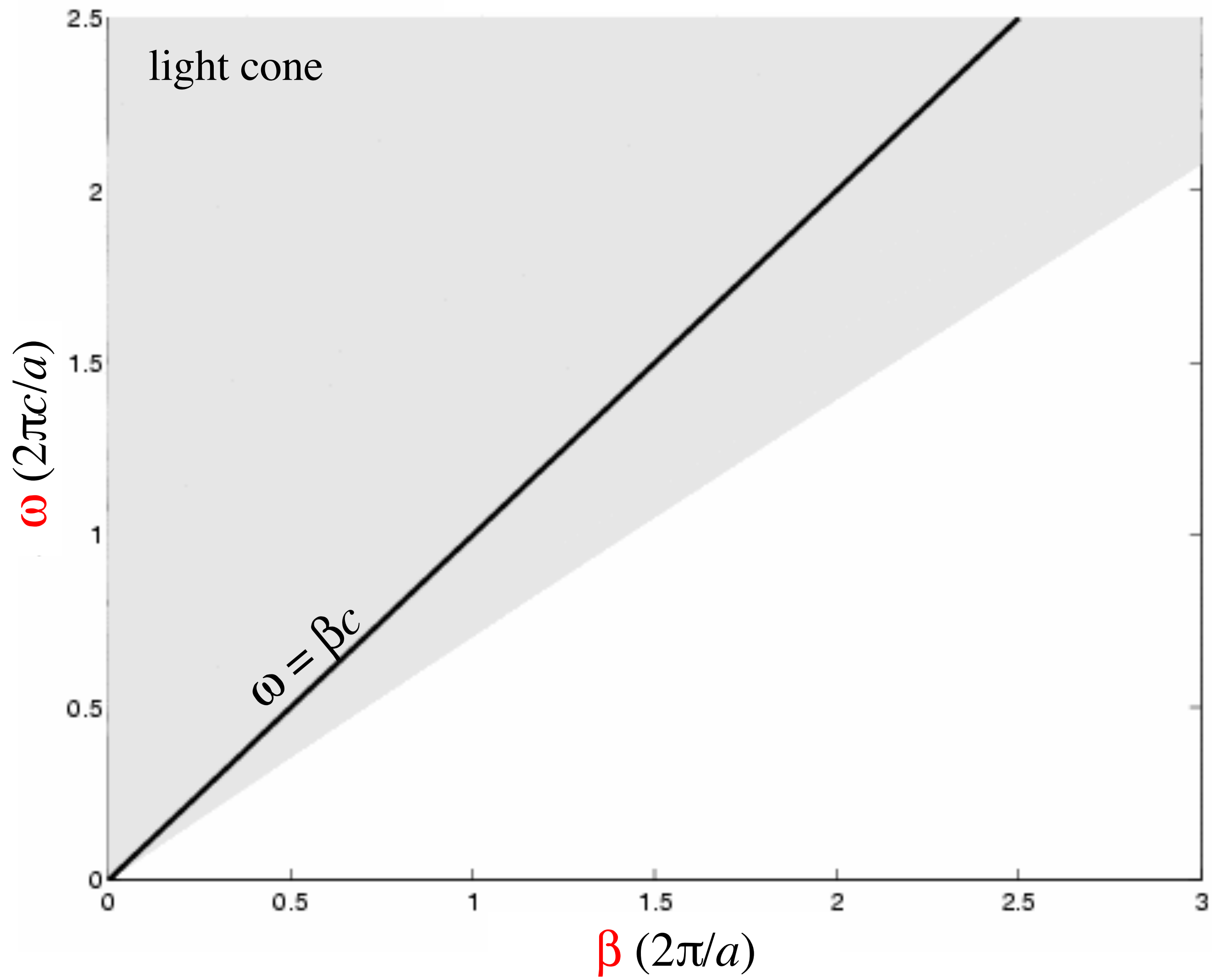
$r = 0.1a$



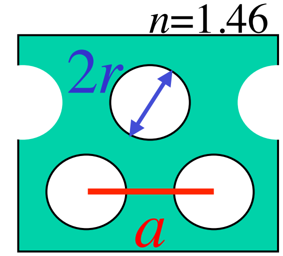
PCF: Holey Silica Cladding



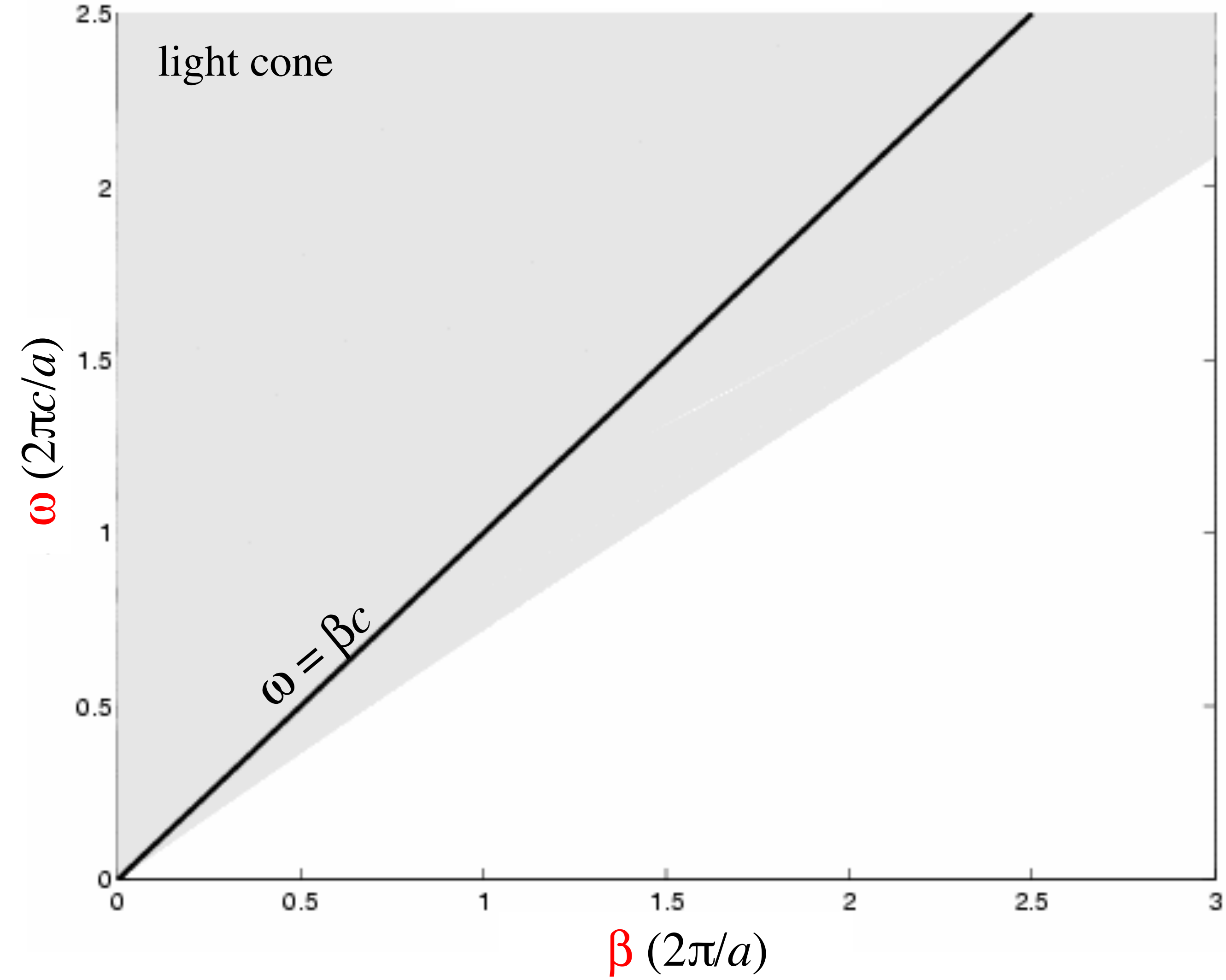
$r = 0.17717a$



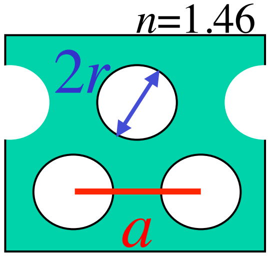
PCF: Holey Silica Cladding

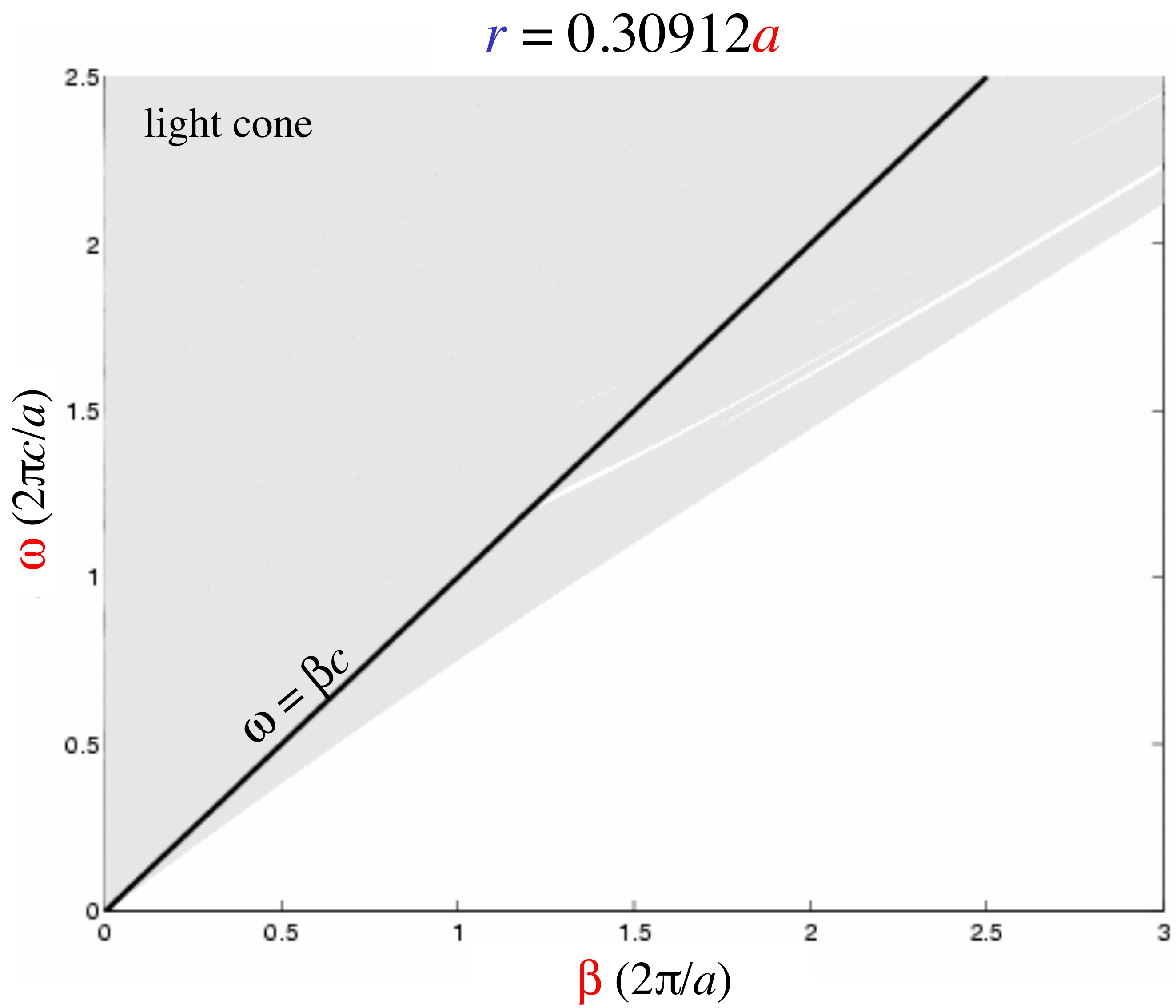


$r = 0.22973a$



PCF: Holey Silica Cladding





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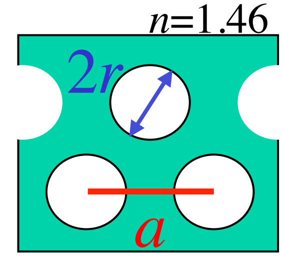

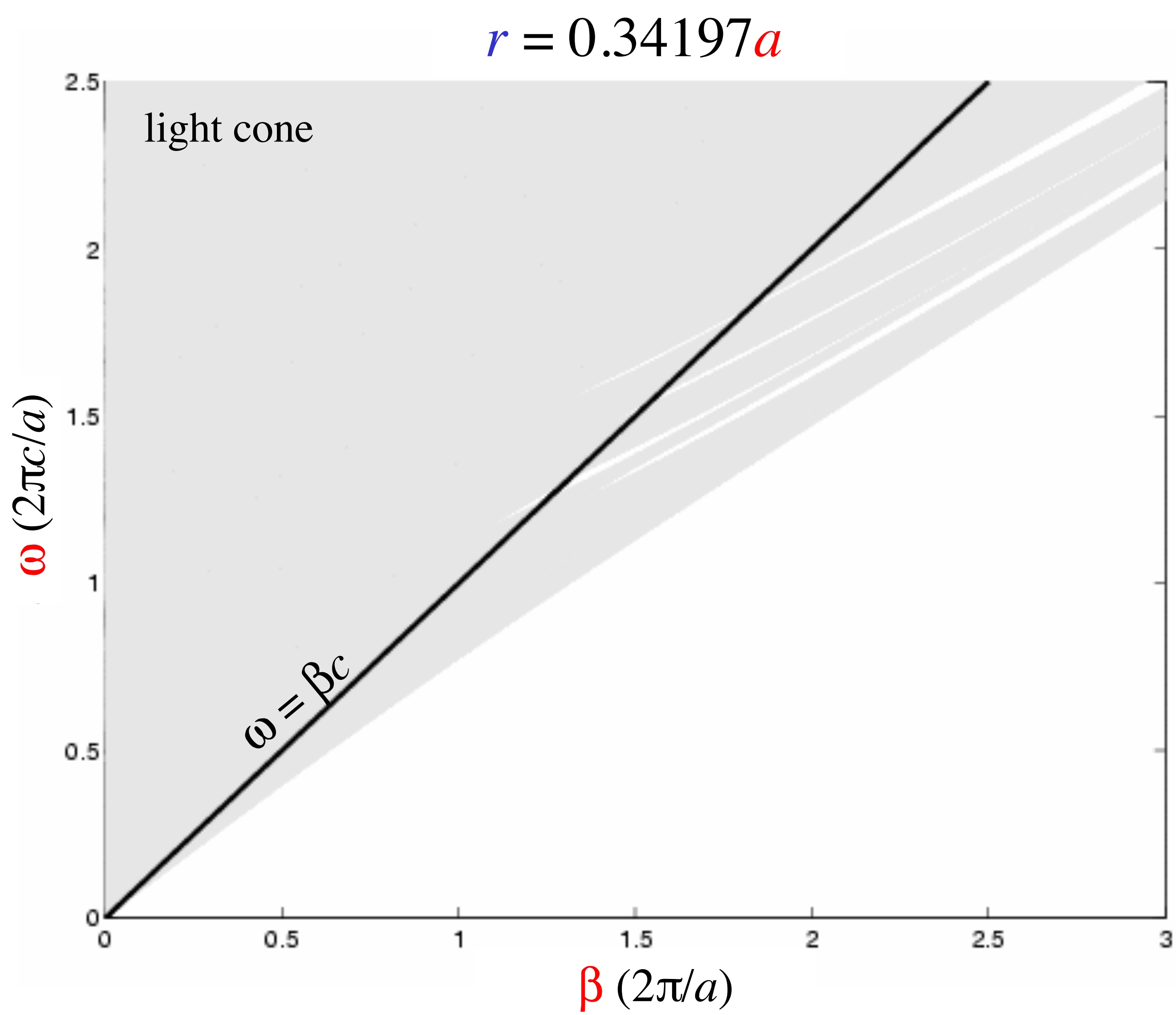

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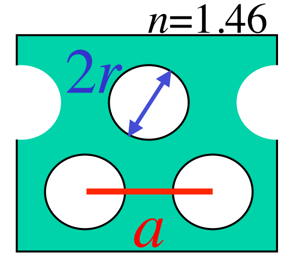
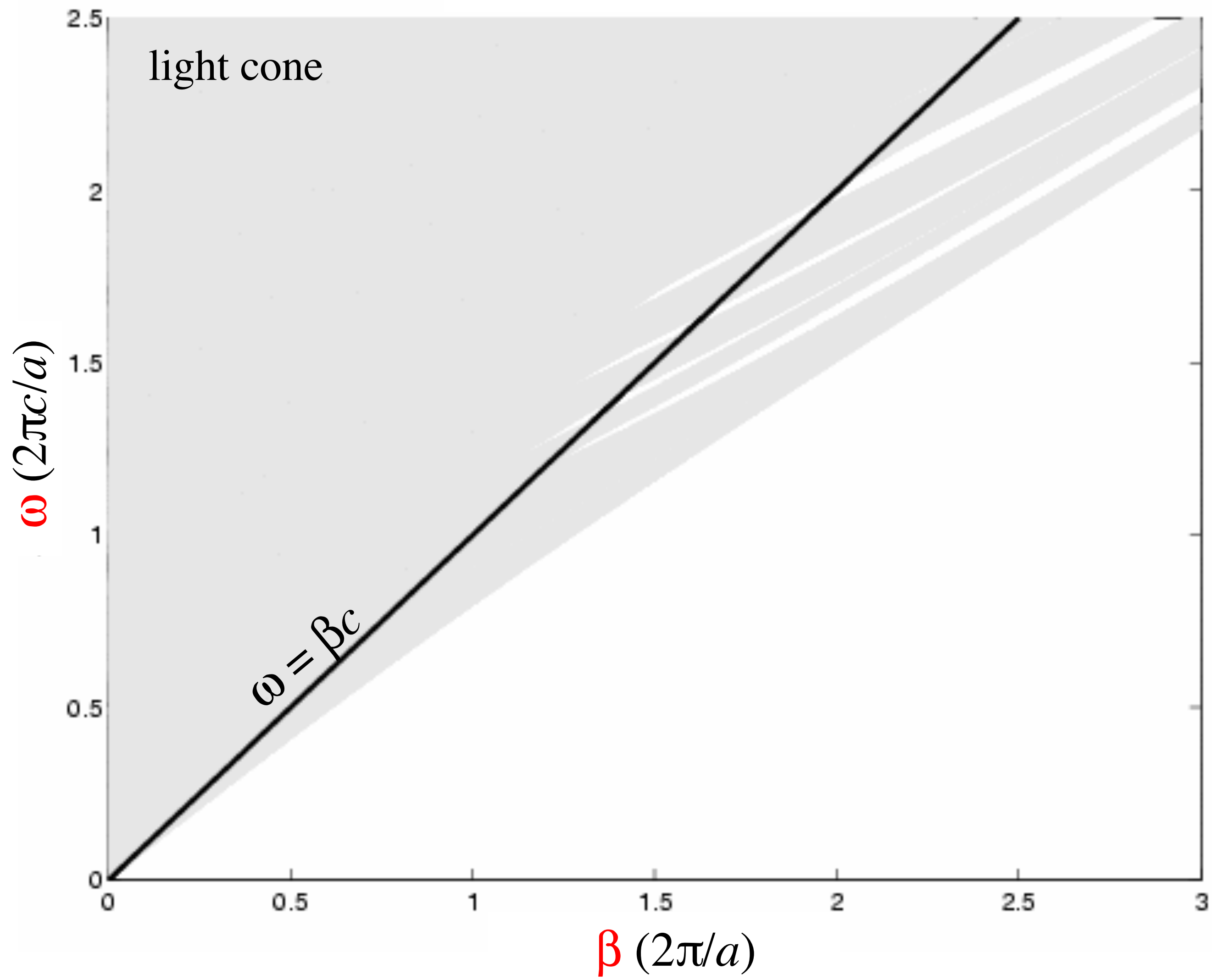
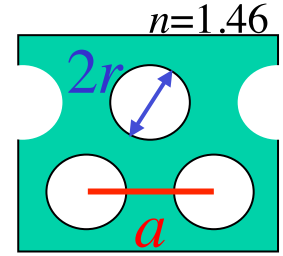


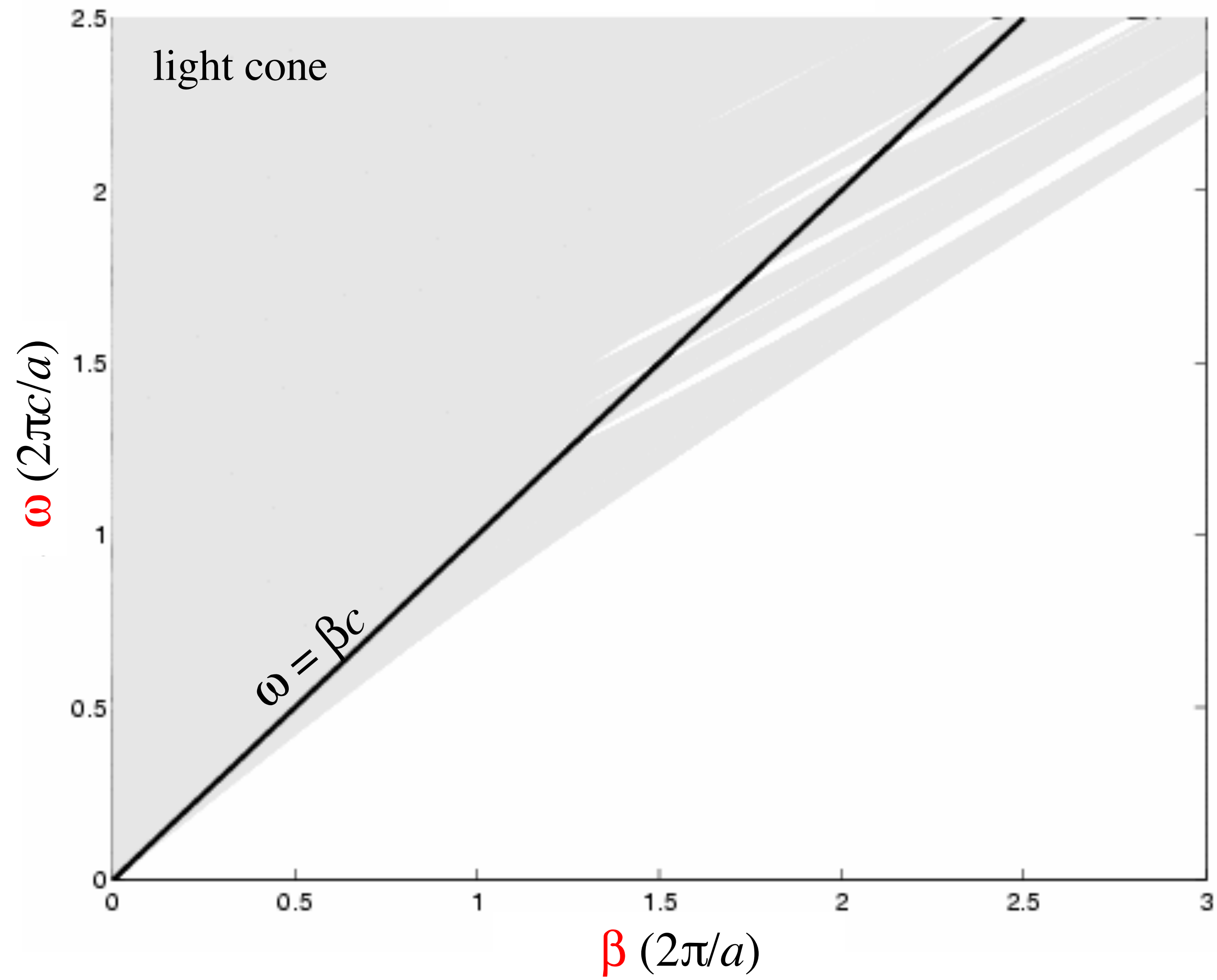
$r = 0.37193a$



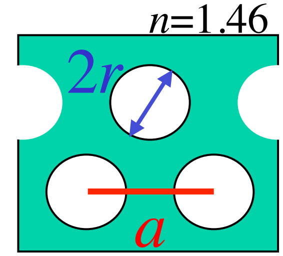
PCF: Holey Silica Cladding

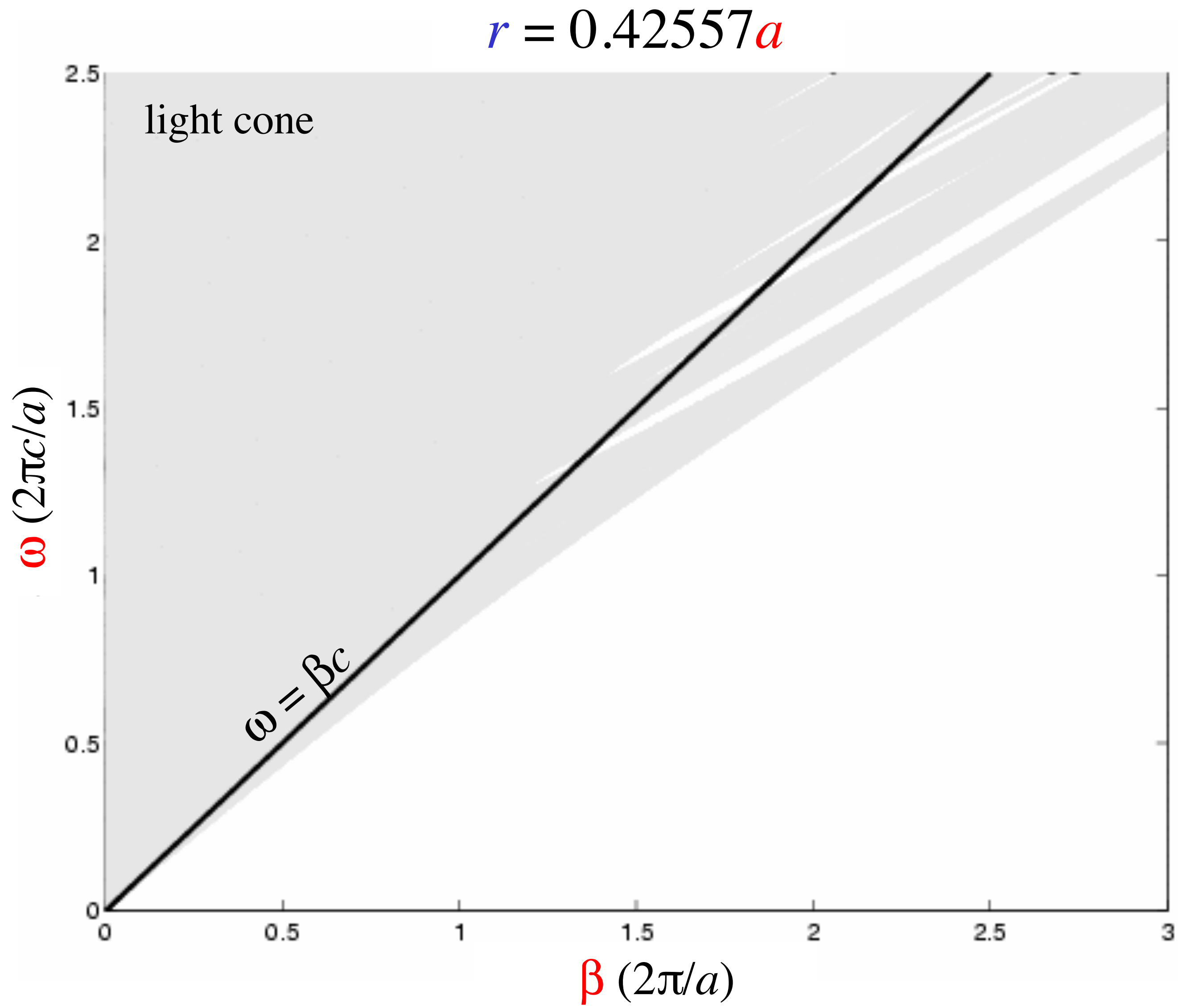


$r = 0.4a$



PCF: Holey Silica Cladding





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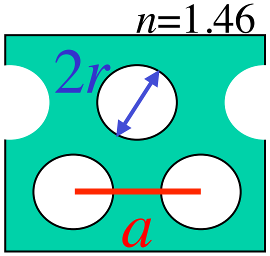
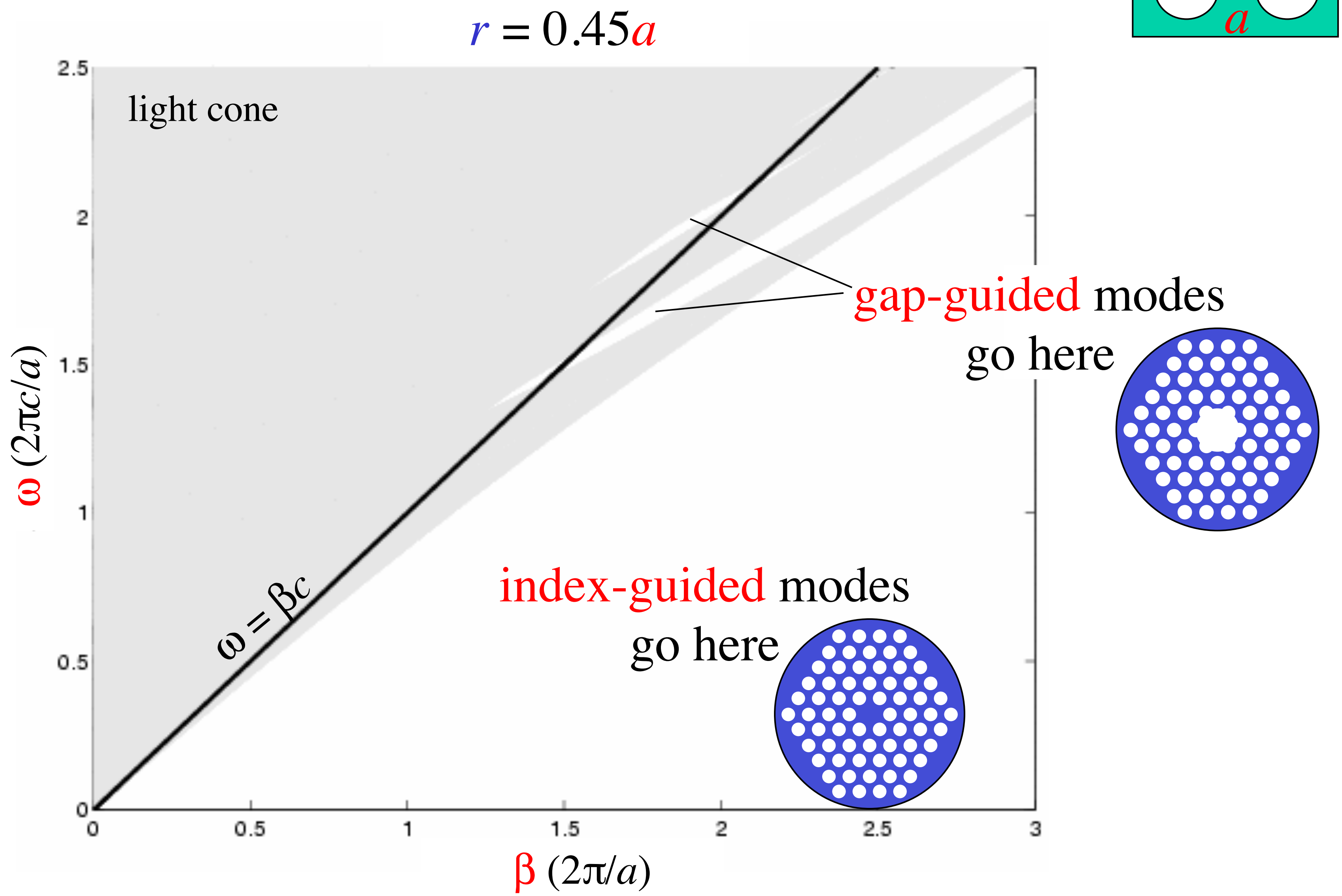
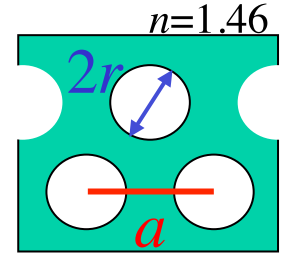
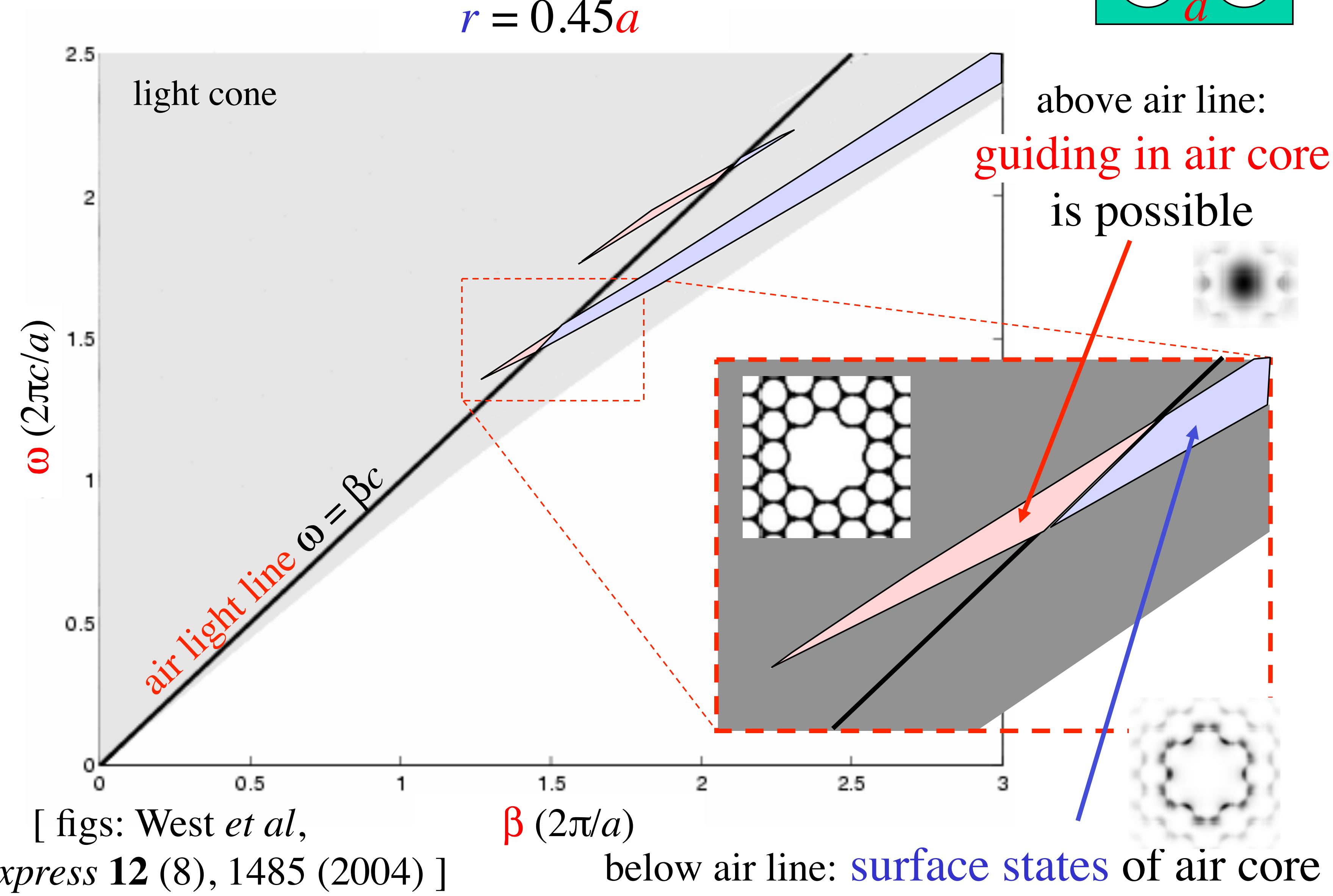




PCF: Holey Silica Cladding



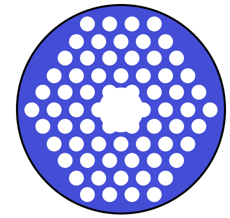
$r = 0.45a$

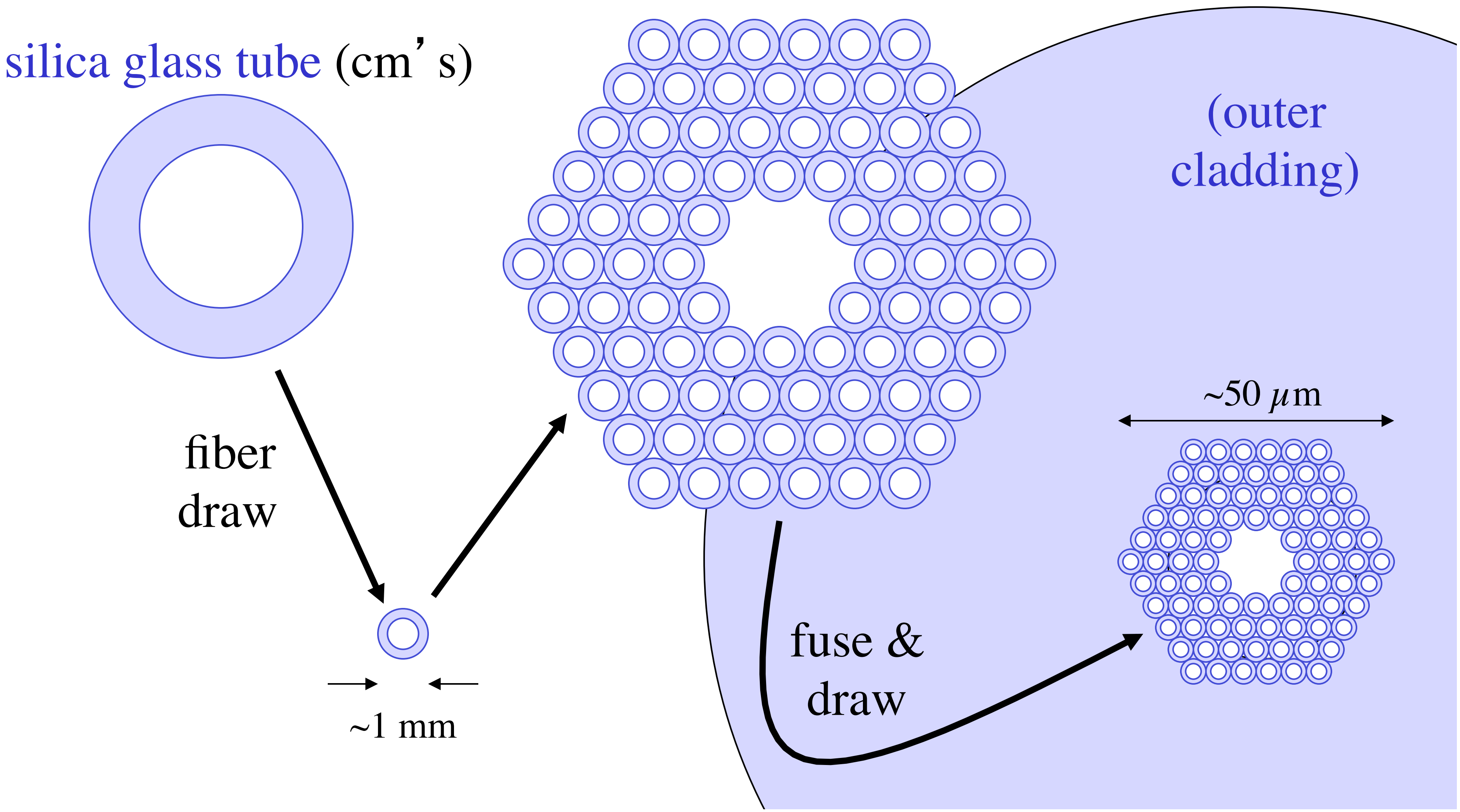


[figs: West *et al*,
Opt. *Express* **12** (8), 1485 (2004)]

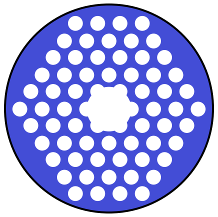
Experimental Air-guiding PCF

Fabrication (e.g.)

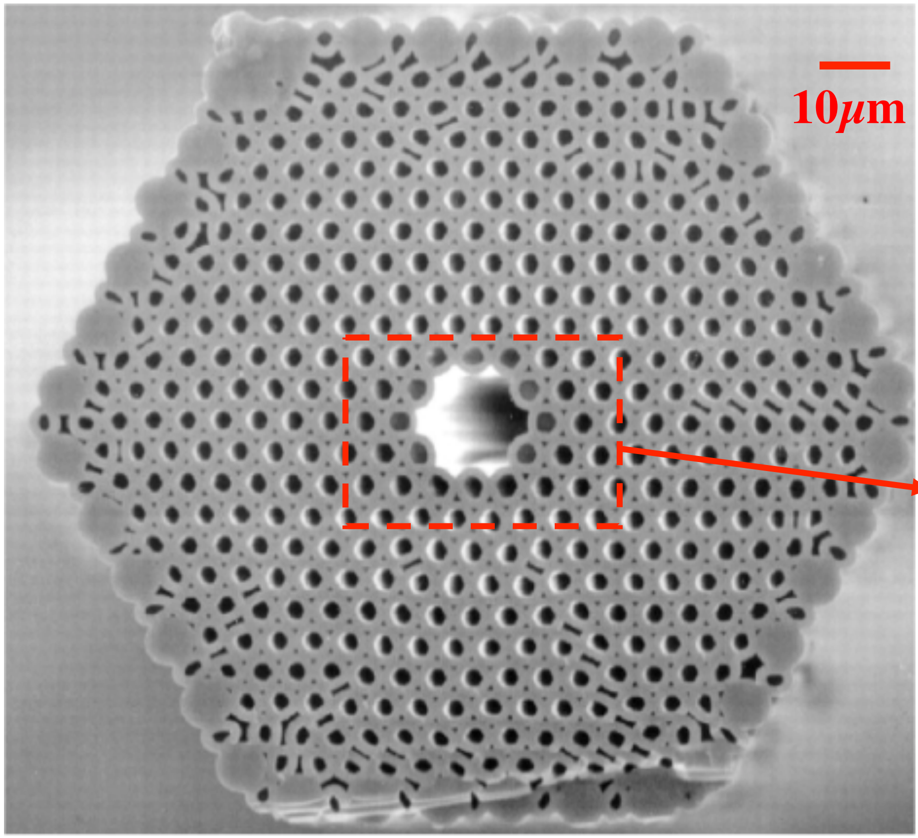


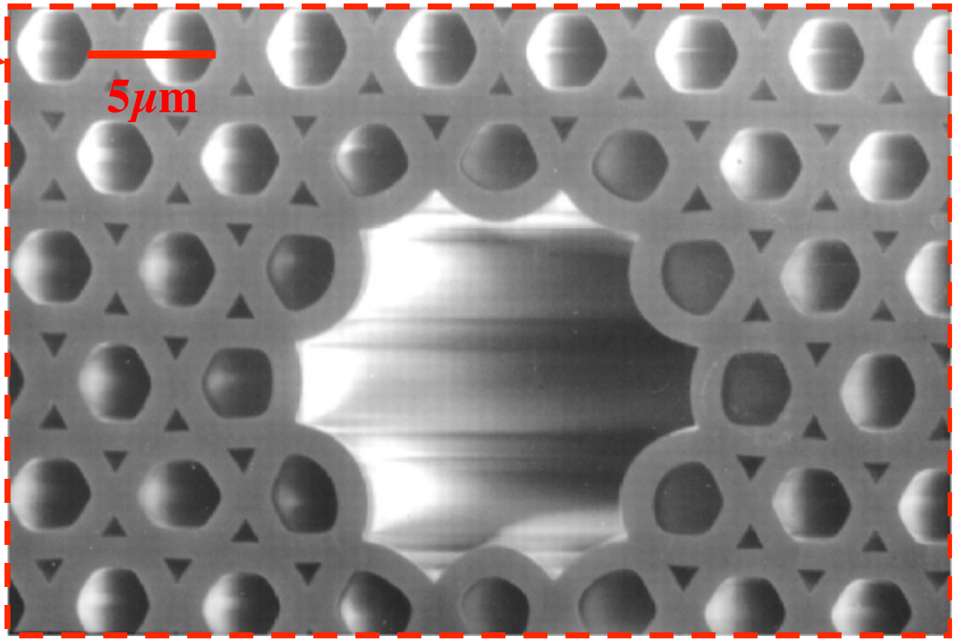


Experimental Air-guiding PCF

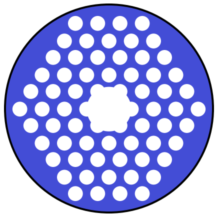


[R. F. Cregan *et al*., *Science* **285**, 1537 (1999)]

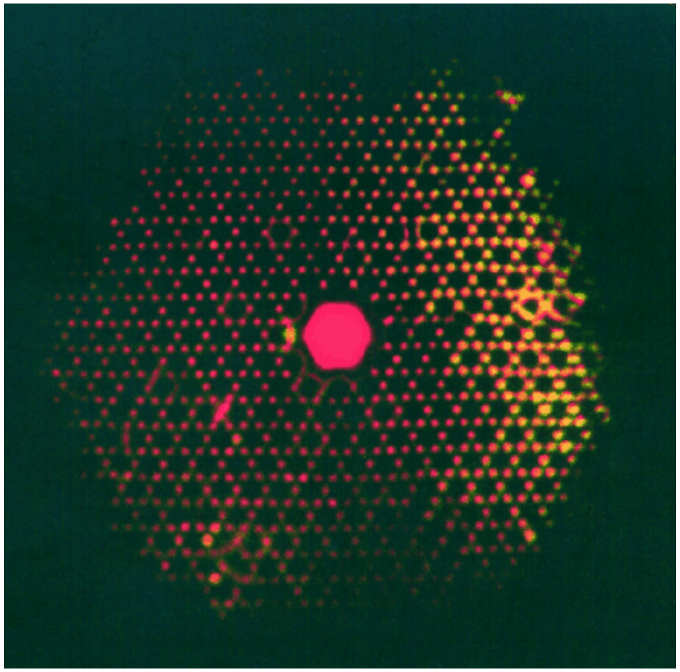




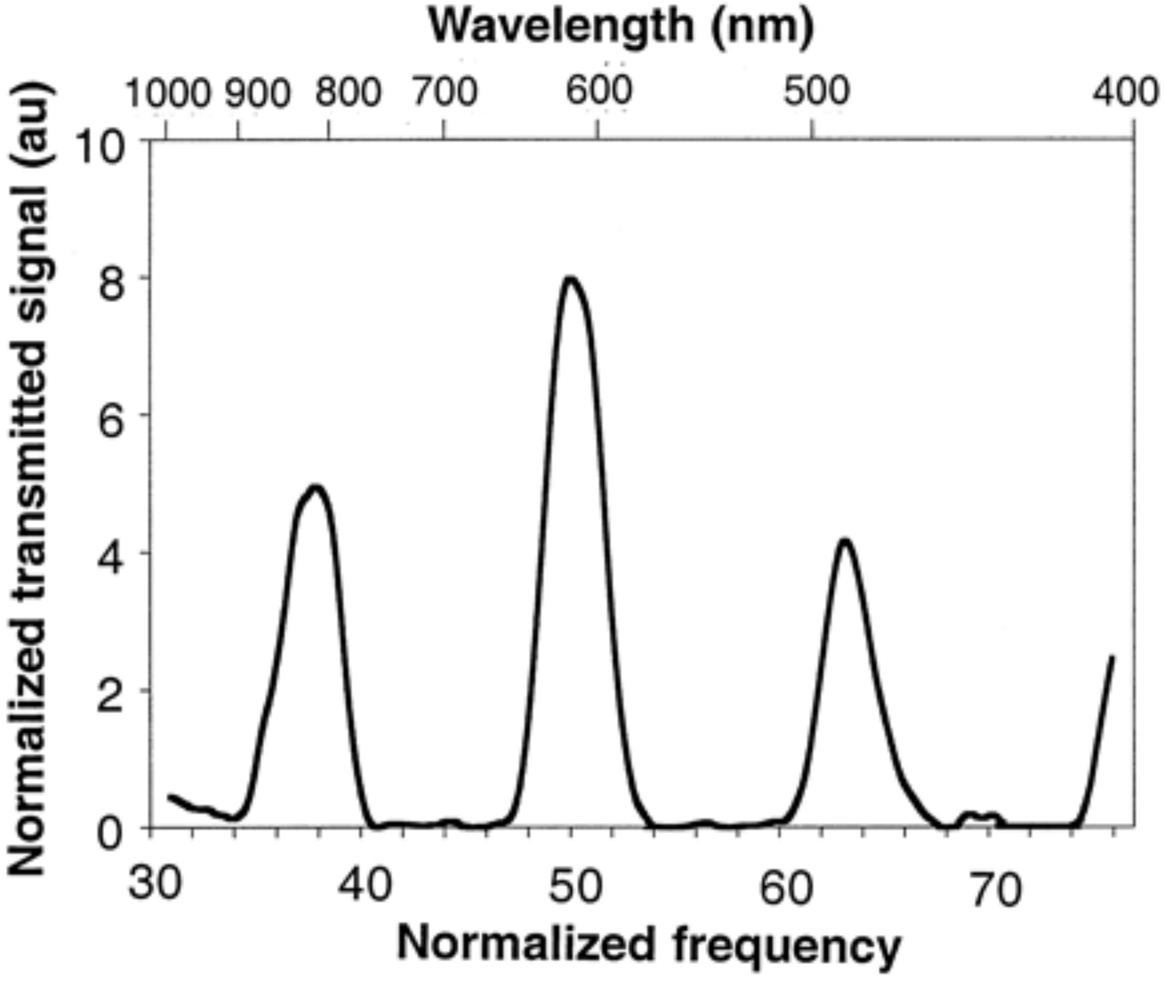
Experimental Air-guiding PCF



[R. F. Cregan *et al.*, *Science* **285**, 1537 (1999)]



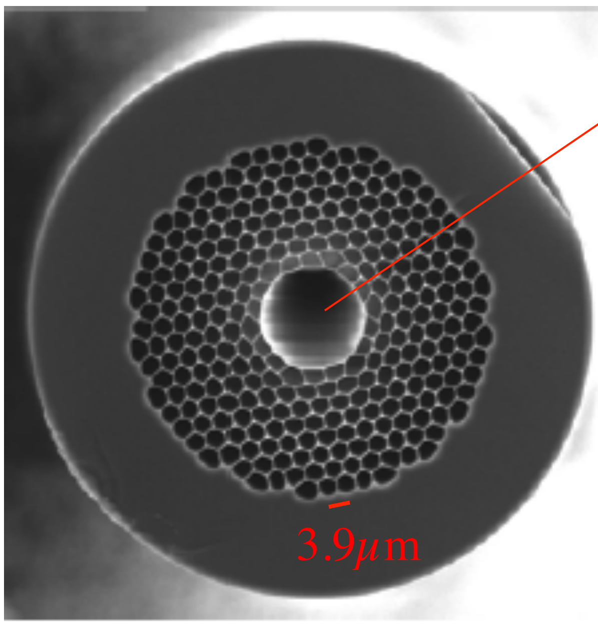
transmitted intensity
after ~ 3cm



ω (c/a) (**not** $2\pi c/a$)

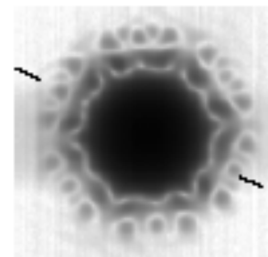
A more recent (lower-loss) example

[Mangan, *et al*., OFC 2004 PDP24]



hollow (air) core (covers 19 holes)

guided field profile:
(flux density)

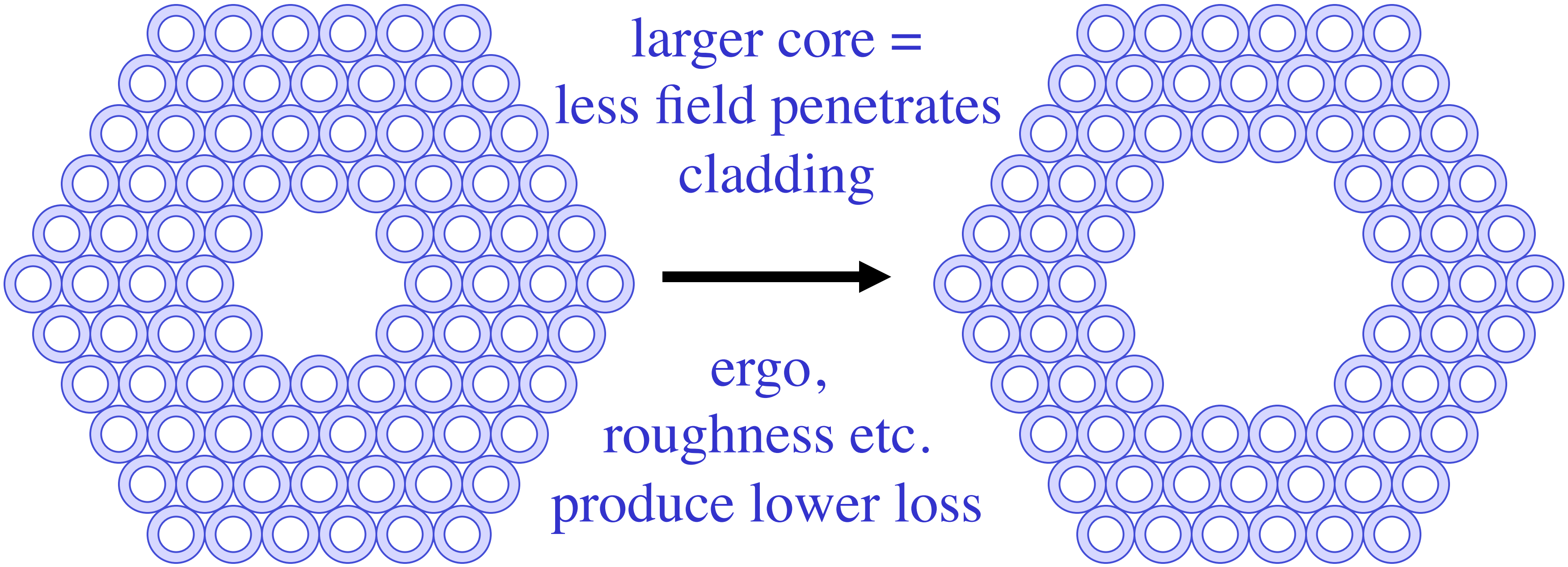


1.7dB/km

BlazePhotonics

over ~ 800m @1.57μm

Improving air-guiding losses



13dB/km

Corning

over ~ 100m @1.5μm

[Smith, *et al*., *Nature* **424**, 657 (2003)]

1.7dB/km

BlazePhotonics

over ~ 800m @1.57μm

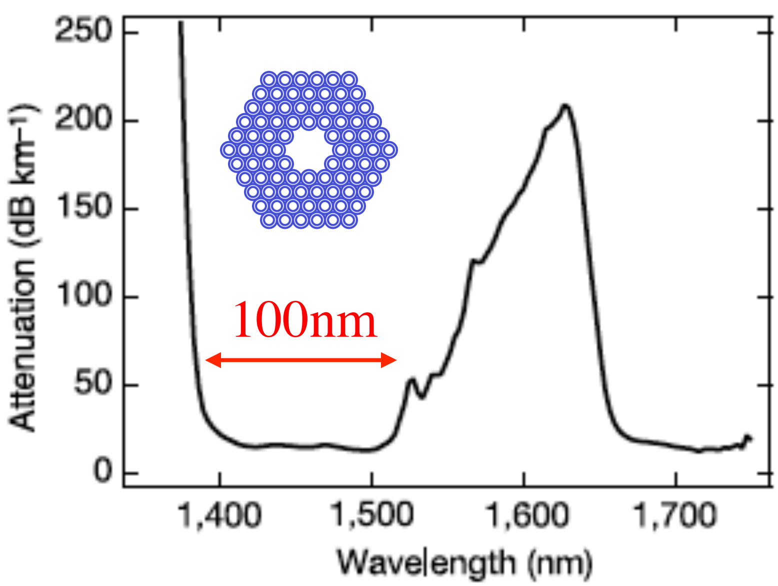
[Mangan, *et al*., *OFC 2004* PDP24]

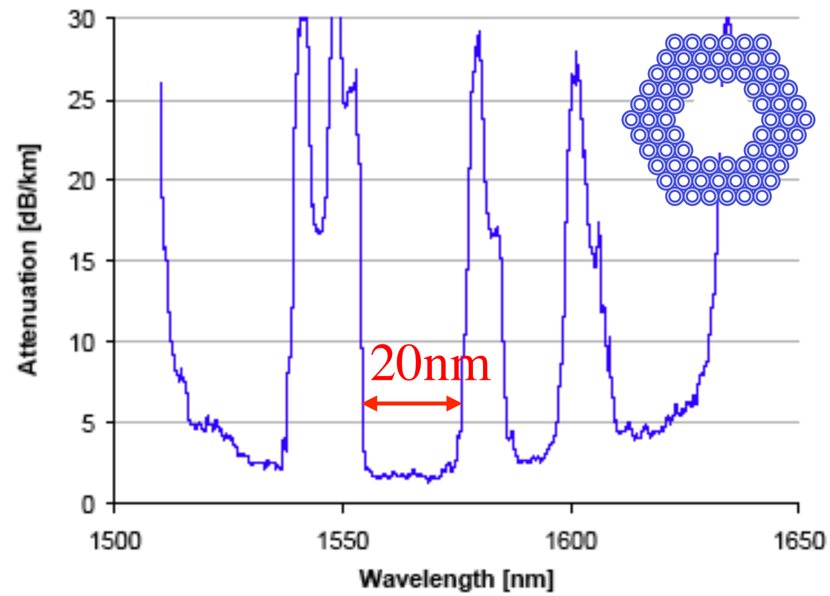
State-of-the-art air-guiding losses

larger core = more surface states crossing guided mode

… but surface states can be removed by proper crystal termination

[West, *Opt. Express* **12** (8), 1485 (2004)]





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[Smith, *et al*., *Nature* **424**, 657 (2003)]

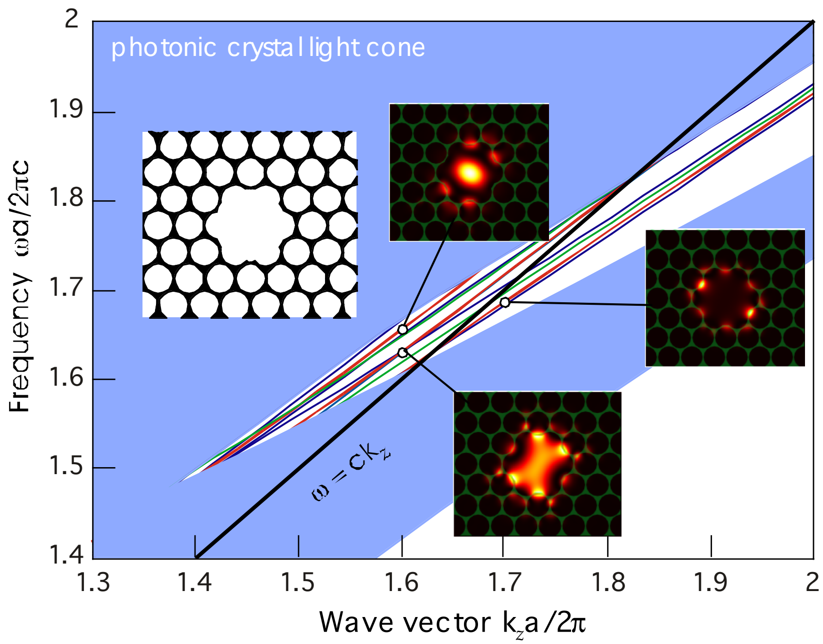
1.7dB/km

BlazePhotonics

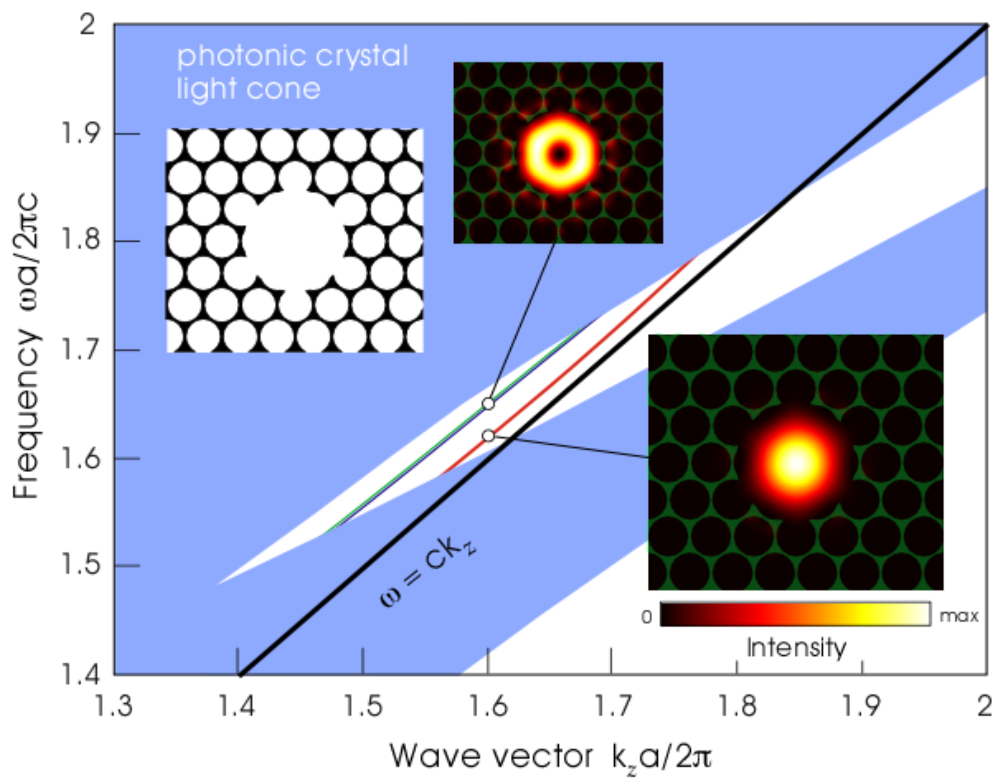
over ~ 800m @1.57μm

[Mangan, *et al*., *OFC 2004* PDP24]

Surface States vs. Termination



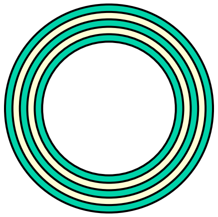
changing the crystal termination
can eliminate surface states



[West, *Opt. Express* **12** (8), 1485 (2004)]

[Saitoh, *Opt. Express* **12** (3), 394 (2004)]

[Kim, *Opt. Express* **12** (15), 3436 (2004)]



Bragg Fiber Cladding

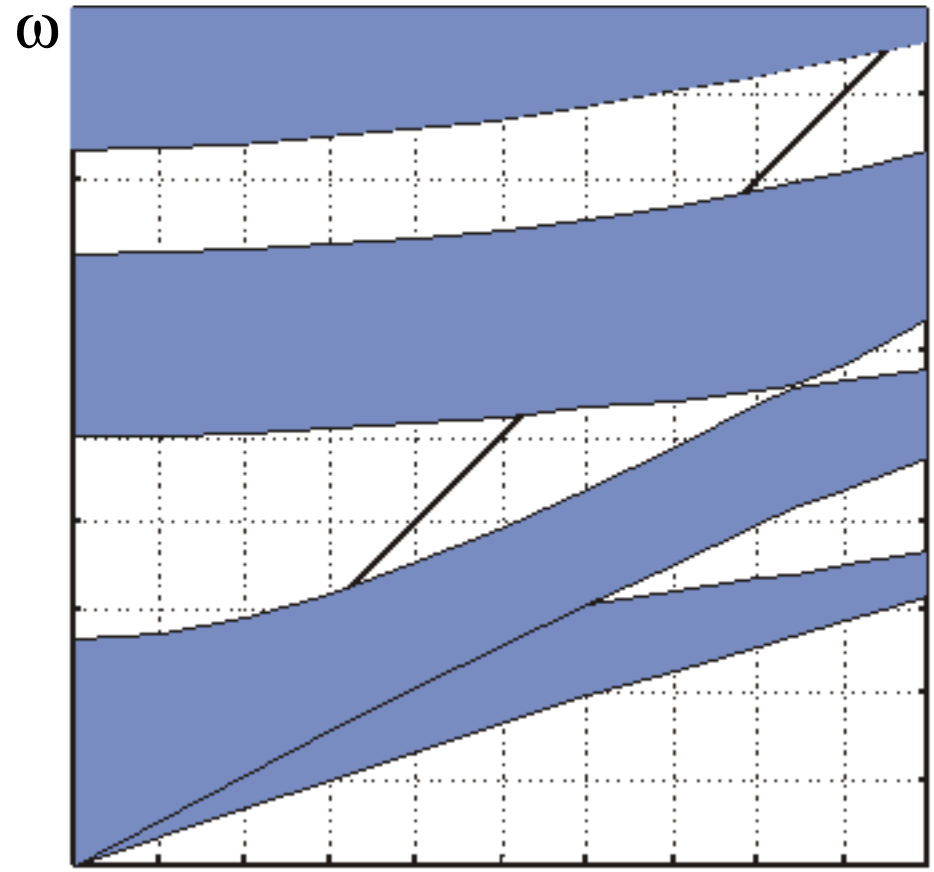
at large radius,
becomes ~ planar

$\beta \odot$

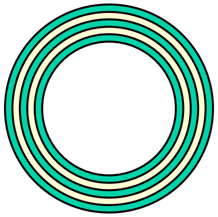
radial k_r
(Bloch wavevector)

$\cancel{k_\phi}$ 0 by conservation of angular momentum

Bragg fiber gaps (1d eigenproblem)

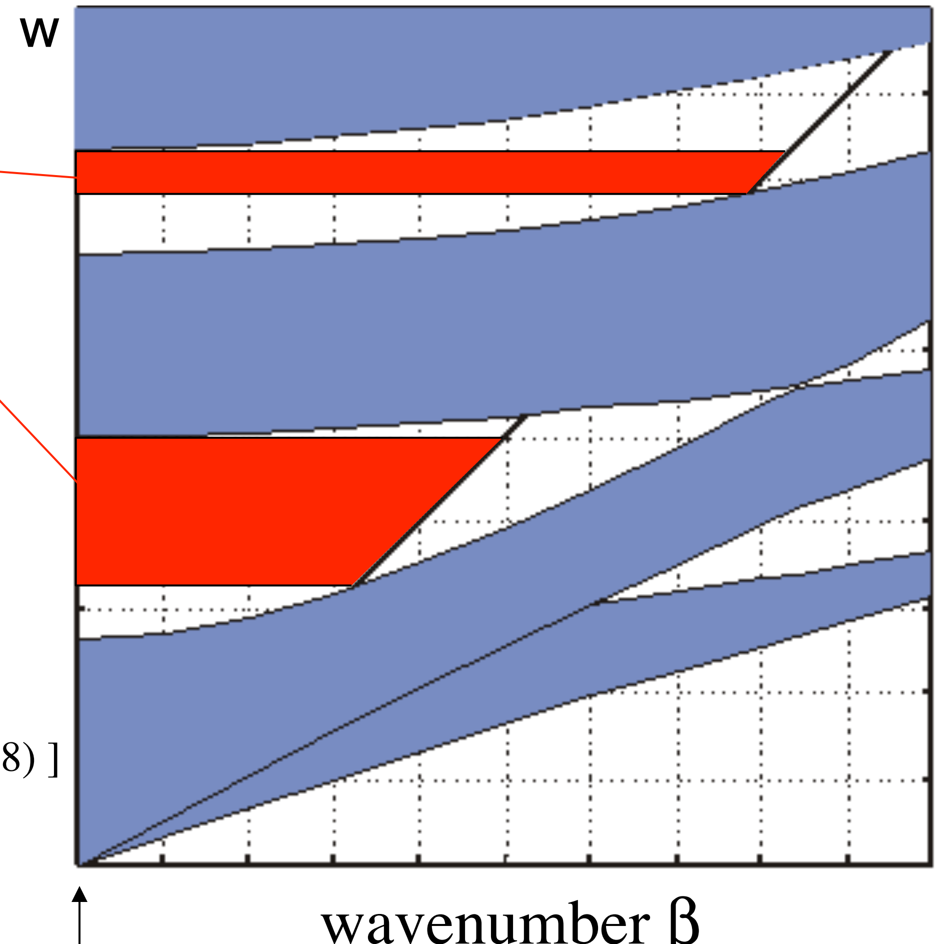


β = 0: normal incidence



Omnidirectional Cladding

Bragg fiber gaps (1d eigenproblem)

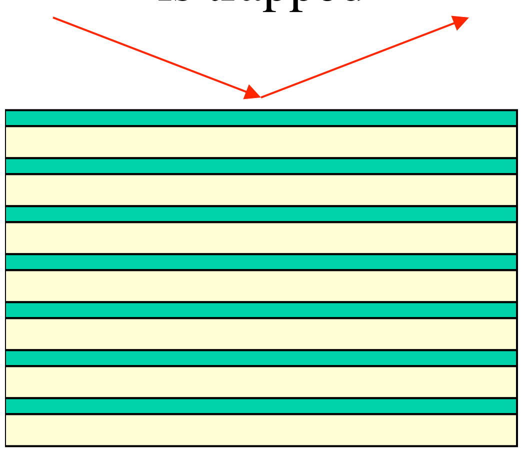


omnidirectional
(planar) reflection

wavenumber β

$\beta = 0$: normal incidence

e.g. light from
fluorescent sources
is trapped

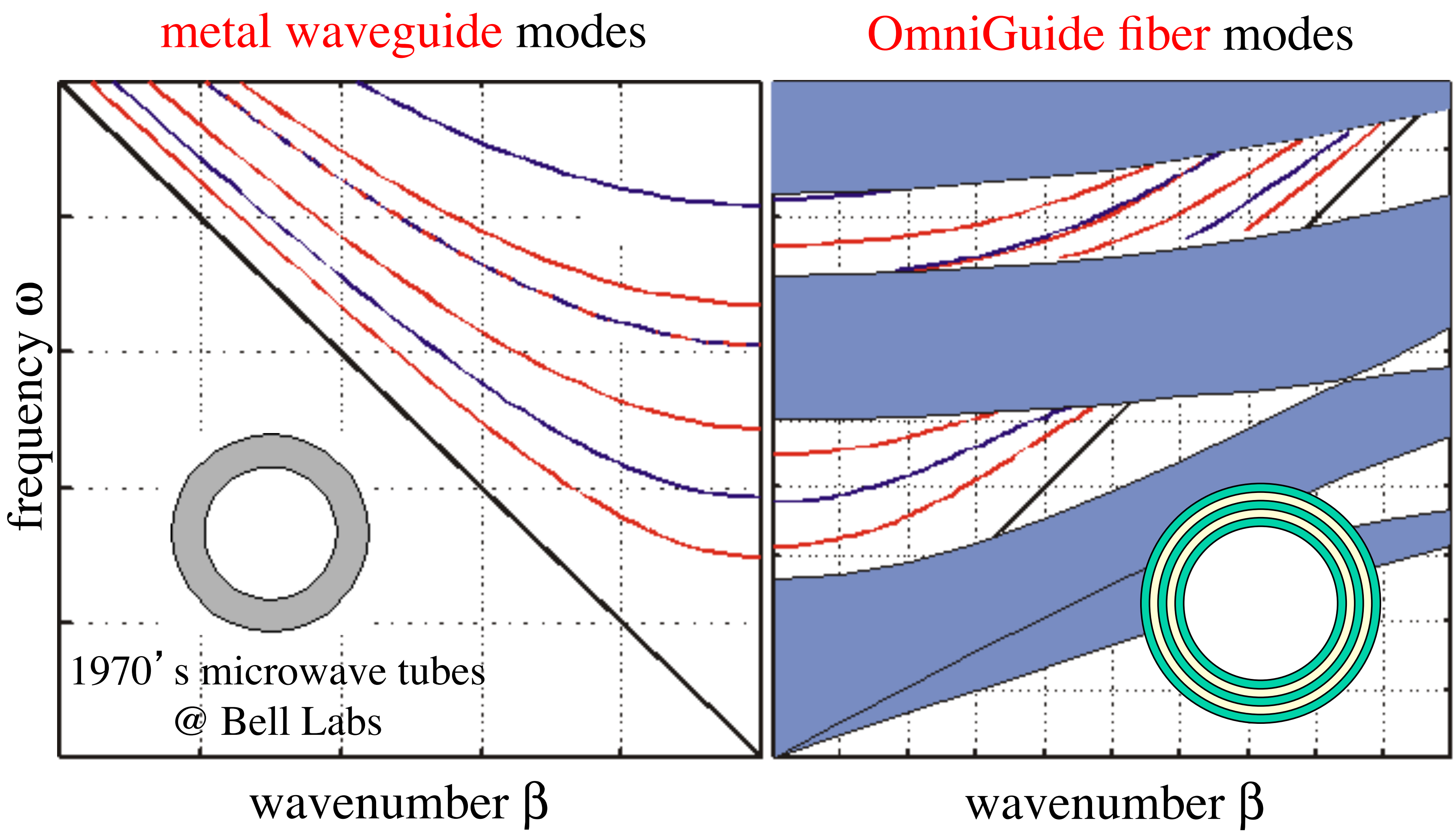


$\beta_{\odot}$

for n_{hi} / n_{lo}
big enough
and $n_{lo} > 1$

[J. N. Winn *et al*,
Opt. Lett. **23**, 1573 (1998)]

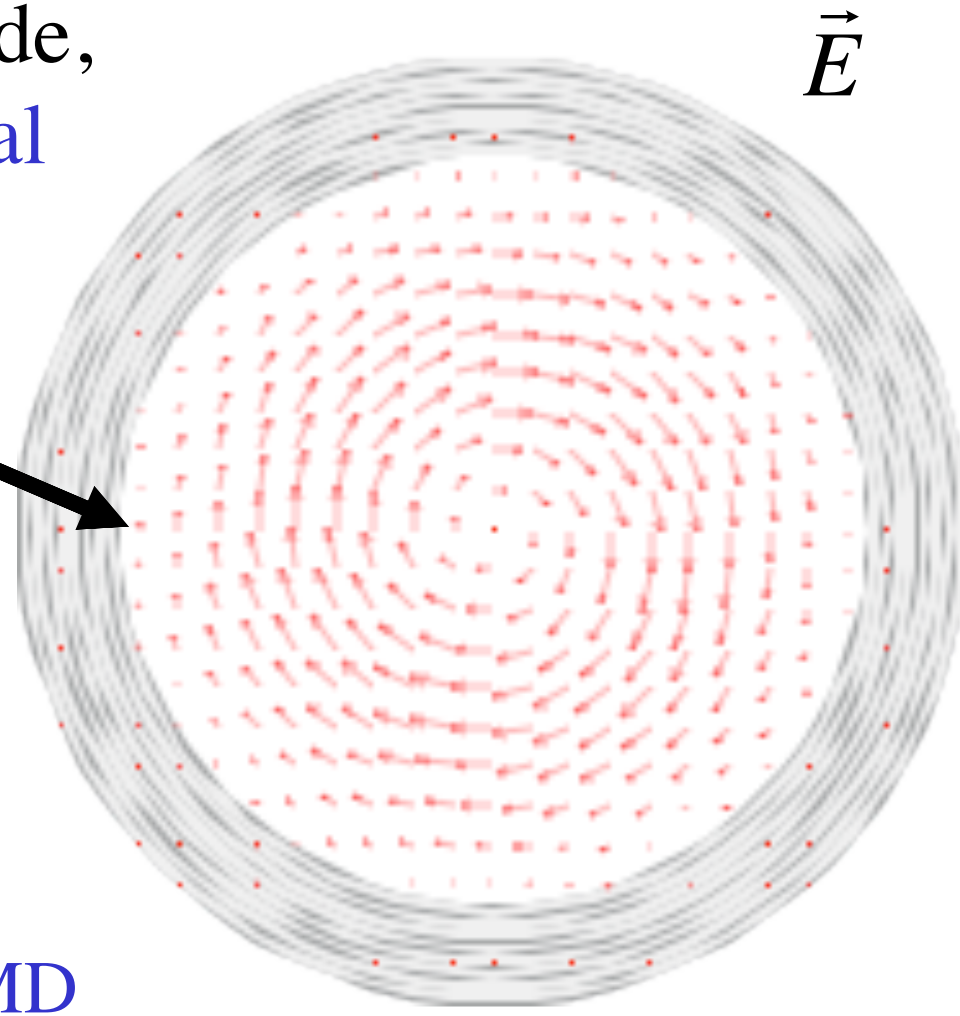
Hollow Metal Waveguides, Reborn



modes are directly analogous to those in hollow metal waveguide

An Old Friend: the TE_{01} mode

lowest-loss mode,
just as in metal



(near) node at interface
= strong confinement
= low losses

non-degenerate mode
— cannot be split
= no birefringence or PMD

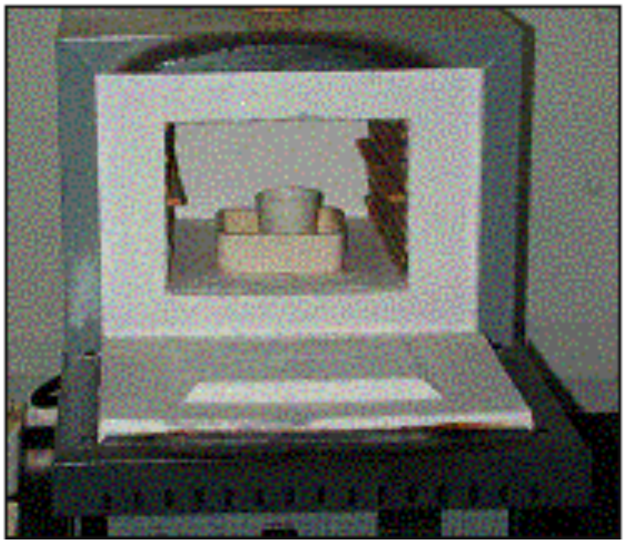
Yes, but how do you make it?

[figs courtesy Y. Fink *et al*., MIT]

1

find compatible materials

(many new possibilities)



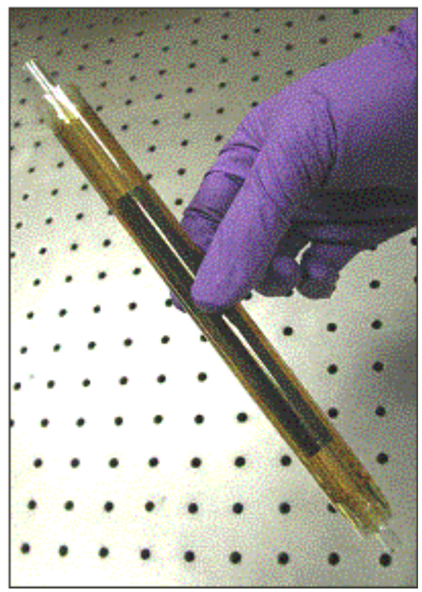
chalcogenide glass, n ~ 2.8

+ polymer (or oxide), n ~ 1.5

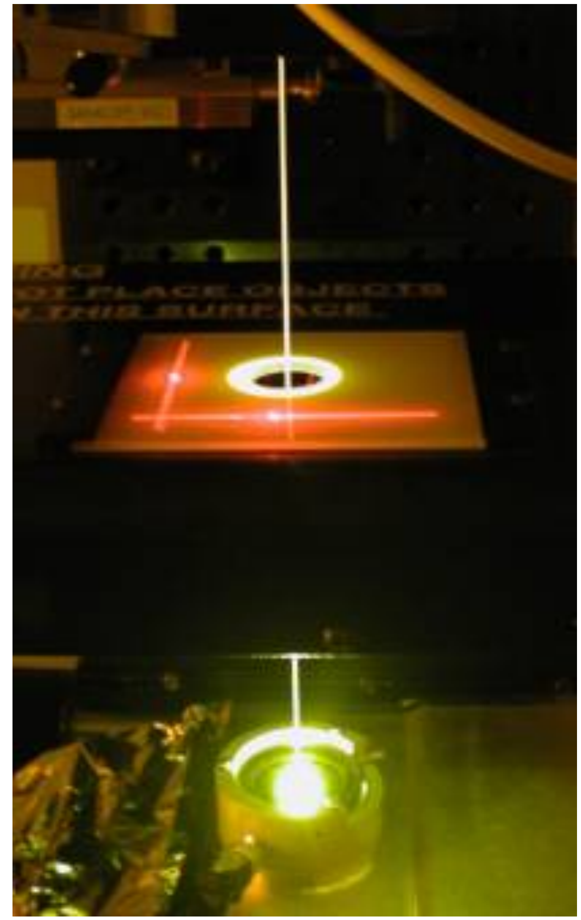
2

Make pre-form

("scale model")



3

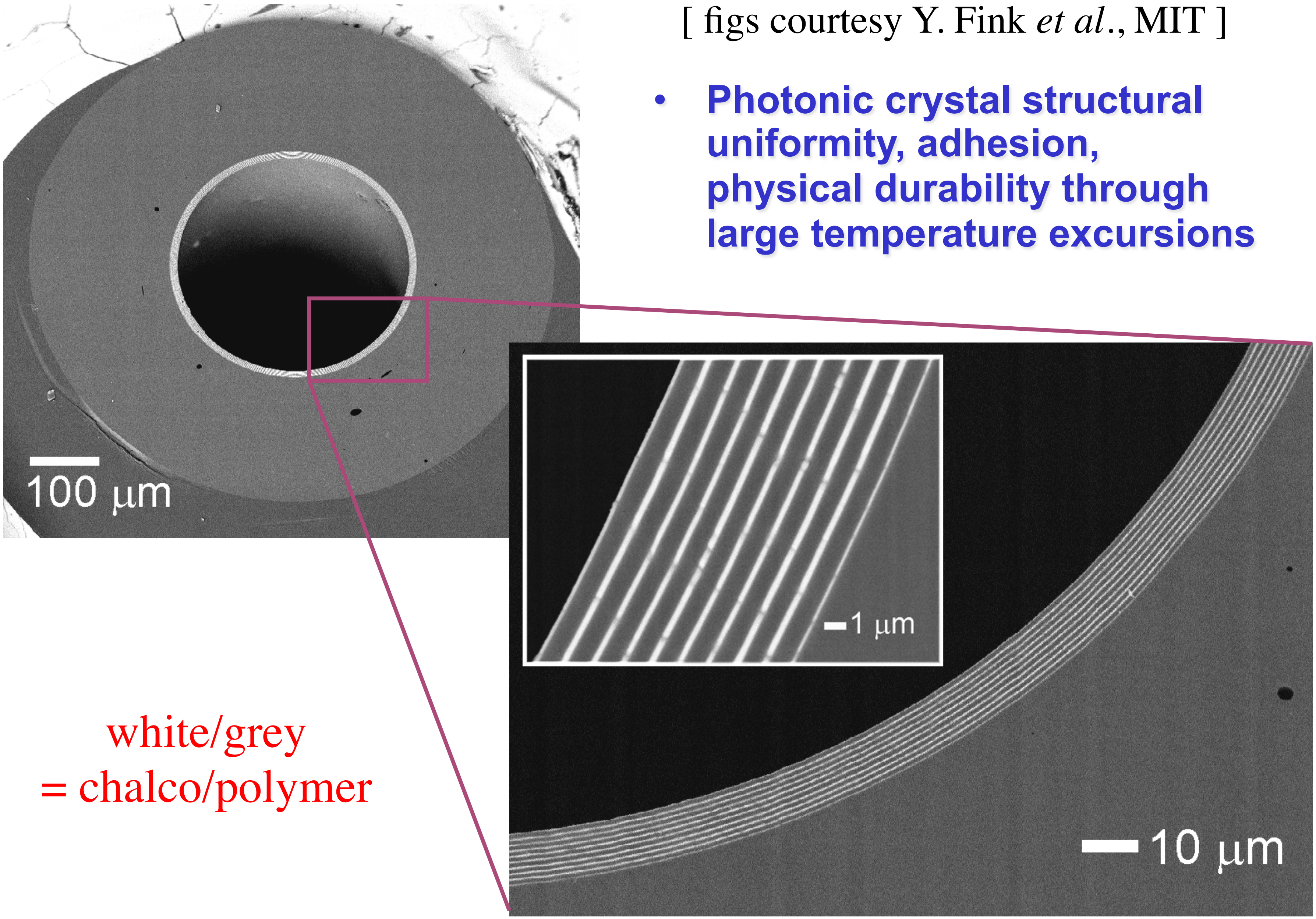


fiber drawing

A Drawn Bandgap Fiber

[figs courtesy Y. Fink *et al*., MIT]

- **Photonic crystal structural uniformity, adhesion, physical durability through large temperature excursions**



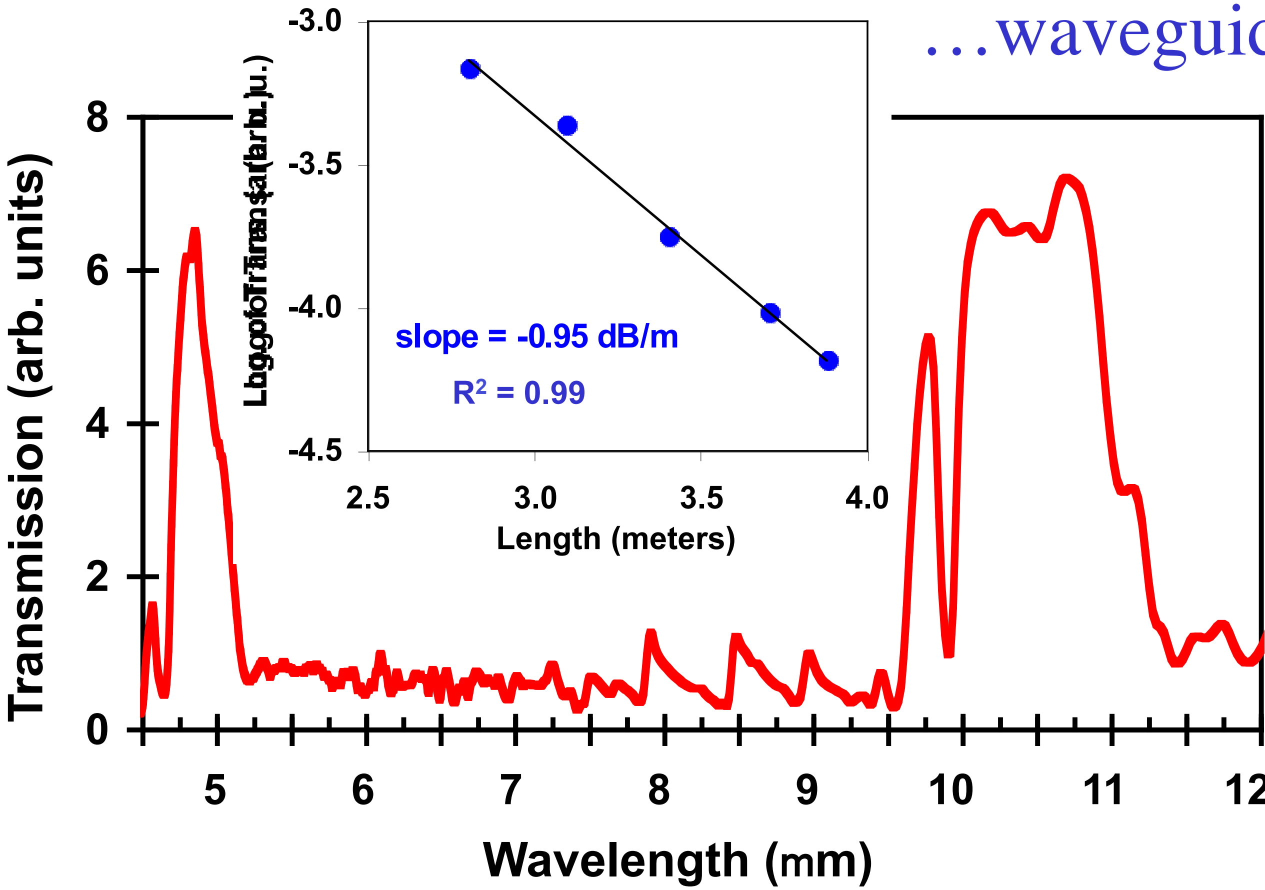
white/grey
= chalco/polymer

High-Power Transmission

at 10.6μm (no previous dielectric waveguide)

Polymer losses @10.6μm ~ 50,000dB/m…

…waveguide losses < 1dB/m

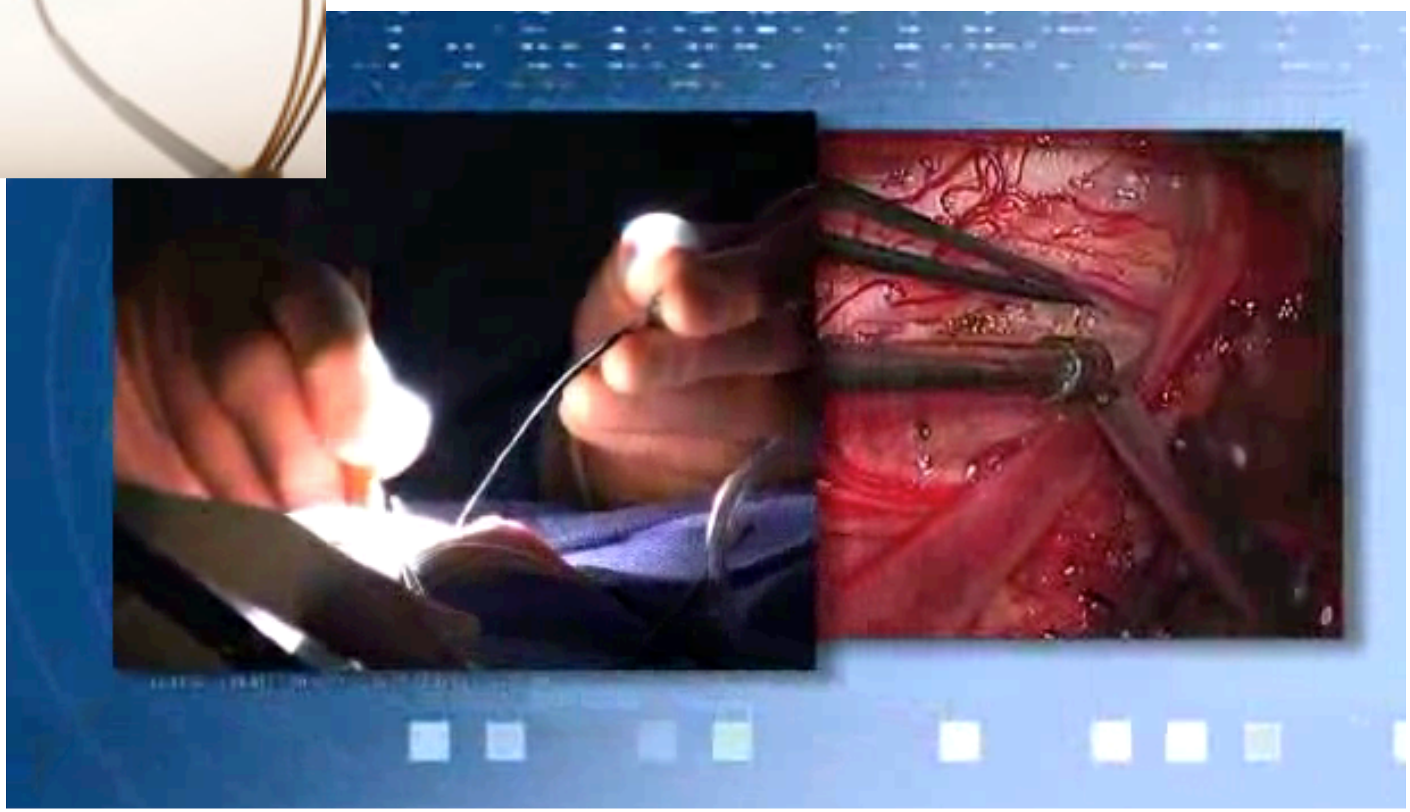


[B. Temelkuran *et al.*, *Nature* **420**, 650 (2002)]

[figs courtesy Y. Fink *et al.*, MIT]

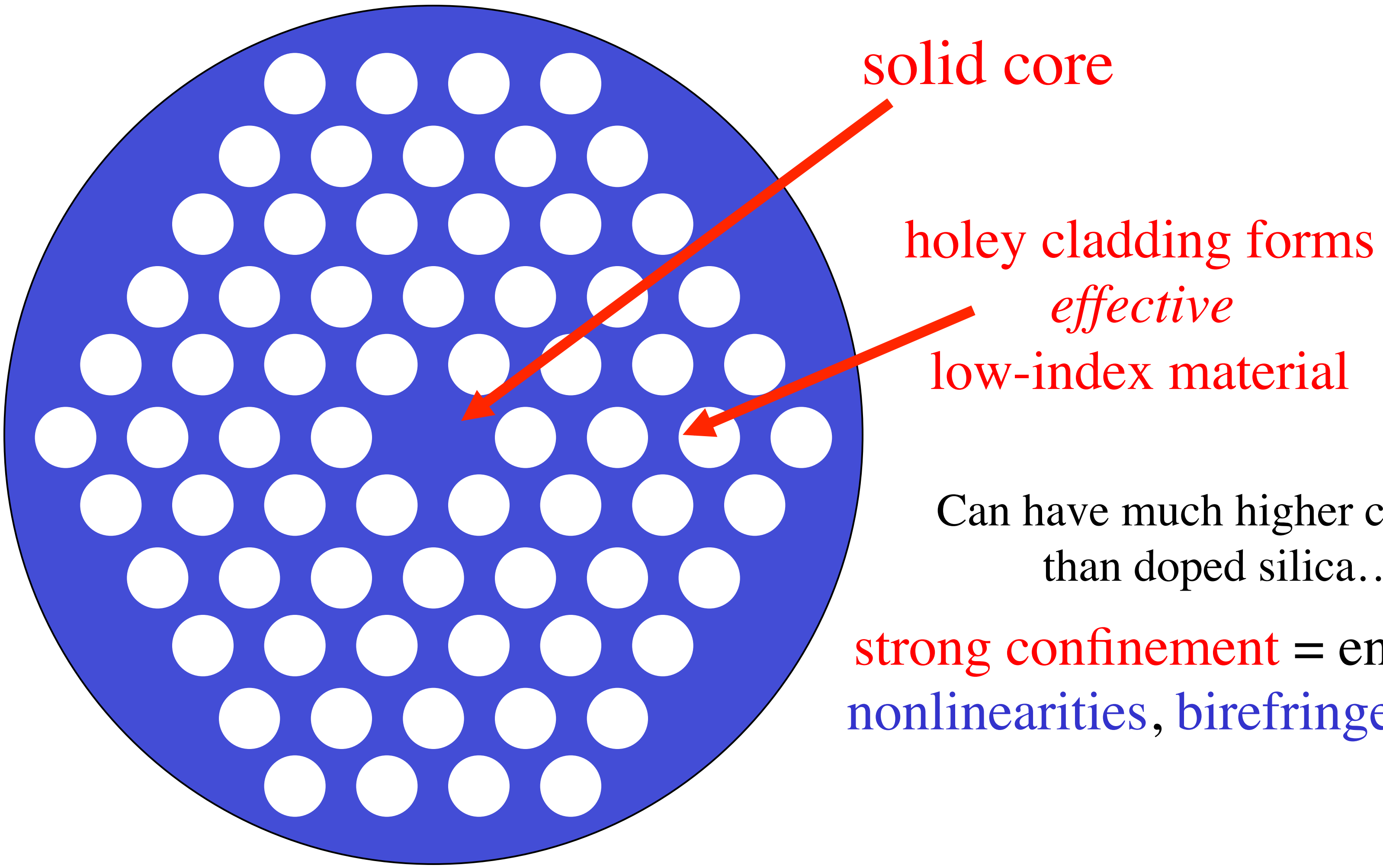
Application: Laser Surgery

[www.omni-guide.com]



Index-Guiding PCF & microstructured fiber:

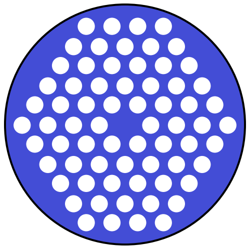
Holey Fibers



Can have much higher contrast than doped silica…

strong confinement = enhanced nonlinearities, birefringence, …

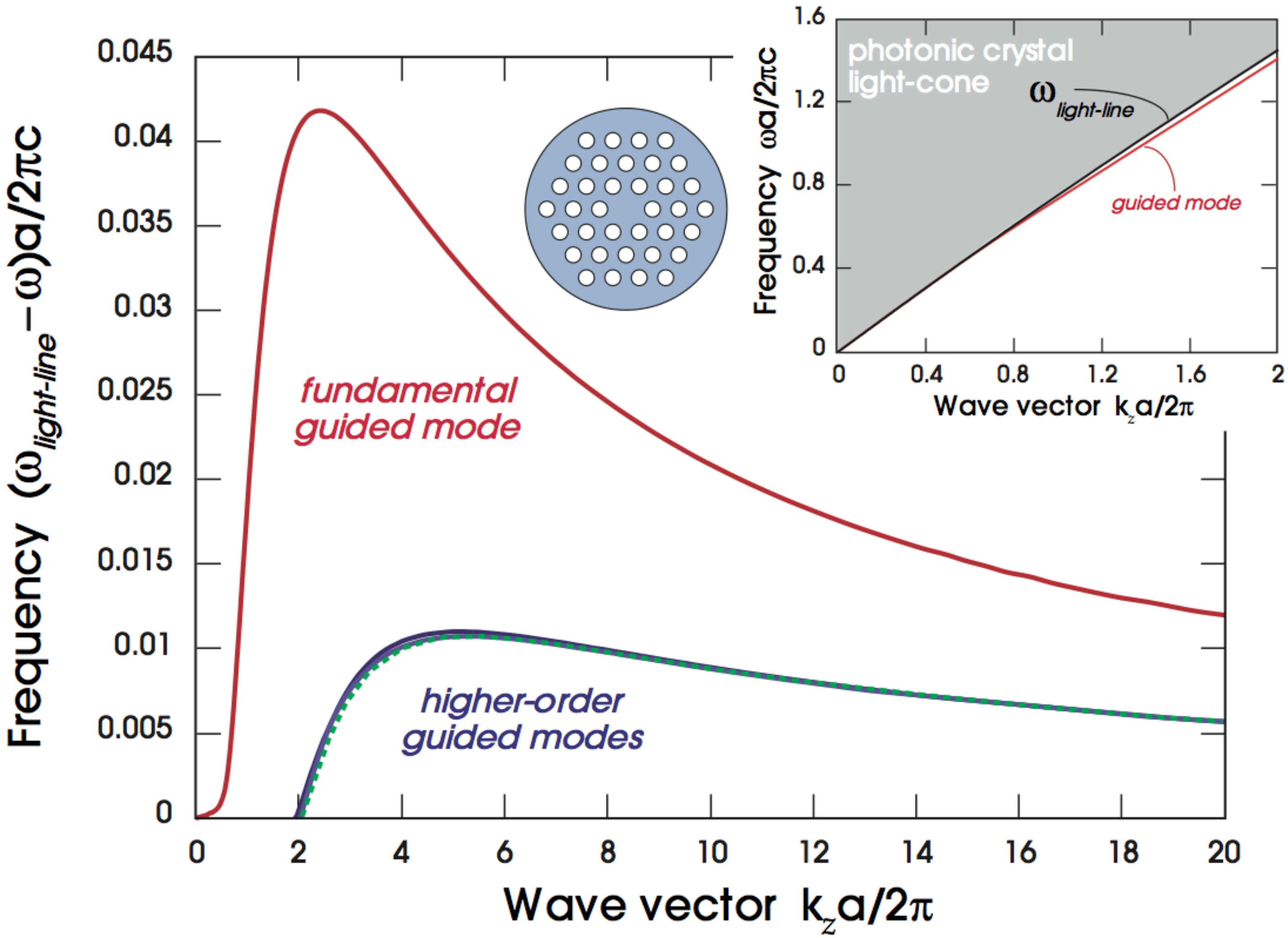
[J. C. Knight *et al*., *Opt*. *Lett*. **21**, 1547 (1996)]



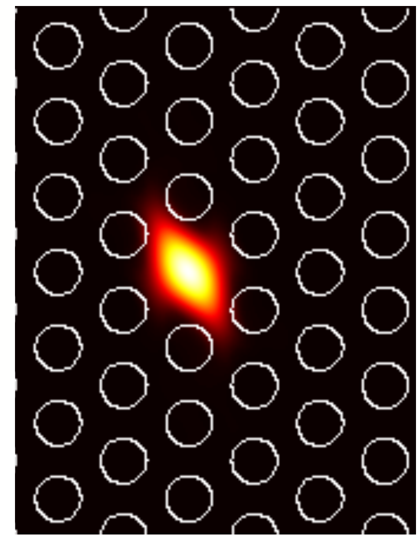
Guided Mode in a Solid Core

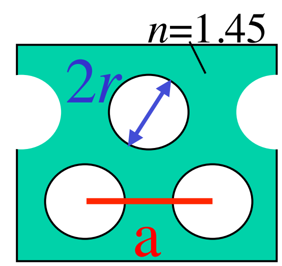
small computation: only lowest-w band!

(~ one minute, planewave)



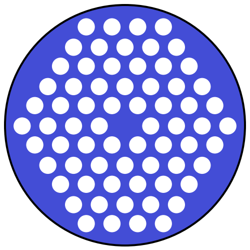
power density

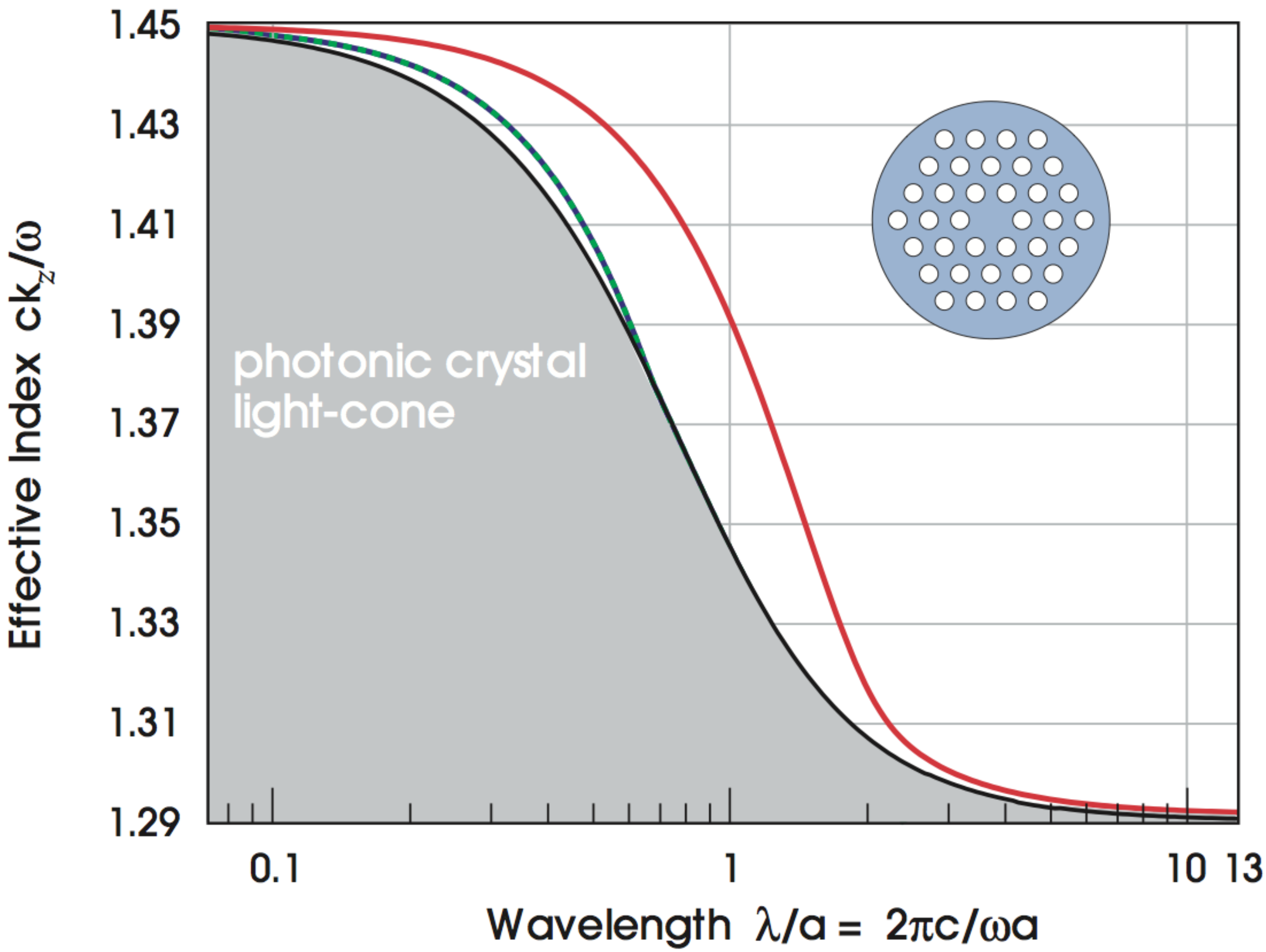


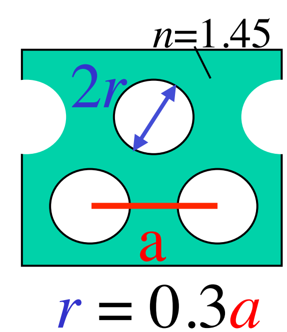


$r = 0.3a$

λ-dependent "index contrast"

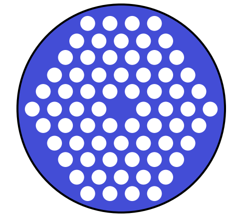


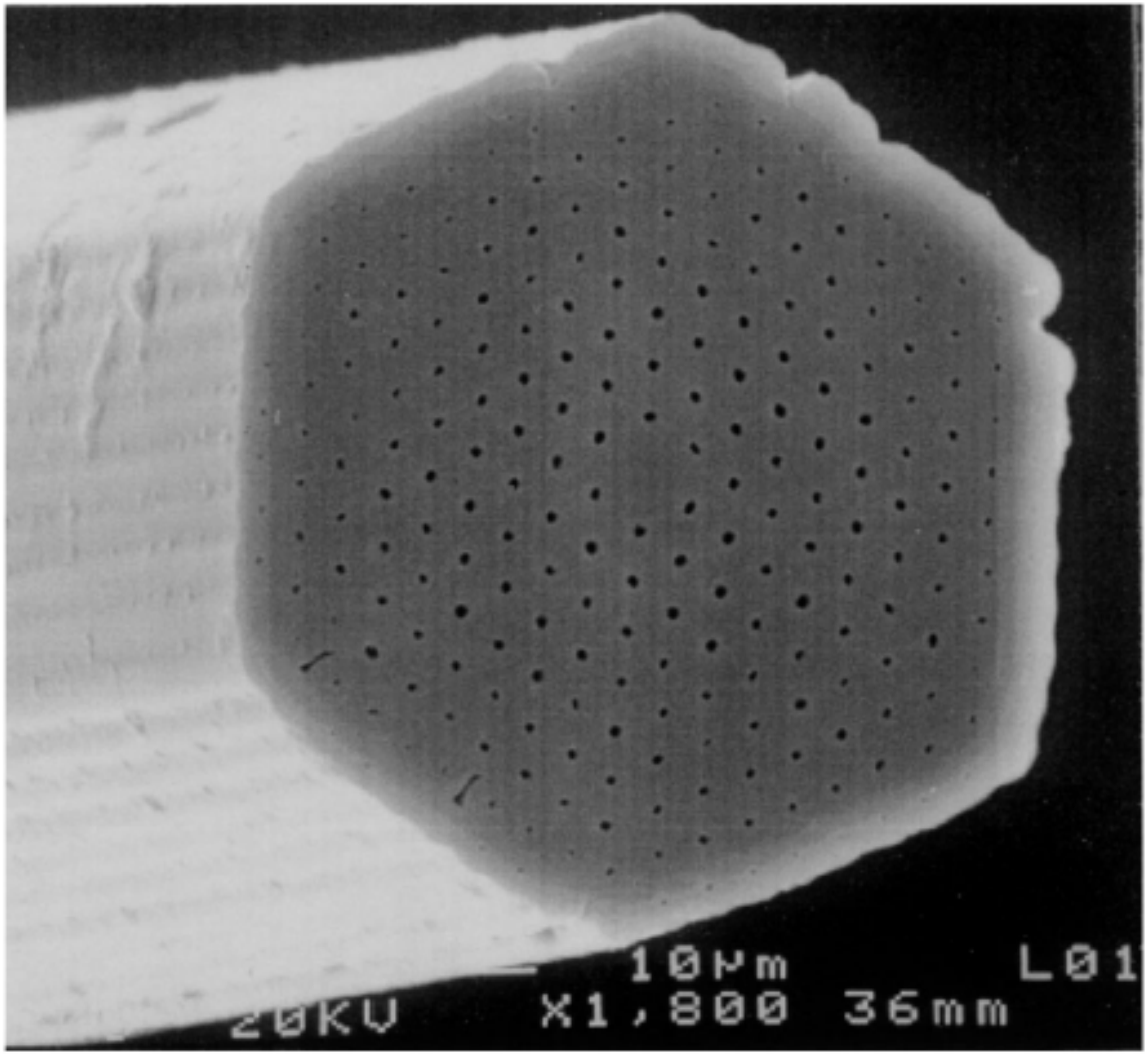




Endlessly Single-Mode

[T. A. Birks *et al*., *Opt. Lett*. **22**, 961 (1997)]

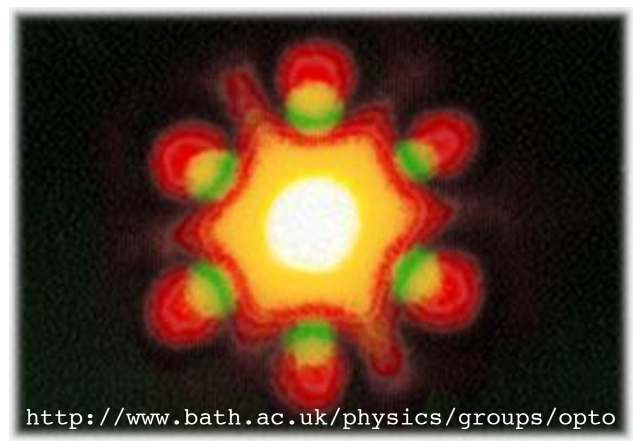




at higher ω
(smaller λ),
the light is more
concentrated in silica

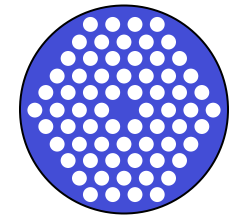
…so the effective
index contrast is less

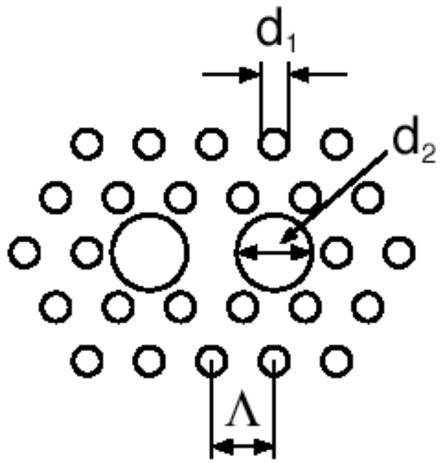
…and the fiber can stay
single mode for all λ!

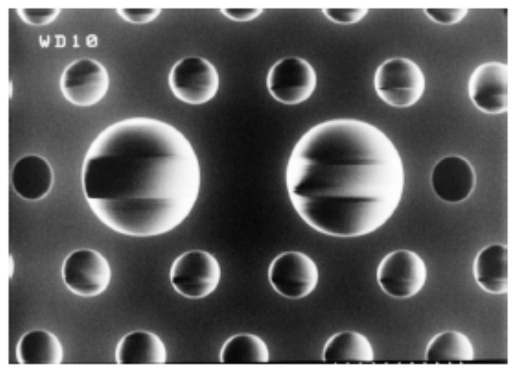


Holey Fiber PMF

(Polarization-Maintaining Fiber)

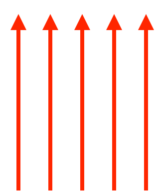
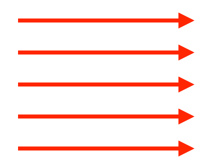






birefringence B = Dbc/w
= 0.0014
(10 times B of silica PMF)

Loss = 1.3 dB/km @ 1.55μm
over 1.5km

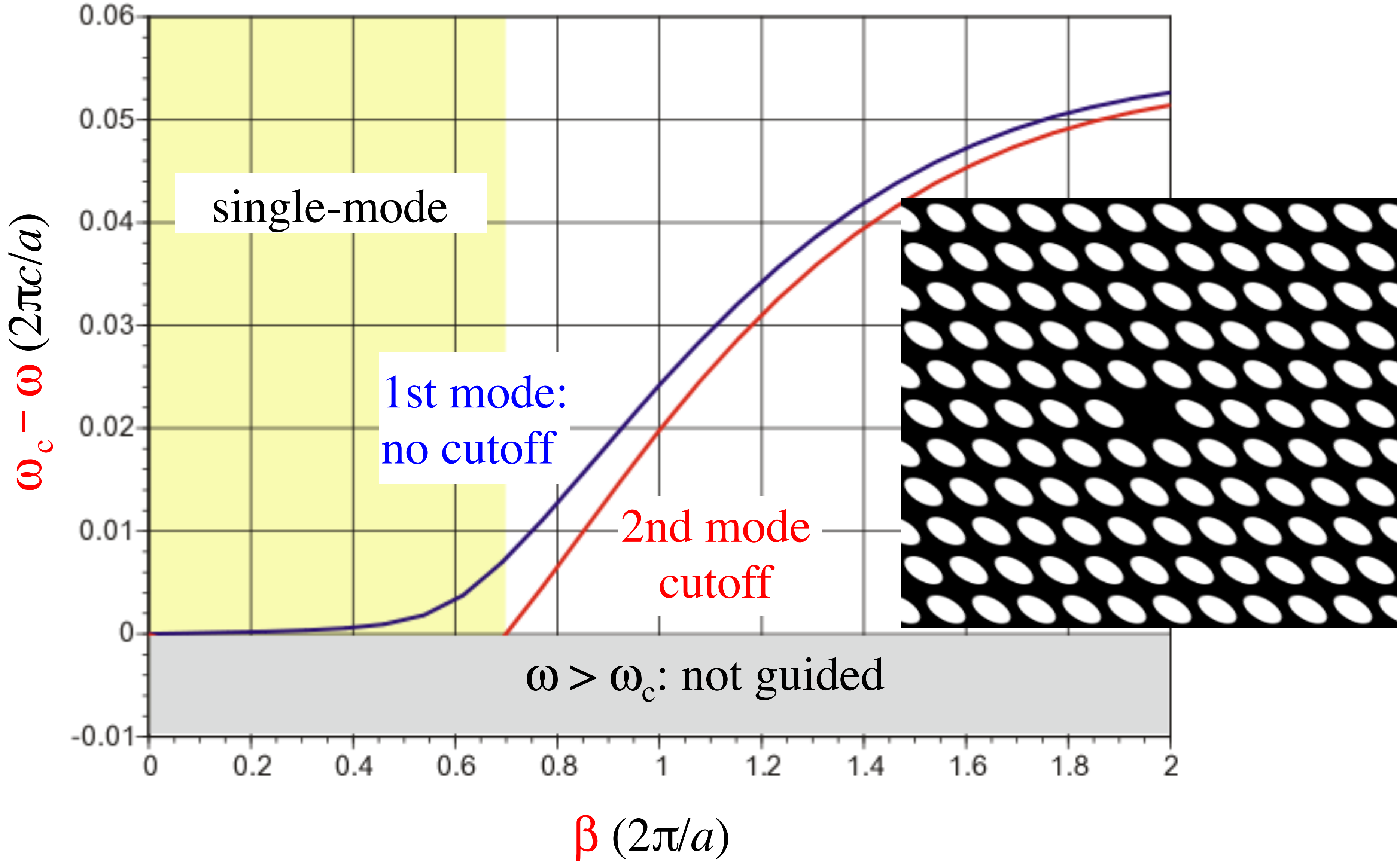
 no longer degenerate with 

Can operate in a single polarization, PMD = 0
(also, known polarization at output)

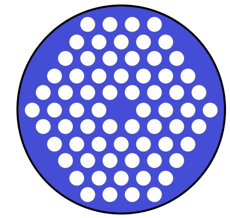
[K. Suzuki, *Opt. Express* **9**, 676 (2001)]

Truly Single-Mode Cutoff-Free Fiber

[Lee et. al., *Optics Express* **16**, 15170-15184 (2008)]



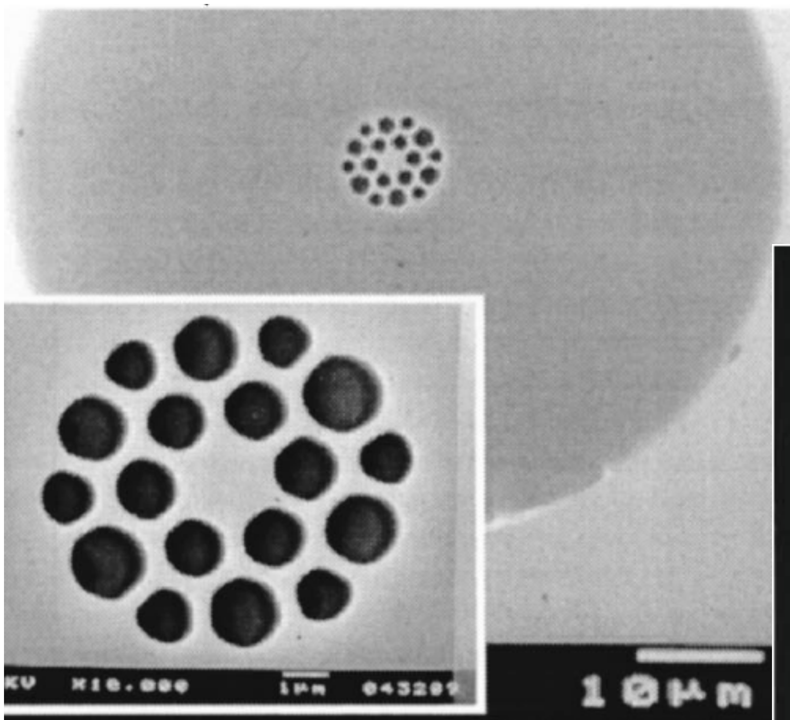
Nonlinear Holey Fibers:

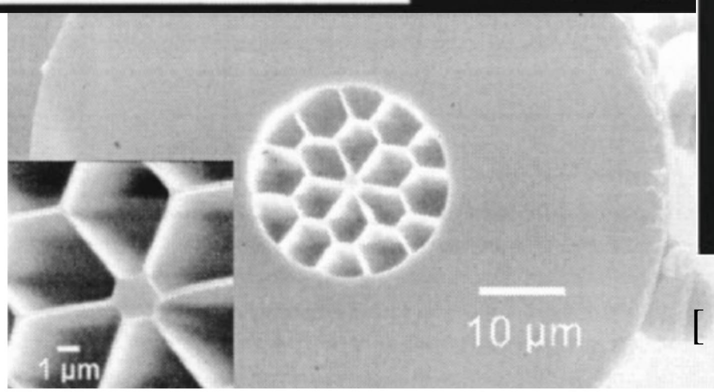


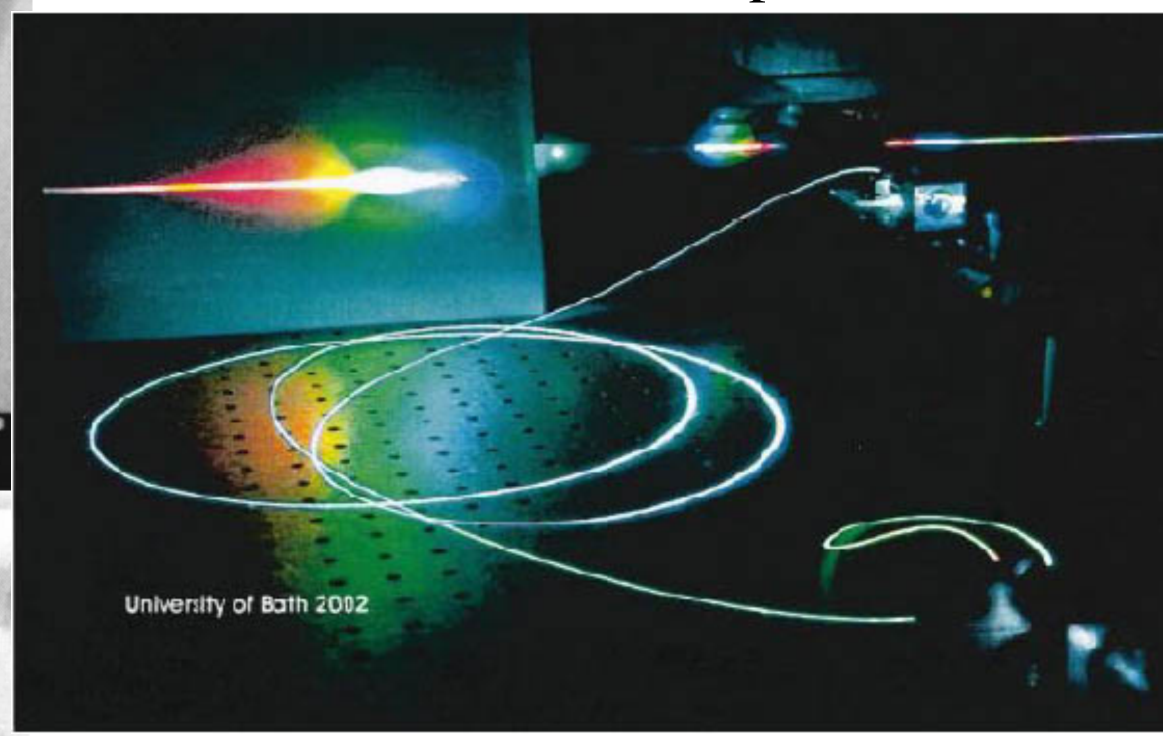
Supercontinuum Generation

(enhanced by strong confinement + unusual dispersion)

e.g. 400–1600nm "white" light:
from 850nm ~200 fs pulses (4 nJ)



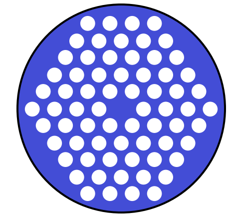




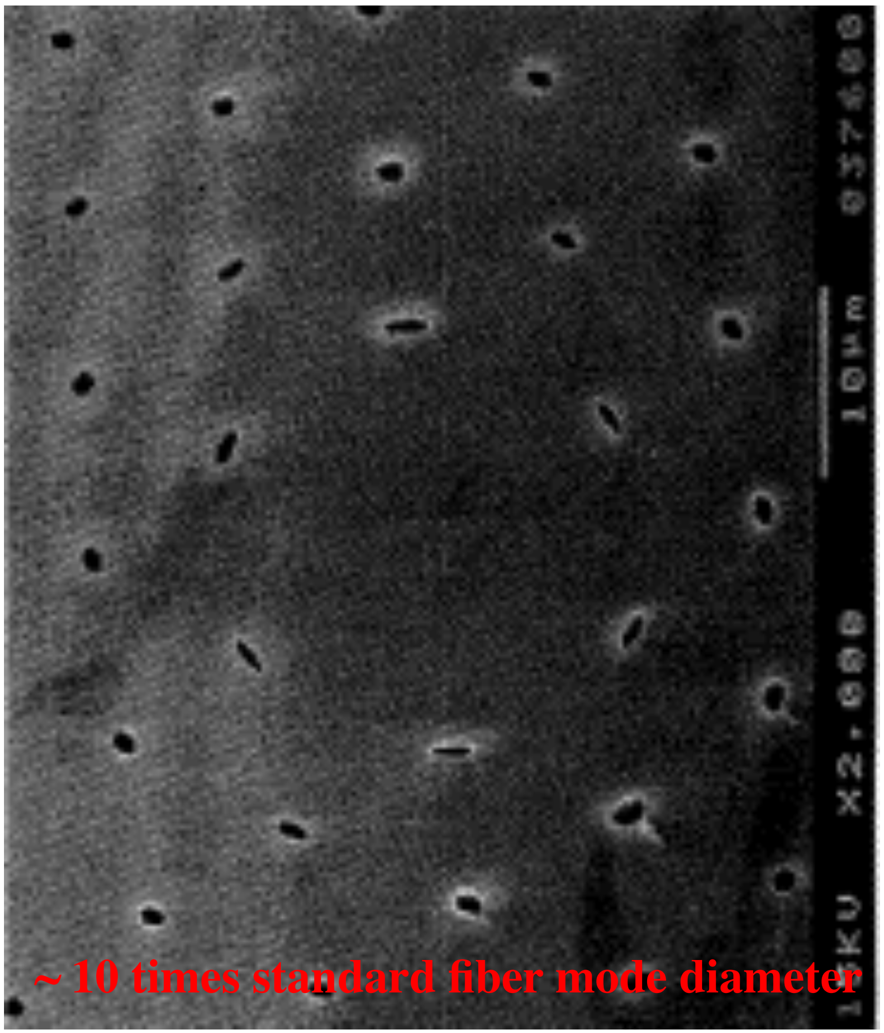
[figs: W. J. Wadsworth *et al.*, *J. Opt. Soc. Am. B* **19**, 2148 (2002)]

[earlier work: J. K. Ranka *et al.*, *Opt. Lett.* **25**, 25 (2000)]

Low Contrast Holey Fibers



[J. C. Knight *et al*., *Elec*. *Lett*. **34**, 1347 (1998)]



The holes can also form an
effective low-contrast medium

i.e. light is only affected slightly
by small, widely-spaced holes

This yields
large-area, single-mode
fibers (low nonlinearities)

…but bending loss is worse

Outline

- Preliminaries: waves in periodic media
- Photonic crystals in theory and practice
- Bulk crystal properties
- Intentional defects and devices
- Index-guiding and incomplete gaps
- Photonic-crystal fibers
- Perturbations, tuning, and disorder

All Imperfections are Small

(or the device wouldn't work)

- Material absorption: small imaginary $\Delta\varepsilon$
- Nonlinearity: small $\Delta\varepsilon \sim |\mathbf{E}|^2$ (Kerr)
- Stress (MEMS): small $\Delta\varepsilon$ or small ε boundary shift
- Tuning by thermal, electro-optic, etc.: small $\Delta\varepsilon$
- Roughness: small $\Delta\varepsilon$ or boundary shift

Weak effects, long distance/time: hard to compute directly

— use semi-analytical methods

Semi-analytical methods for small perturbations

- Brute force methods (FDTD, *etc*.):
  expensive and give limited insight

- Semi-analytical methods
  — numerical solutions for perfect system
  + analytically bootstrap to imperfections

… coupling-of-modes, perturbation theory,
Green's functions, coupled-wave theory, …

Perturbation Theory

for Hermitian eigenproblems

given eigenvectors/values: $\hat{O}|u\rangle = u|u\rangle$

…find change Δu & $\Delta|u\rangle$ for small $\Delta\hat{O}$

Solution:

expand as power series in $\Delta\hat{O}$

$$\Delta u = 0 + \Delta u^{(1)} + \Delta u^{(2)} + \ldots$$

& $\Delta|u\rangle = 0 + \Delta|u\rangle^{(1)} + \ldots$

$$\boxed{\Delta u^{(1)} = \frac{\langle u|\Delta\hat{O}|u\rangle}{\langle u|u\rangle}}$$

(first order is usually enough)

Perturbation Theory

for electromagnetism

$$\Delta\omega^{(1)} = \frac{c^2}{2\omega} \frac{\langle \mathbf{H} | \Delta\hat{A} | \mathbf{H} \rangle}{\langle \mathbf{H} | \mathbf{H} \rangle}$$

$$= -\frac{\omega}{2} \frac{\int \Delta\varepsilon |\mathbf{E}|^2}{\int \varepsilon |\mathbf{E}|^2}$$

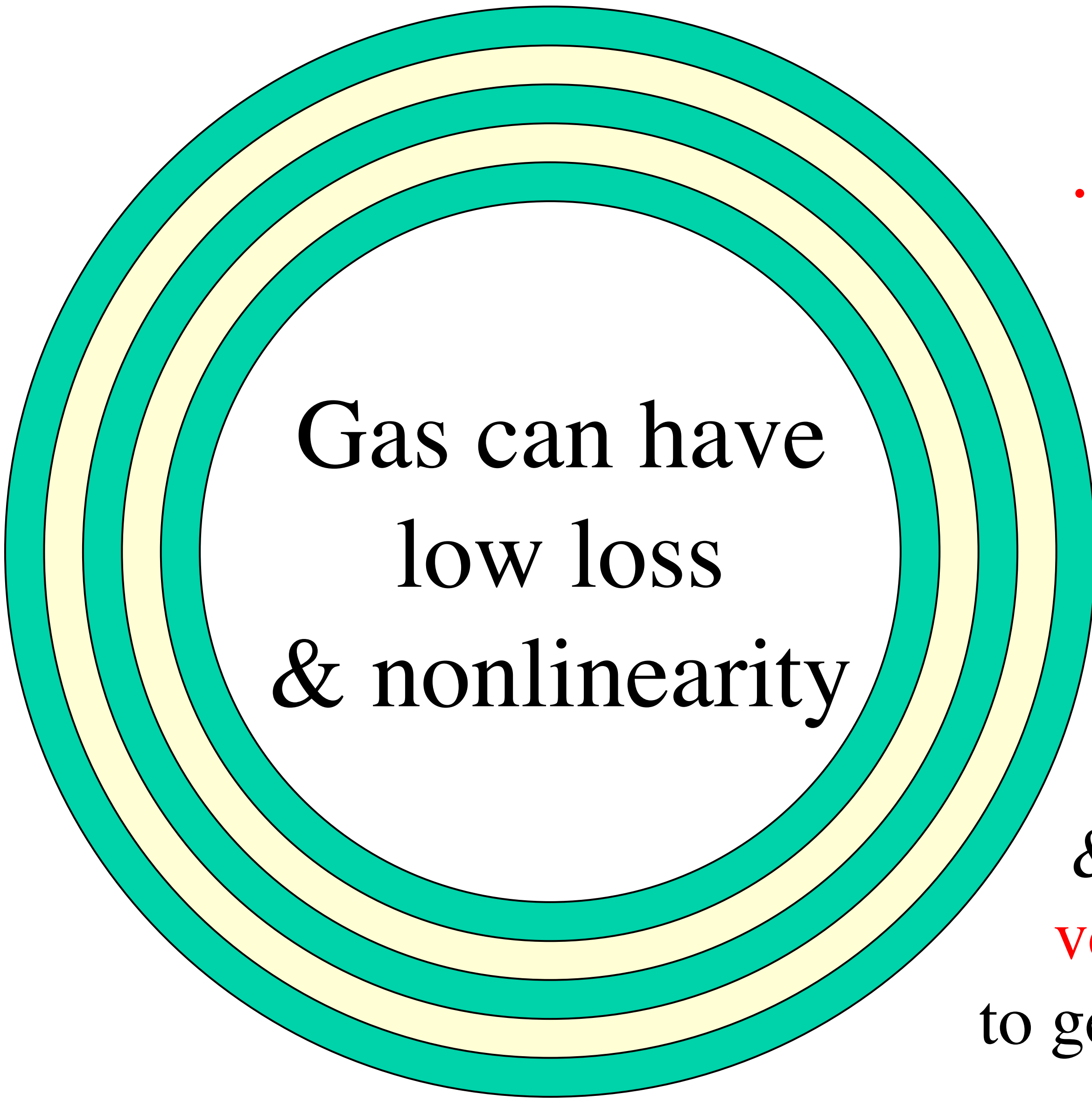
…e.g. absorption gives imaginary Dw = decay!

$$or: \quad \Delta k^{(1)} = \Delta\omega^{(1)} / v_g$$

$$v_g = \frac{d\omega}{dk}$$

$$\Rightarrow \frac{\Delta\omega^{(1)}}{\omega} = -\frac{\Delta n}{n} \cdot (\text{fraction of } \varepsilon|\mathbf{E}|^2 \text{ in } \Delta n)$$

A Quantitative Example



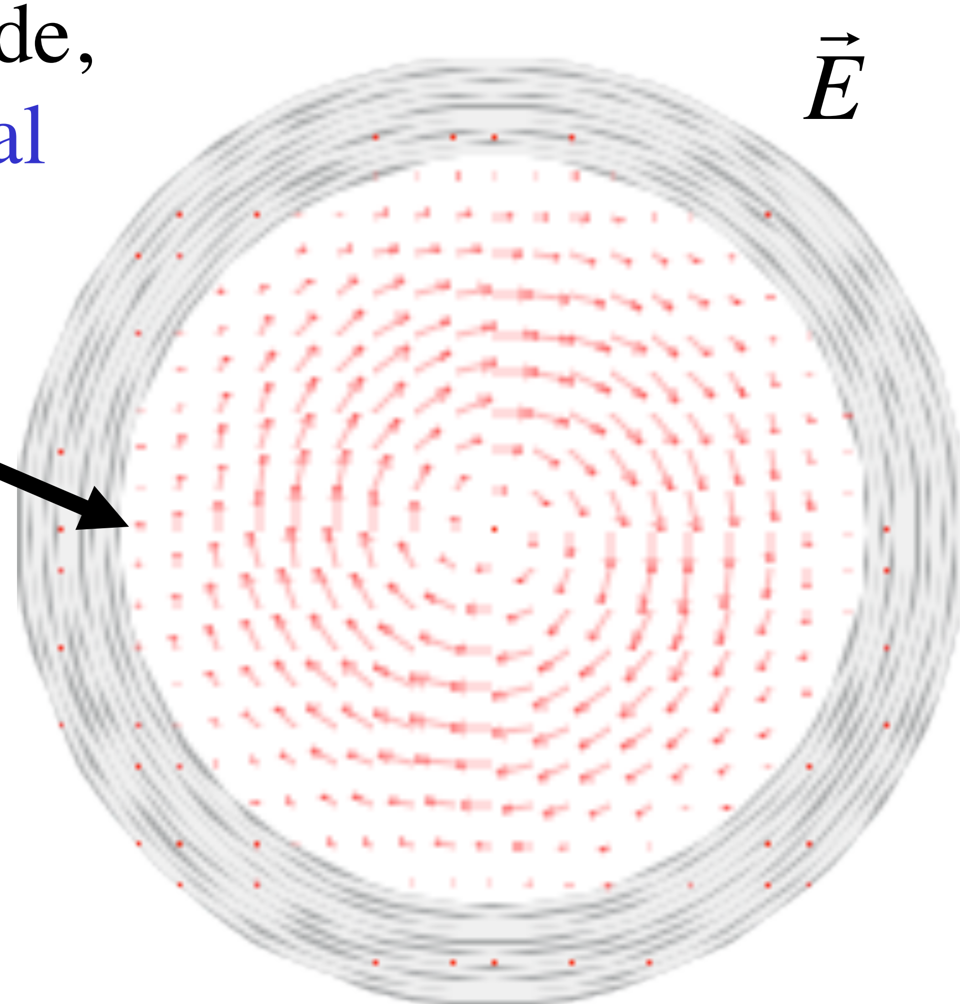
…but what about the cladding?

…*some* field penetrates!

& may need to use very “bad” material to get high index contrast

Review: the TE_{01} mode

lowest-loss mode,
just as in metal

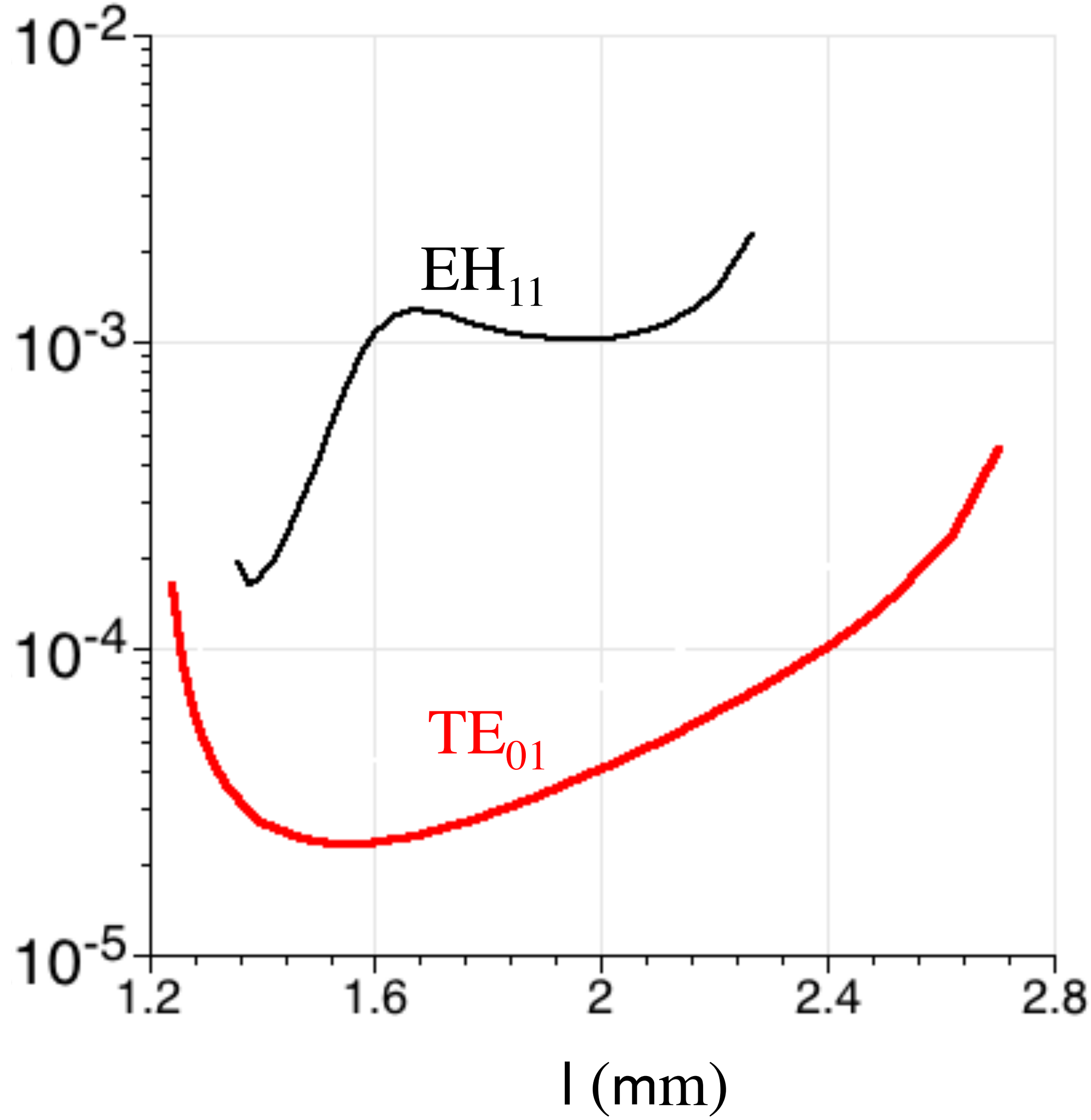


(near) node at interface
= strong confinement
= low losses

[Johnson, *Opt. Express* **9**, 748 (2001)]

Suppressing Cladding Losses

Mode Losses
÷
Bulk Cladding Losses



Large differential loss

TE_{01} strongly suppresses cladding absorption

(like ohmic loss, for metal)

[Johnson, *Opt. Express* **9**, 748 (2001)]

Quantifying Nonlinearity

$\Delta\beta \sim$ power $P \sim 1$ / lengthscale for nonlinear effects

$$\gamma = \Delta\beta / P$$

= nonlinear-strength parameter determining
self-phase modulation (SPM), four-wave mixing (FWM), …

(unlike "effective area,"
tells *where* the field is,
not just how big)

[Johnson, *Opt. Express* **9**, 748 (2001)] [R. Ramaswami & K. N. Sivarajan, *Optical Networks: A Practical Perspective*]

Suppressing Cladding Nonlinearity

[Johnson, *Opt. Express* **9**, 748 (2001)]

Mode Nonlinearity*

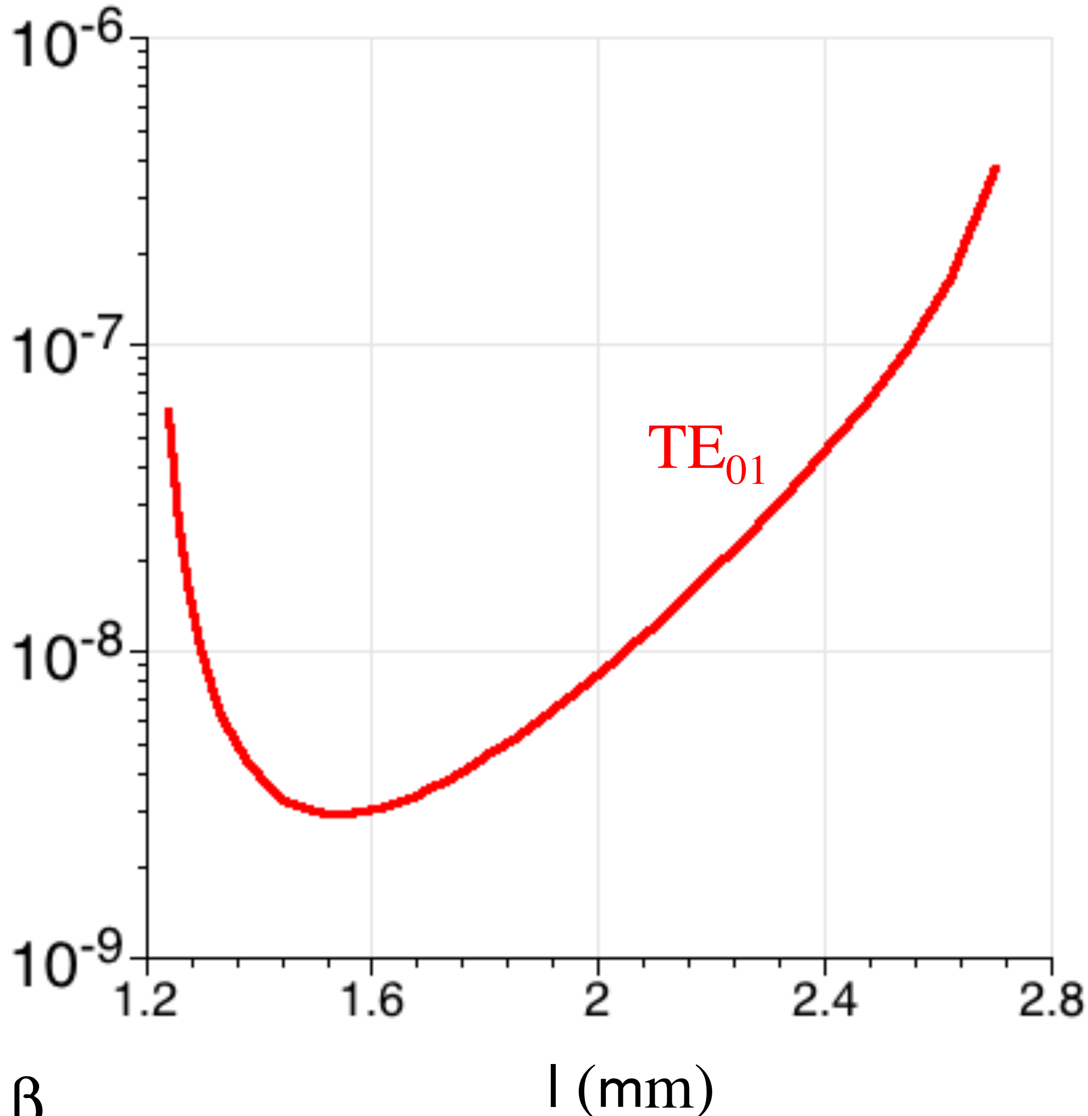
÷

Cladding Nonlinearity

Will be dominated by nonlinearity of air

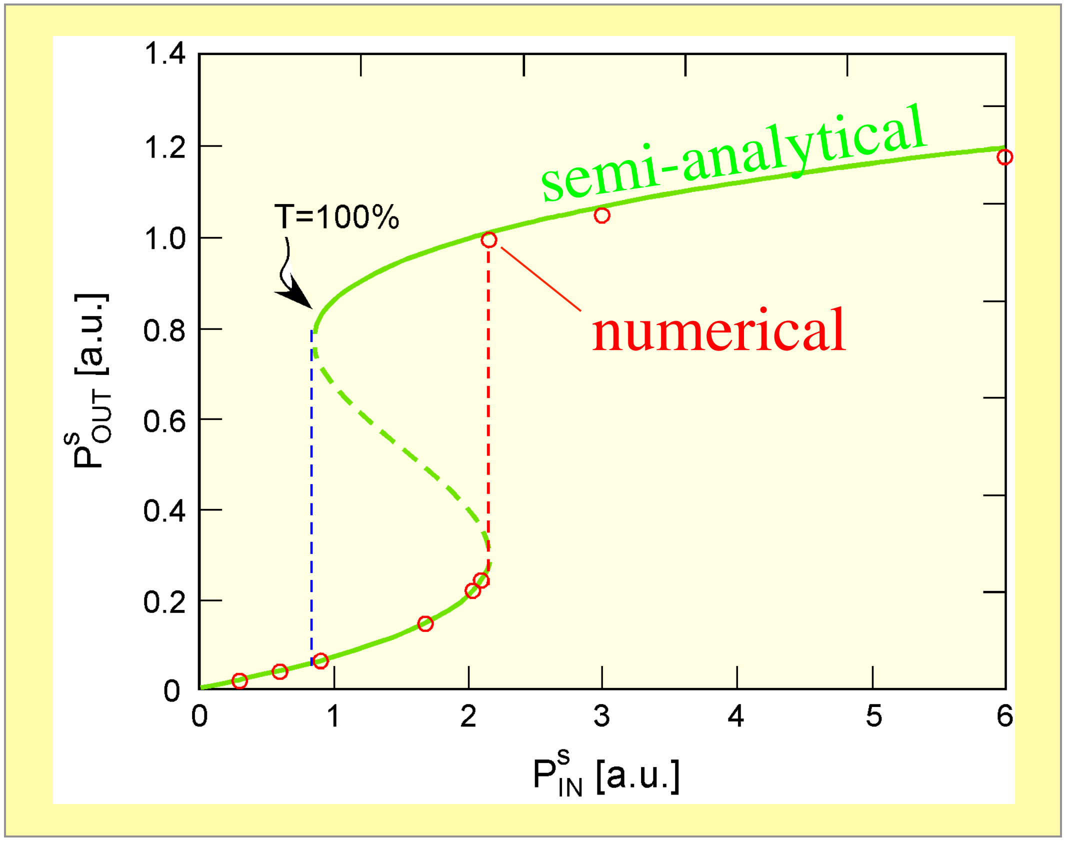
~10,000 times weaker than in silica fiber

(including factor of 10 in area)



* “nonlinearity” = $\Delta\beta^{(1)} / P = \beta$

A ~~Linear~~ *Nonlinear* "Transistor"



Bistable (hysteresis) response

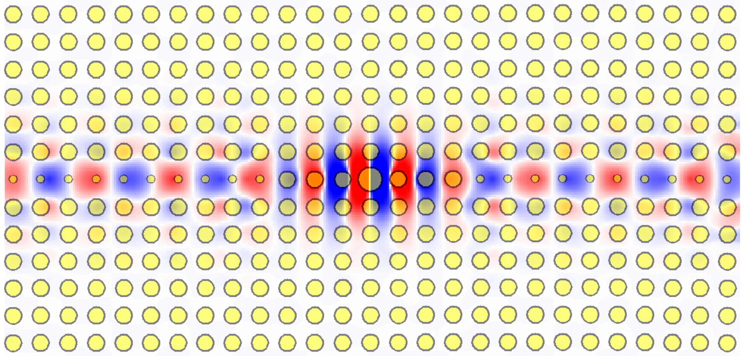
Entire nonlinear response from *one* linear calculation:

Lorentzian mode w, Q

+

Kerr $\Delta\omega \sim |\mathbf{E}|^2$

(to first order)



[Soljacic *et al.*, *PRE Rapid. Comm.* **66**, 055601 (2002).]

Tuning Microcavities

- Correcting for fabrication error:

  — narrow-band filters require 10^{-3} or better accuracy

  $\Rightarrow$ fabricate "close enough" and tune post-fabrication

  … want: large tunability, slow speeds

- Switching/routing:

  — require small tunability (e.g. by bandwidth: 10^{-3})

  — need high speeds (ideally, ns or better)

Many mechanisms to change cavity index or shape:

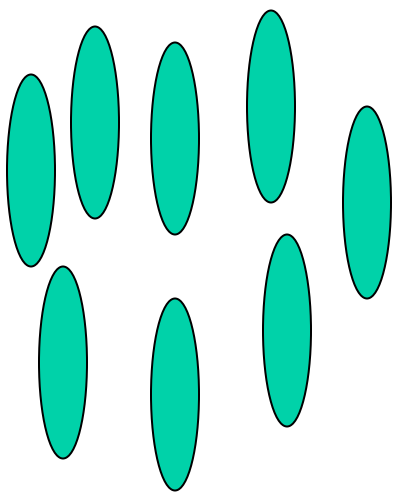
liquid crystal, thermal,

nonlinearities, carrier density, MEMS…

"easy" theory for Δn tuning: $\frac{\Delta\omega^{(1)}}{\omega} = -\frac{\Delta n}{n} \cdot (\text{fraction of } \varepsilon|\mathbf{E}|^2 \text{ in } \Delta n)$

Liquid-crystal Tuning

One of the earliest proposals: [Busch & John, *PRL* **83**, 967 (1999).]



Asymmetric particles oriented by external field:
— n on (two) "ordinary" axes can differ
from "extraordinary-axis" n by Δn ~15%

Response time: 20–200μs [Shimoda, *APL* **79**, 3627 (2001).]

Difficulty: filling entire photonic crystal
with liquid ($n \sim 1.5$) usually destroys the gap

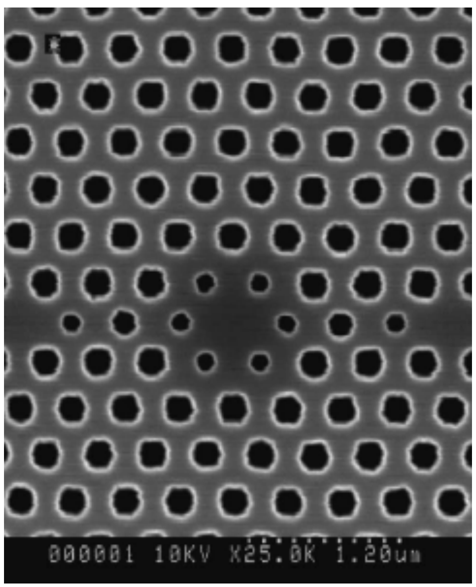
Possible solutions:

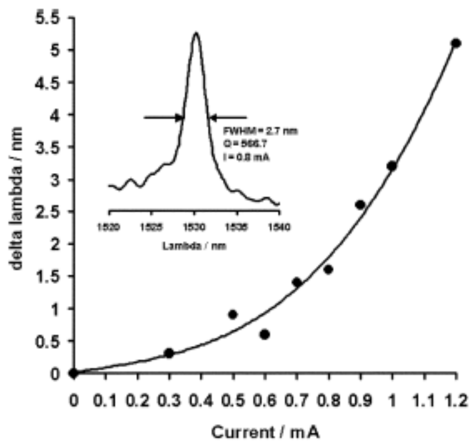
- use thin LC coating [Busch, 1999], but small Δfrequency
- use micro-fluidic droplet only in cavity?

Thermal tuning

using thermal expansion, phase transitions,
or most successfully, thermo-optic coefficient (dn/dT)

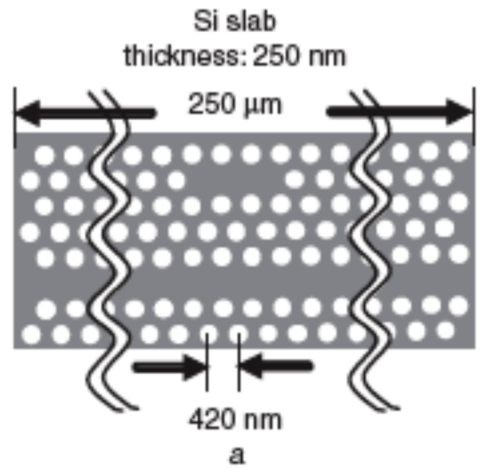
[Chong, *PTL* **16**, 1528 (2004).]

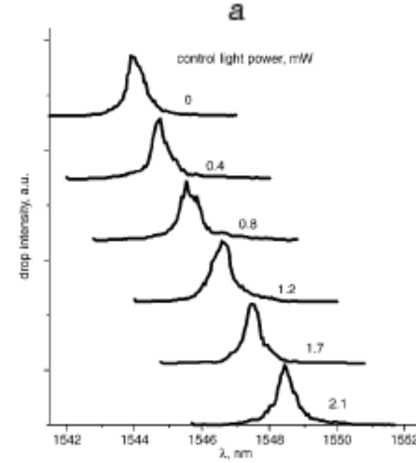




5 nm tuning (0.3%) in Si
time (estimated) < 1 ms

[Asano, *Elec. Lett.* **41** (1) (2005).]

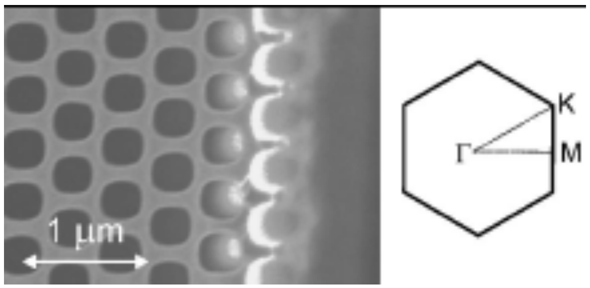




5 nm tuning
(0.3%)
time ~ 20μs

Tuning by Free-carrier Injection

[Leonard, *PRB* **66**, 161102 (2002).]

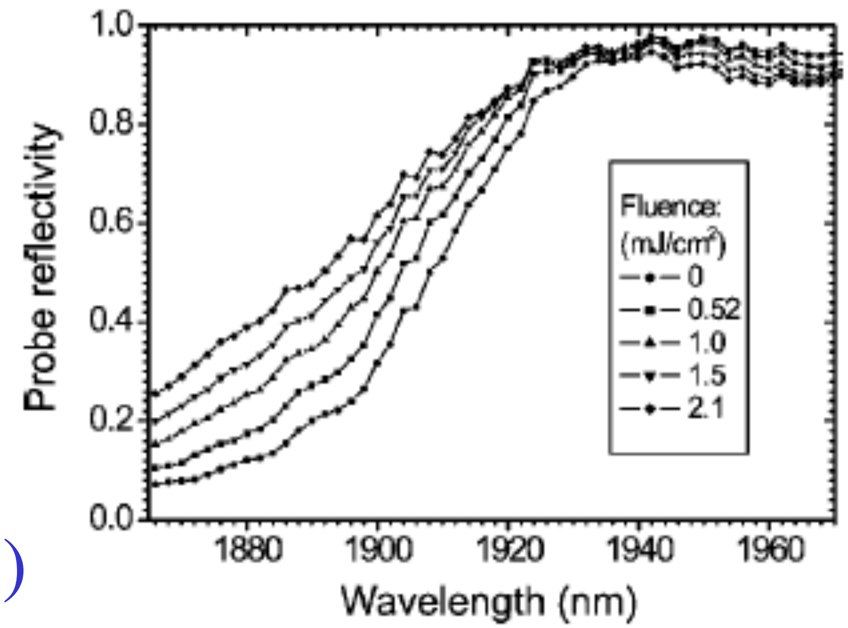


macroporous Si

optical carrier injection
by 300fs pulses
at 800nm pump wavelength

31 nm wavelength shift (2%)
rise time ~ 500 fs
***but** affects absorption too*

Measured Δreflectivity from
band-edge shift at 1.9μm



Tuning by Optical Nonlinearities

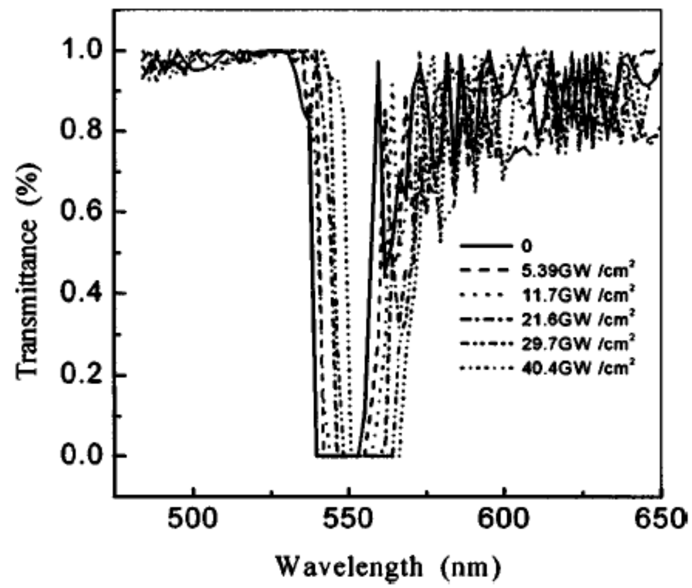
Pockels effect ($\Delta n \sim \mathbf{E}$)

[Takeda, *PRE* **69**, 016605 (2004).]

Theory only

Kerr effect ($\Delta n \sim |\mathbf{E}|^2$)

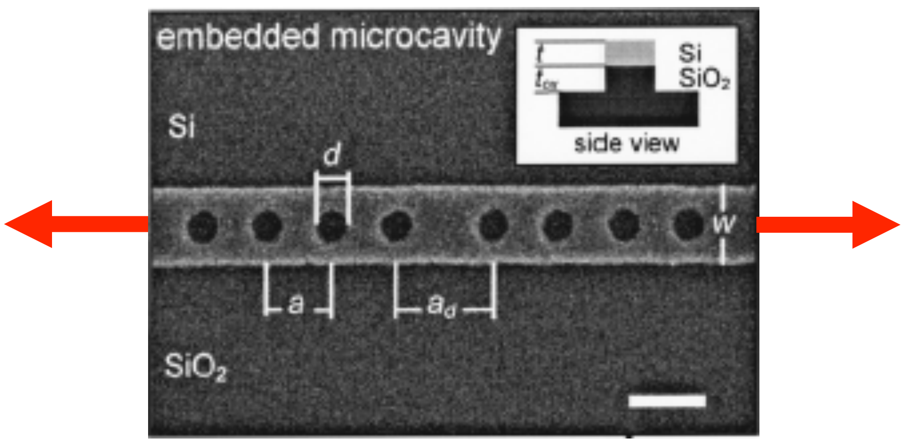
[Hu, *APL* **83**, 2518 (2003).]



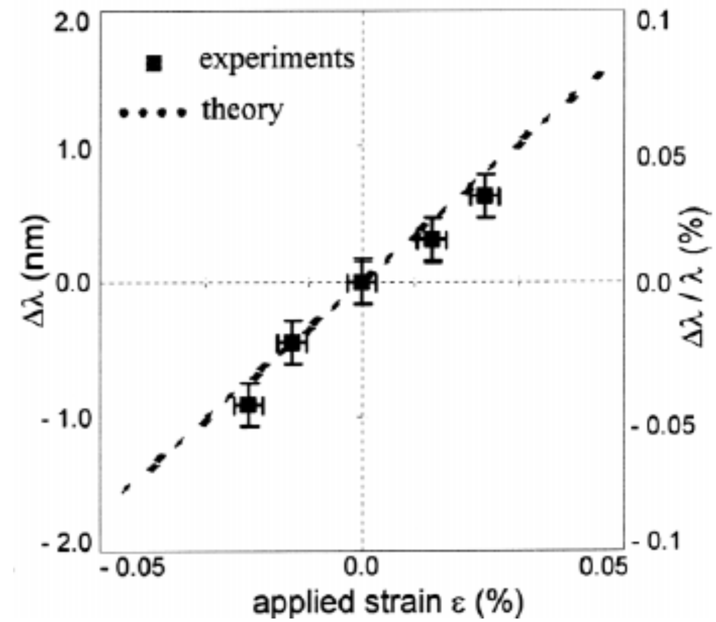
fcc lattice of polystyrene spheres
(*incomplete gap*)
13nm shift @ 540nm (2.4%)
response time ~ 10 ps

Tuning by MEMS deformation

[C.-W. Wong, *Appl. Phys. Lett.* **84**, 1242 (2004).]



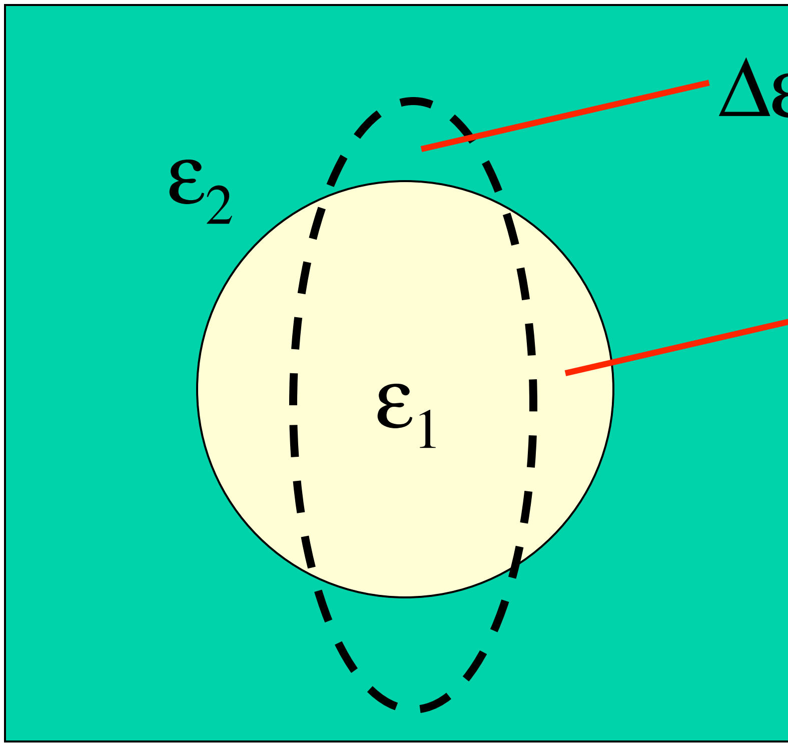
stretch piezo-electrically
(MEMS)



1.5 nm shift @ 1.5μm (0.1%)
response-time not measured, expected in "microseconds" range

Theory tricky: *not* a Δn shift

Boundary-perturbation theory



$\Delta\varepsilon = \varepsilon_1 - \varepsilon_2$

$\Delta\varepsilon = \varepsilon_2 - \varepsilon_1$

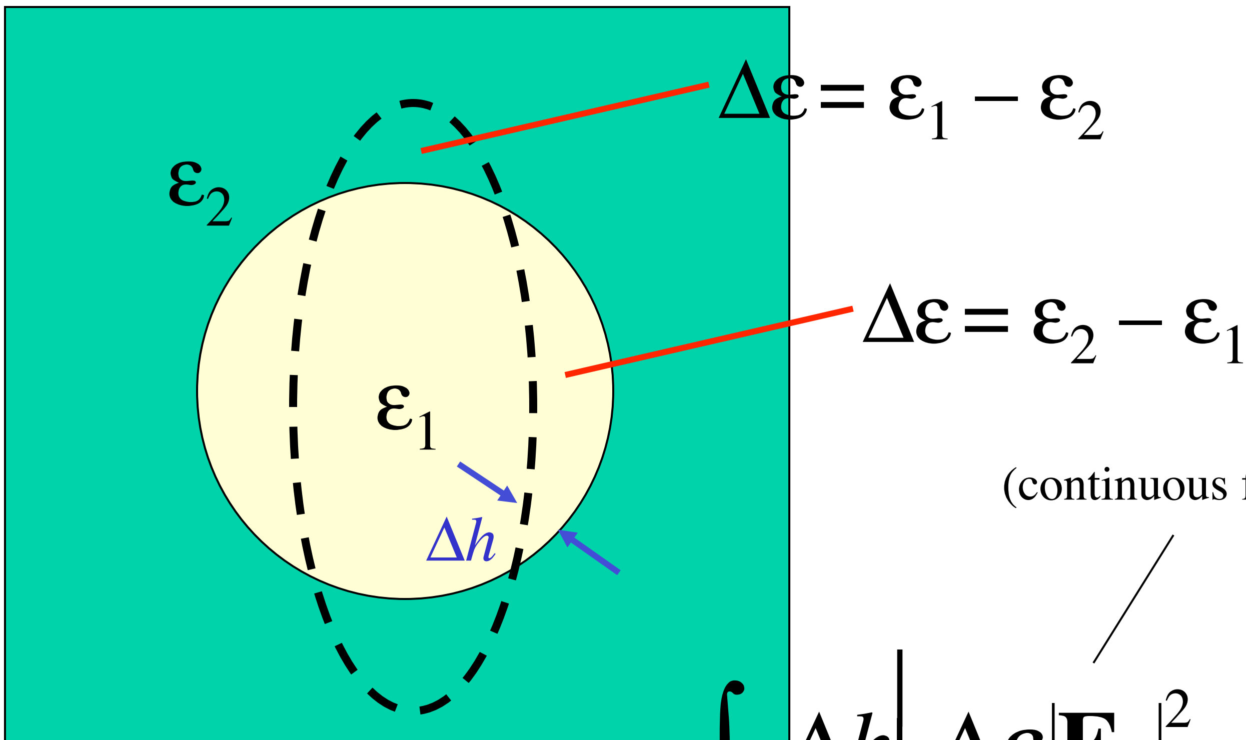
… just plug $\Delta\varepsilon$ into perturbation formulas?

FAILS for high index contrast!

beware field discontinuity…
fortunately, a simple correction exists

[S. G. Johnson *et al.*, *PRE* **65**, 066611 (2002)]

Boundary-perturbation theory



(continuous field components)

$$\Delta\omega^{(1)} = -\frac{\omega}{2} \frac{\int_{\text{surf.}} \Delta h \left[\Delta\varepsilon \left| \mathbf{E}_{\parallel} \right|^2 - \Delta \frac{1}{\varepsilon} \left| D_{\perp} \right|^2 \right]}{\int \varepsilon \left| \mathbf{E} \right|^2}$$

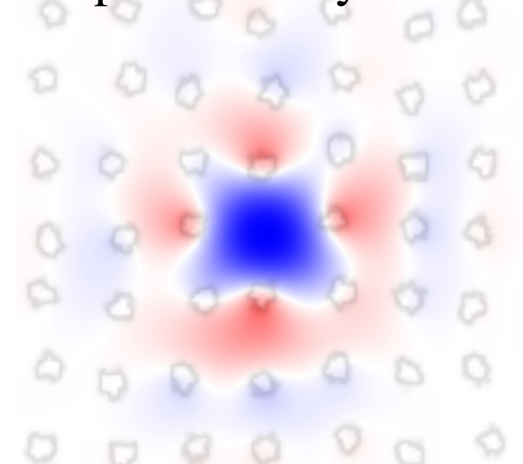
[S. G. Johnson *et al*.,
PRE **65**, 066611 (2002)]

Surface roughness disorder?

[http://www.physik.uni-wuerzburg.de/TEP/Website/groups/opto/etching.htm]

loss limited by disorder
(in addition to bending)

disordered
photonic crystal



[A. Rodriguez, MIT]

[S. Fan *et. al.*, *J. Appl. Phys.* **78**, 1415 (1995).]

small (bounded) disorder does not destroy the bandgap

[A. Rodriguez *et. al.*, *Opt. Lett.* **30**, 3192 (2005).]

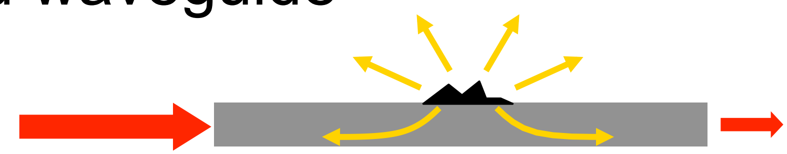
Q limited only by crystal size (for a 3d complete gap) …

… but waveguides have more trouble …

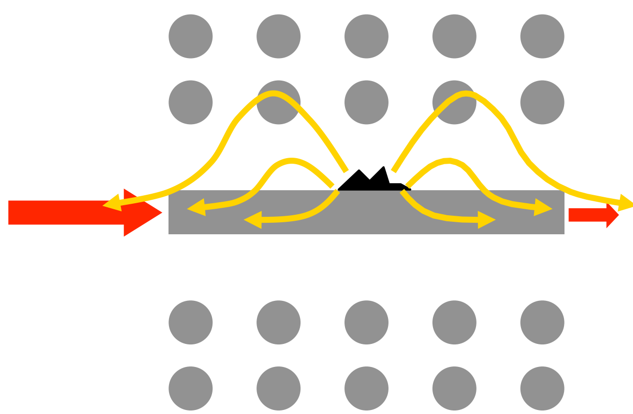
Effect of Gap on Disorder (e.g. Roughness) Loss?

[with M. Povinelli]

index-guided waveguide

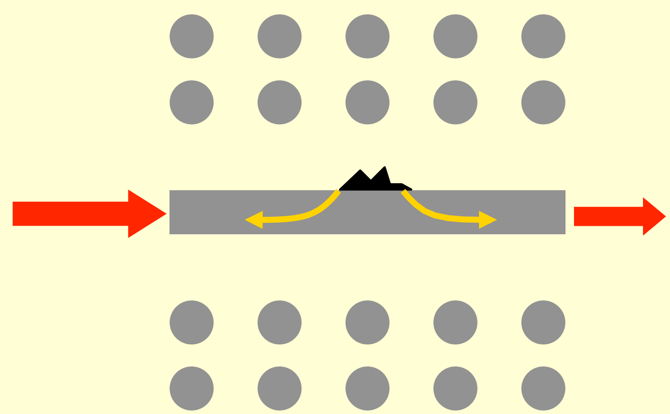


photonic-crystal waveguide: which picture is correct?



radiation blocked
increased reflection

OR



radiation blocked
no increase in reflection

Coupled-mode theory

Expand state in ideal eigenmodes, for constant w:

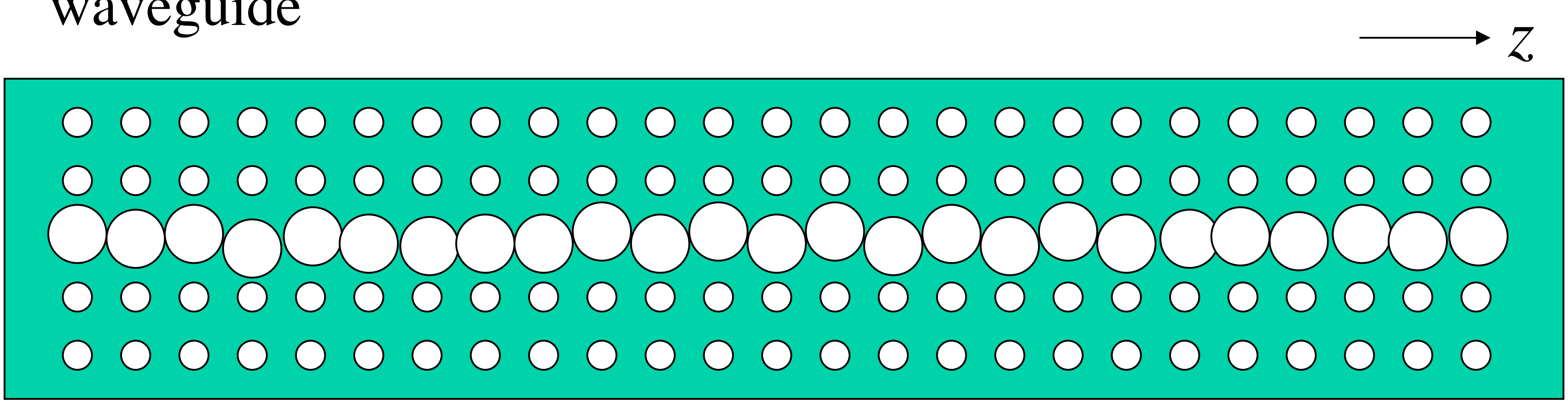
$$|\psi\rangle = \sum_n c_n(z) \boxed{|n\rangle} e^{i\beta_n z}$$

state (field) of disordered waveguide

expansion coefficient

eigenstate of perfect waveguide

wavenumber



What's New in Coupled-Mode Theory?

- Traditional methods (Marcuse, 1970): weak periodicity only
- Strong perodicity (Bloch modes expansion):
  - — de Sterke *et al*. (1996): coupling in *time* (nonlinearities)
  - — Russell (1986): weak perturbations, slowly varying only

scalar

2002+: exact extension, for *z*-dependent (constant ω), and:
arbitrary periodicity,
arbitrary index contrast (full vector),
arbitrary disorder [and/or tapers]

full-vector

[S. G. Johnson *et al*., *PRE* **66**, 066608 (2002).]
[M. L. Povinelli *et al*., *APL* **84**, 3639 (2004).]

[M. Skorobogatiy *et al*., *Opt. Express* **10**, 1227 (2002).]

Coupled-wave Theory

(skipping all the math…)

$$\frac{dc_n}{dz} = \sum_{m \neq n} [\text{coupling}]_{m,n} \, e^{i\Delta\beta z} \, c_m$$

mode expansion coefficients

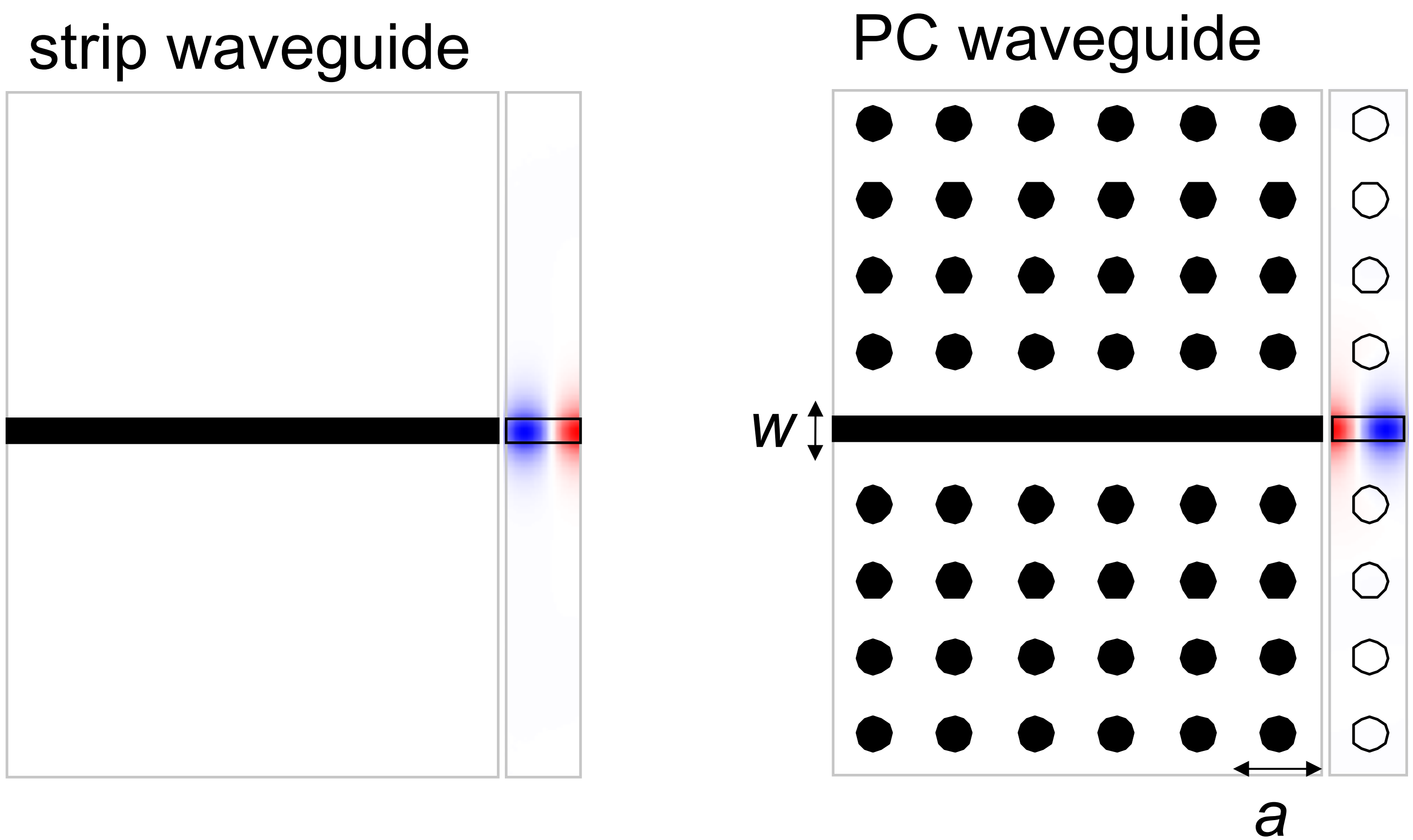
Depends only on: [M. L. Povinelli *et al*., *APL* **84**, 3639 (2004).]

- strength of disorder
- mode field at disorder
- group velocities

Weak disorder, short correlations: refl. ~ |coupling|^2
if disorder and modes are "same,"
then reflection is the same

A Test Case

[M. L. Povinelli *et al*., *APL* **84**, 3639 (2004).]



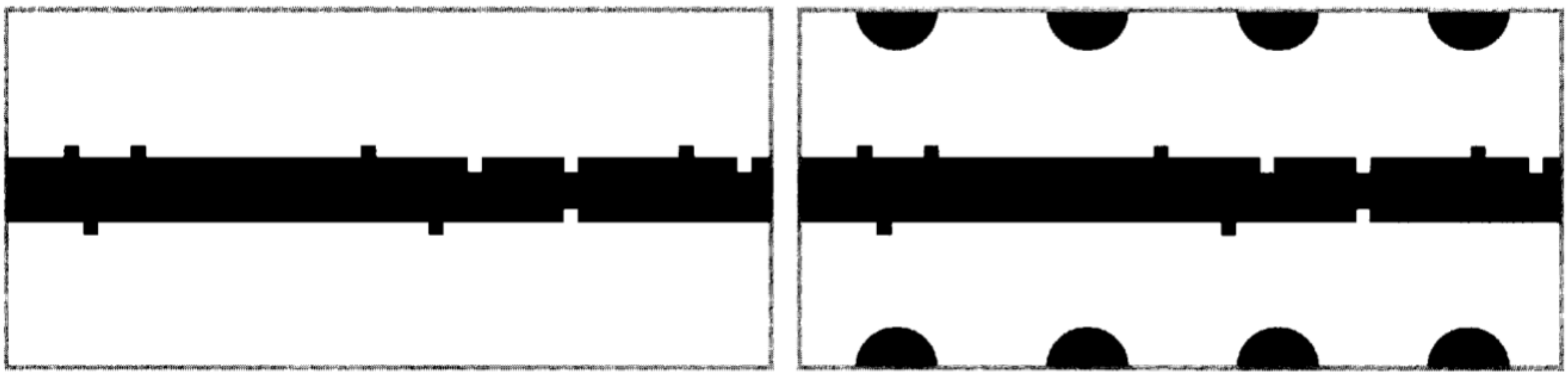
index-guided

gap-guided, same ω(β)

A *controlled comparison**: gap is the only difference.*

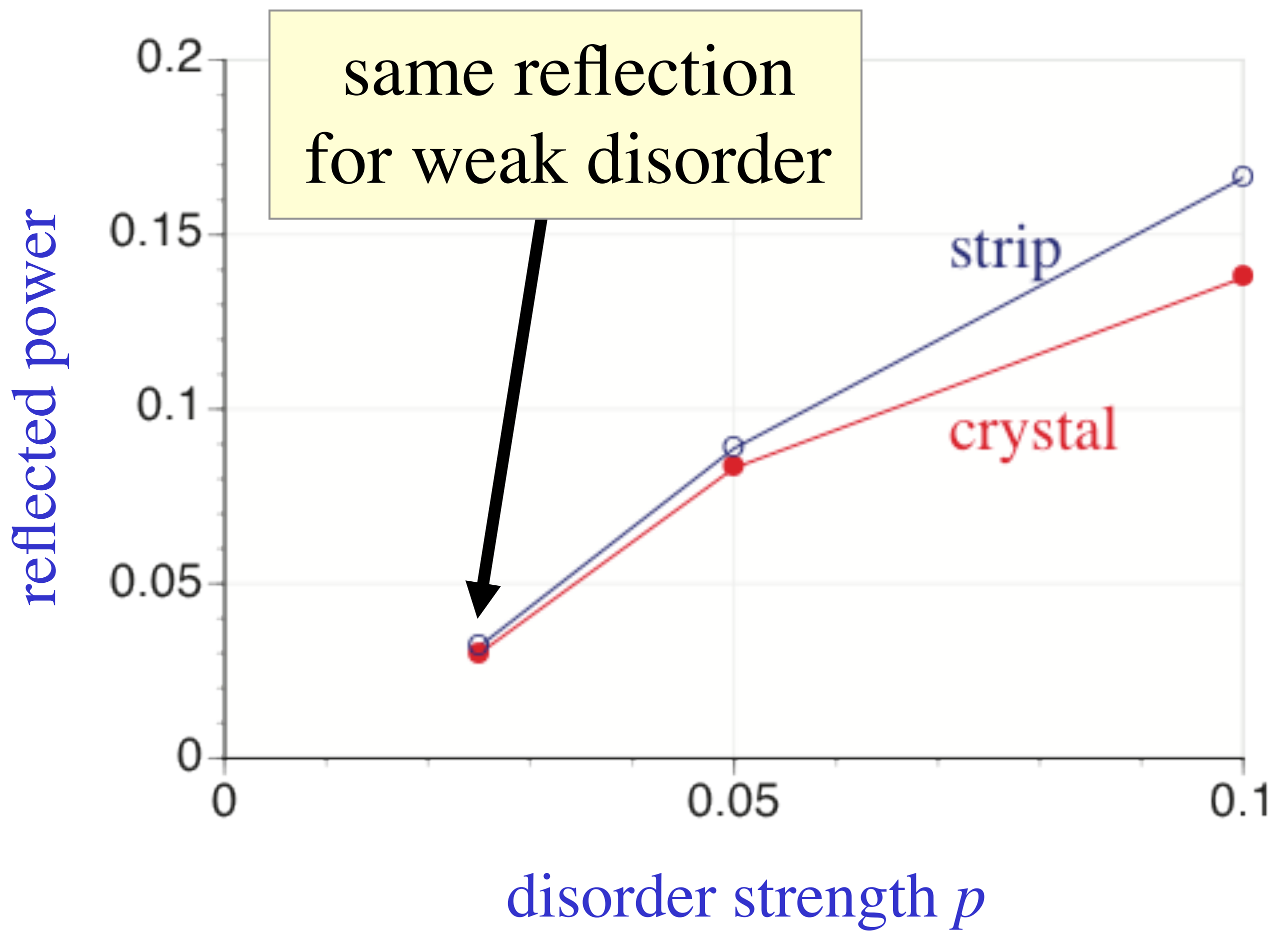
A Test Case

pixels added/removed with probability p

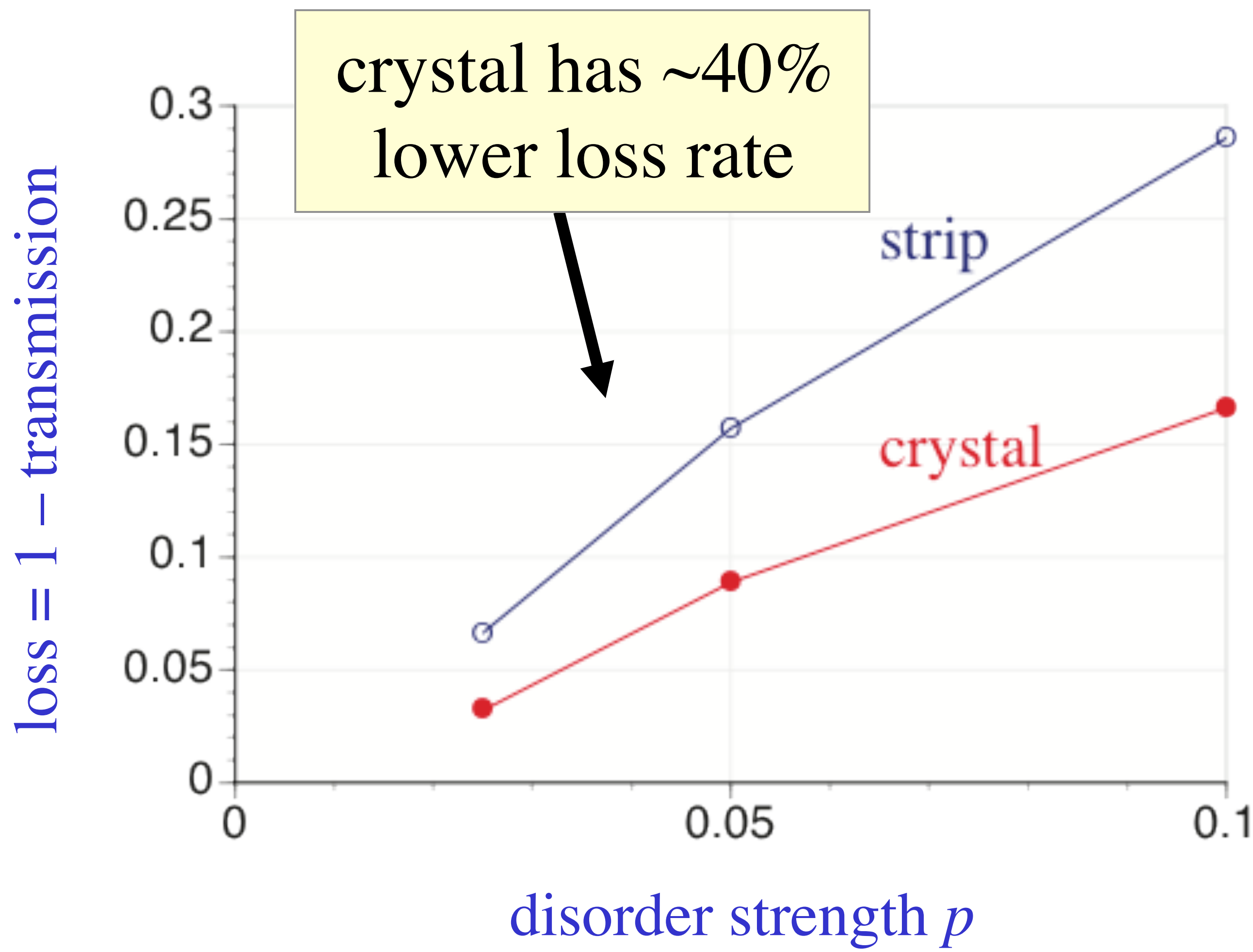


same disorder in both cases, averaged over many FDTD runs

Test Case Results: Reflection



Test Case Results: Total Loss



photonic bandgap
(all other things equal)
= unambiguous improvement

But, the news isn't all good…

Group-velocity (v) dependence
other things being equal

[S. G. Johnson *et al*., *Proc. 2003 Europ. Symp. Phot. Cryst.* **1**, 103.]
[S. Hughes *et al*., *Phys. Rev. Lett.* **94**, 033903 (2005).]

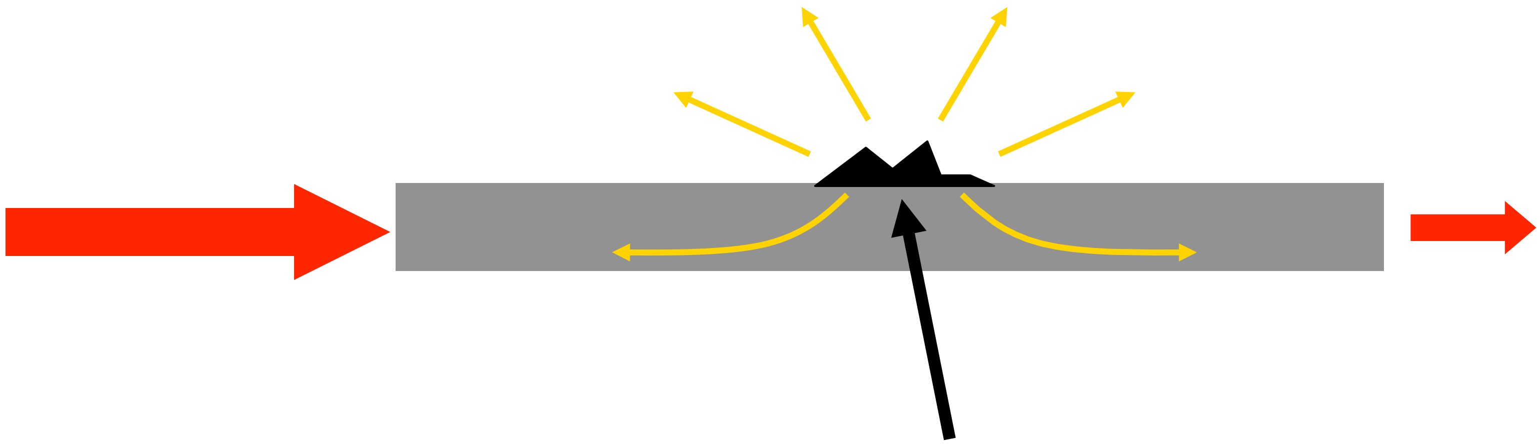
absorption/radiation-scattering loss
(per distance) $\sim 1/v$

reflection loss
(per distance) $\sim 1/v^2$

(per time) $\sim 1/v$

Losses a challenge for slow light…

An Easier Way to Compute Loss



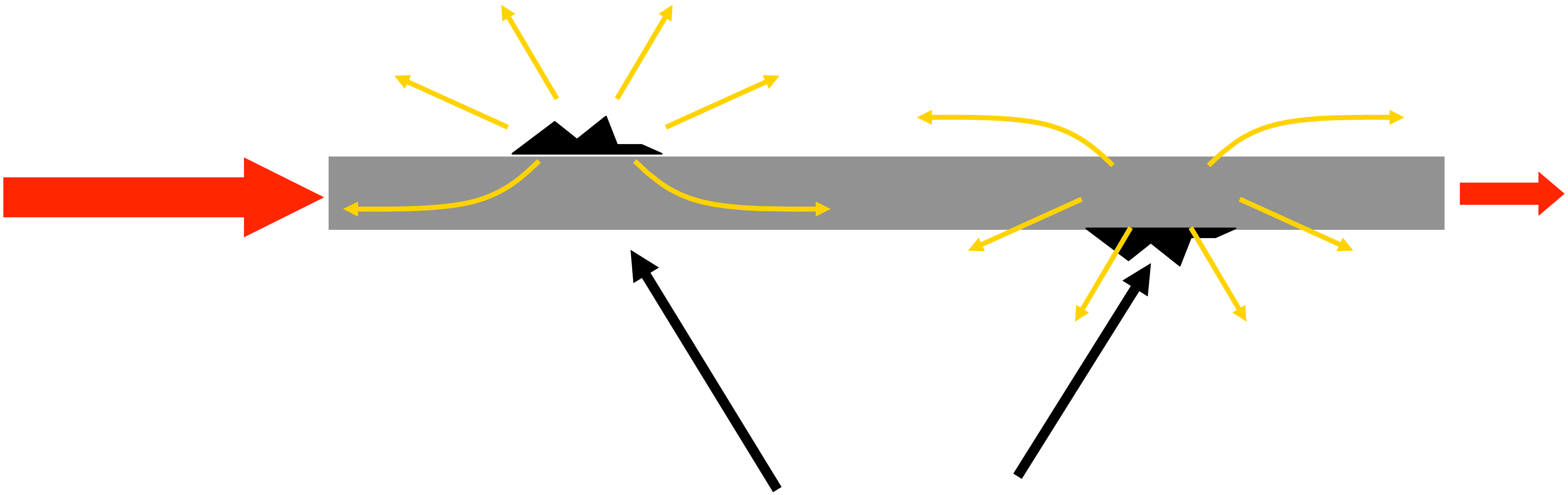
imperfection acts like a volume current

$$\vec{J} \sim \Delta\varepsilon \, \vec{E}_0$$

volume-current method

(i.e., first Born approx. to Green's function)

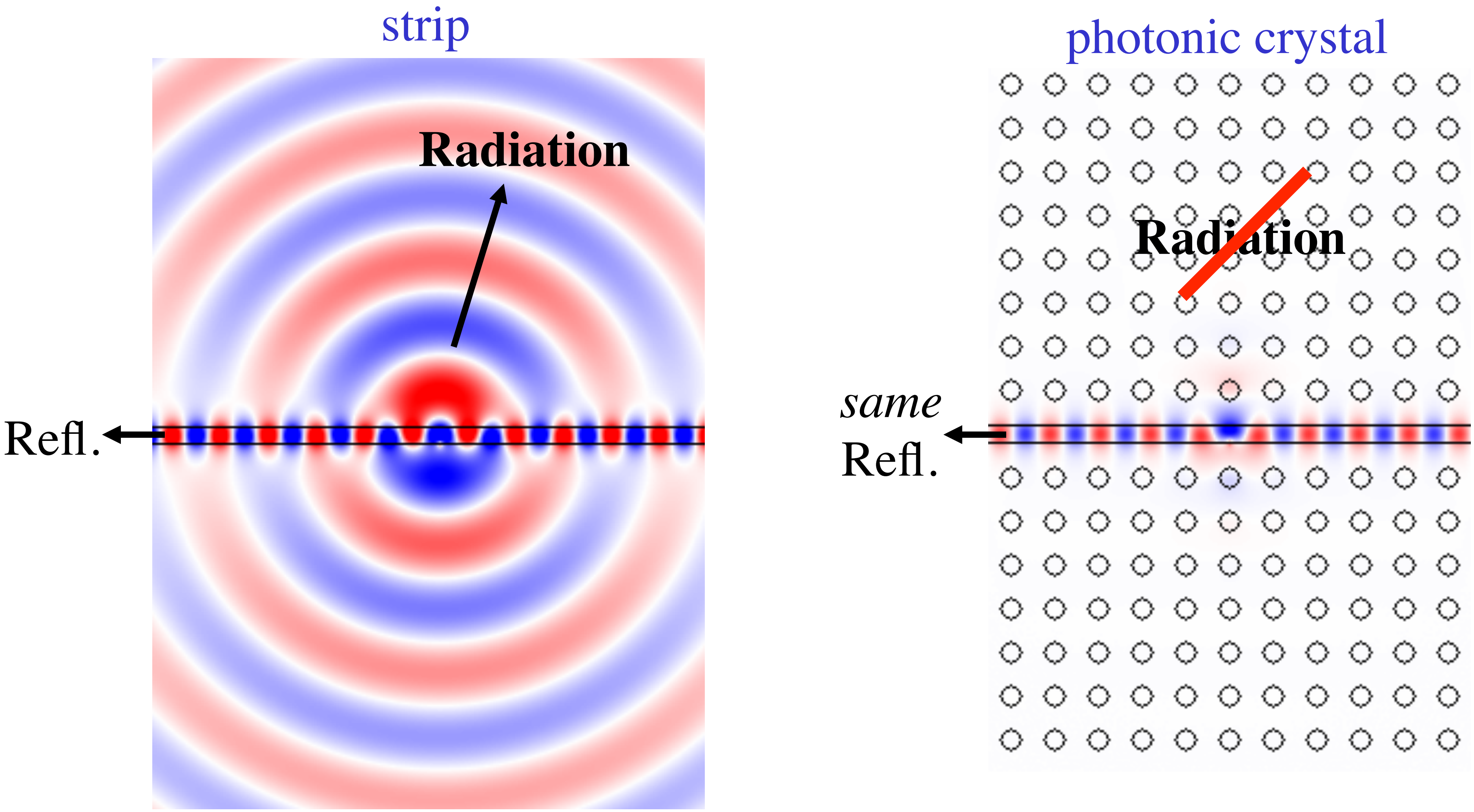
An Easier Way to Compute Loss



uncorrelated disorder adds *incoherently*

So, compute power P radiated by *one* localized source *J*,
and loss rate ~ P * (mean disorder strength)

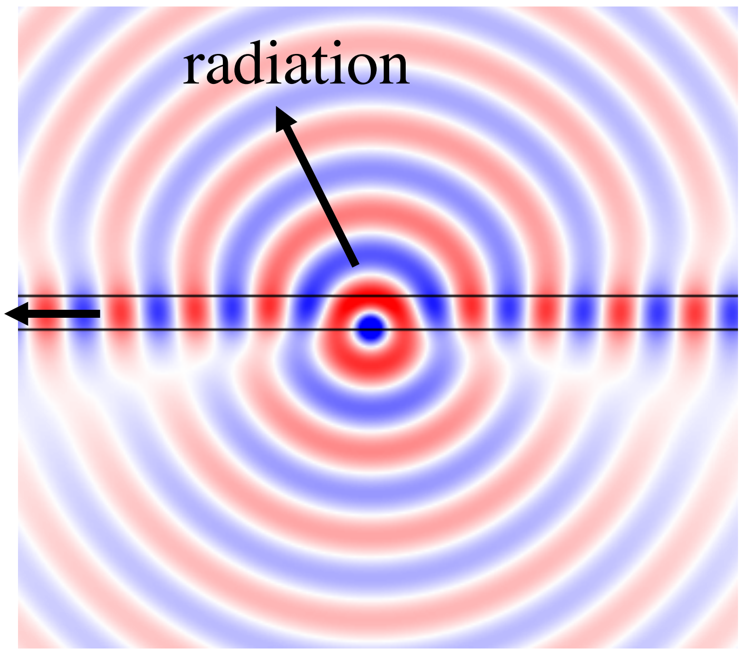
Losses from Point Scatterers



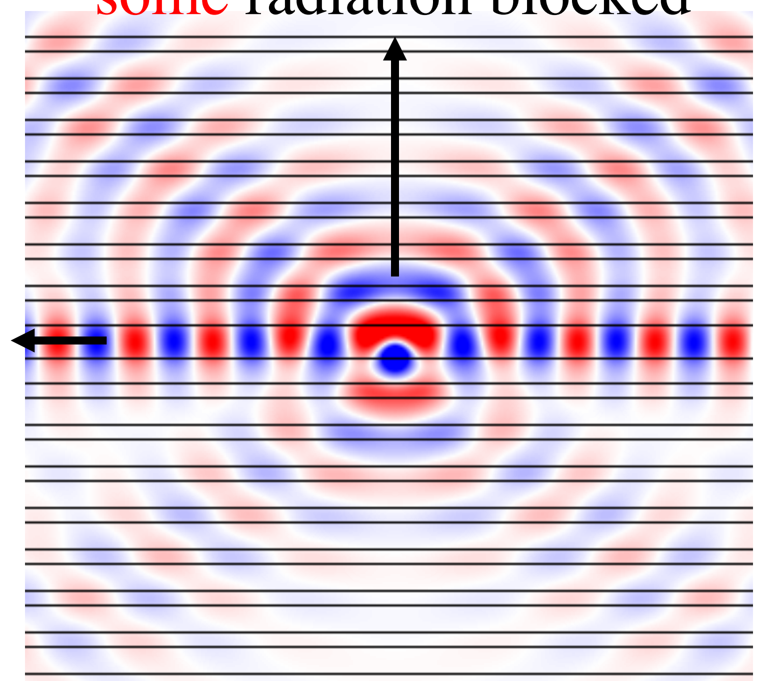
Loss rate ratio = (Refl. only) / (Refl. + Radiation) = 60%

Effect of an *Incomplete* Gap

on uncorrelated surface roughness

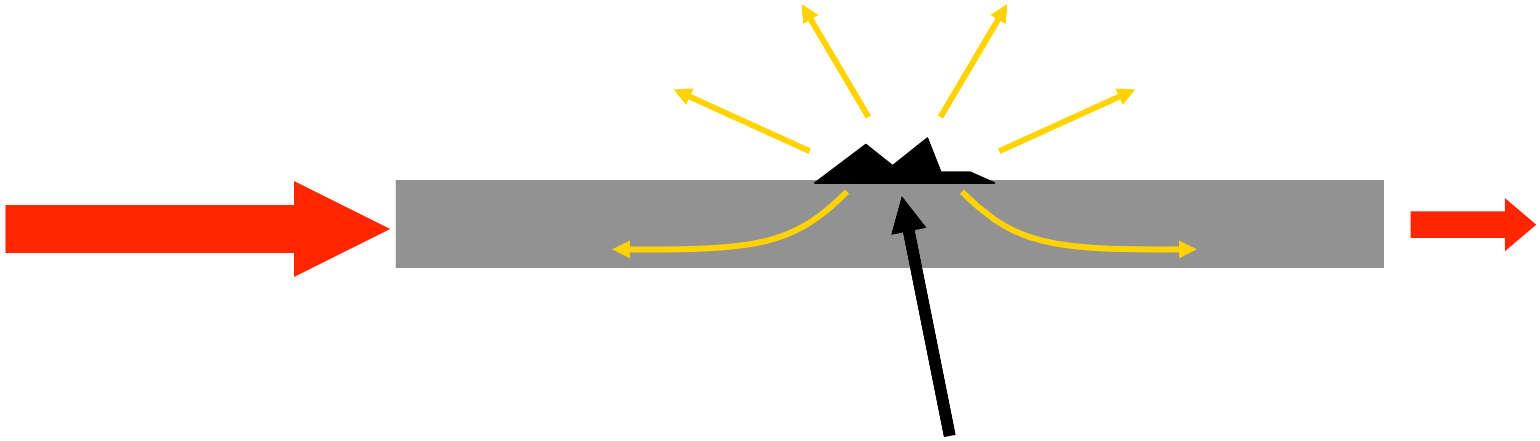


Conventional waveguide
(matching modal area)



…with Si/SiO_2 Bragg mirrors (1D gap)
50% lower losses (in dB)
same reflection

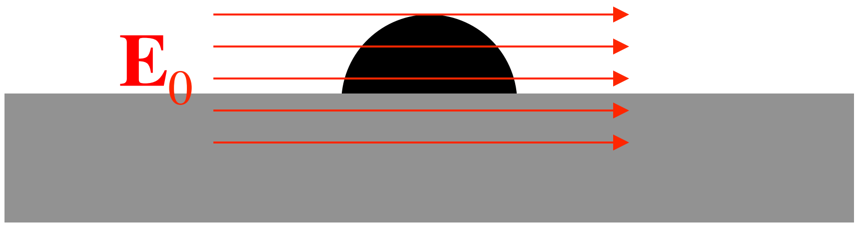
Failure of the Volume-current Method



imperfection acts like a volume current

$$\vec{J} \sim \cancel{\Delta\varepsilon \vec{E}_0}$$

Incorrect for large $\Delta\varepsilon$ (except in 2d TM polarization)

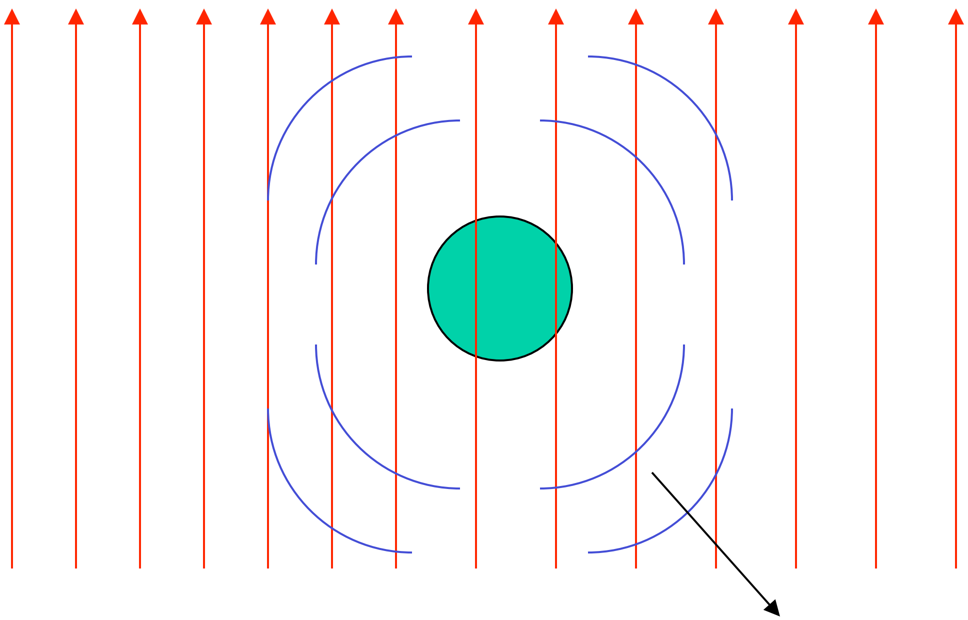


$\Delta\varepsilon$ "bump" *changes* **E**

($E_\perp$ is *discontinuous*)

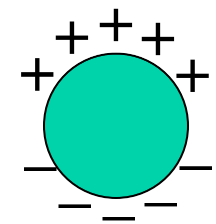
Scattering Theory (for small scatterers)

[e.g. Jackson, *Classical Electrodynamics*]



=

(quasi-static)



dipole $\mathbf{p} = a\,\mathbf{E}_0$

(scattered wave)

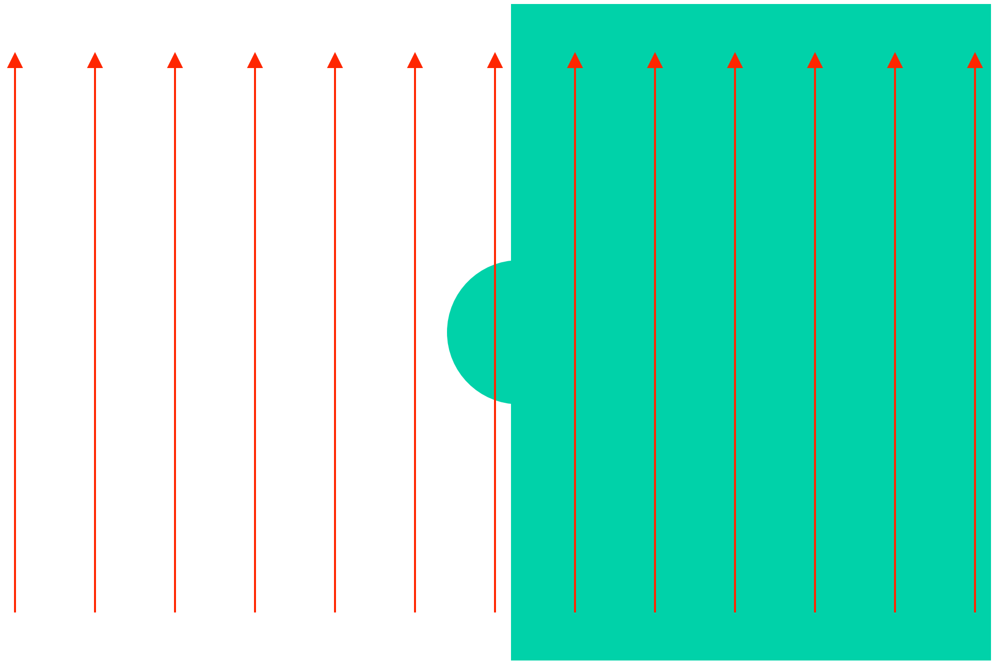
incident wave $\mathbf{E}_0$ ($\lambda >> d$)

sphere: *effective* point current $\mathbf{J} \sim \mathbf{p} / \Delta V$

$= 3\,\Delta\varepsilon\,\mathbf{E}_0 / (\Delta\varepsilon + 3)$

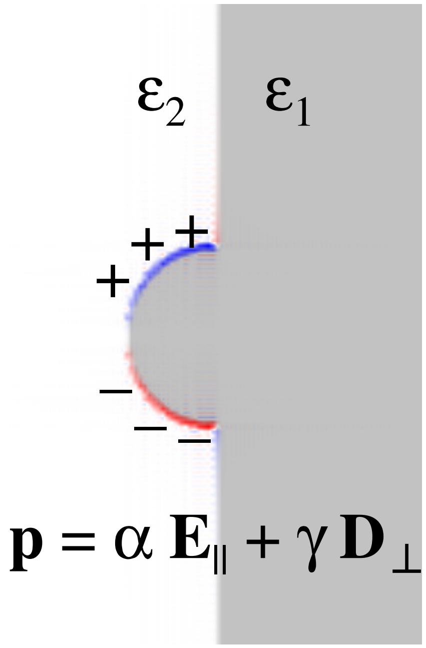
$= \Delta\varepsilon\,\mathbf{E}_0$ for small $\Delta\varepsilon$, but very different for large $\Delta\varepsilon$

Corrected Volume Current for Large De



unperturbed field **E**

=



dipole $\mathbf{p} = \alpha\,\mathbf{E}_{||} + \gamma\,\mathbf{D}_{\perp}$

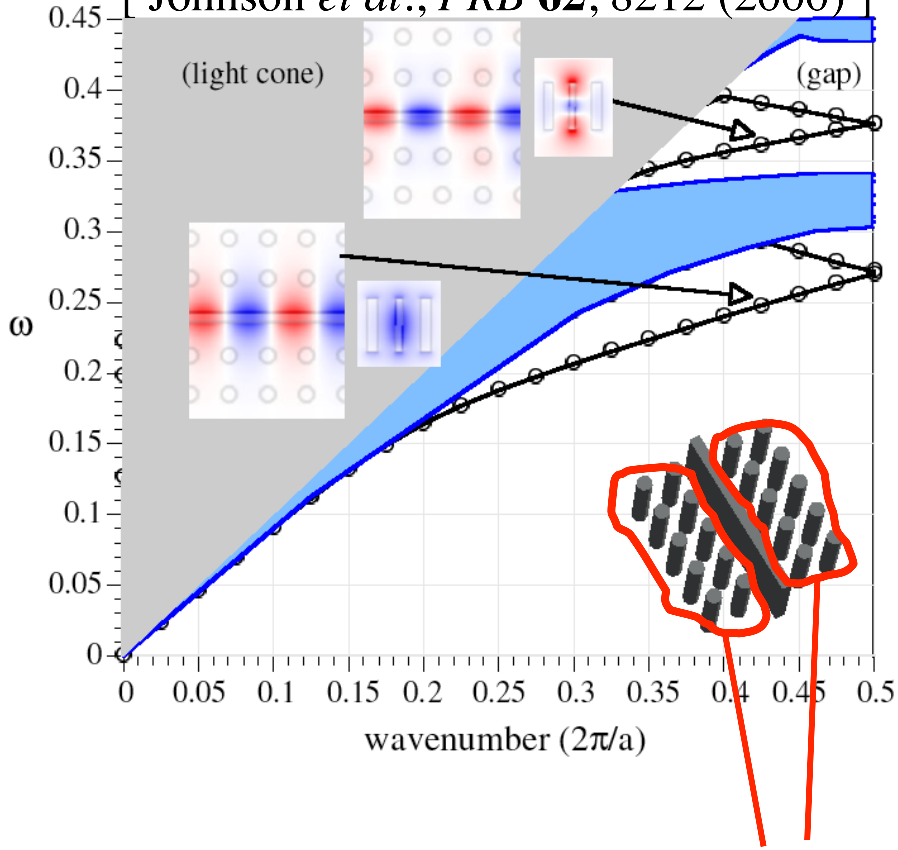
(compute polarizability *numerically*)

effective point current $\mathbf{J} \sim \left(\frac{\varepsilon_1+\varepsilon_2}{2}\mathbf{p}_{||} + \varepsilon\mathbf{p}_{\perp}\right) / \Delta V$

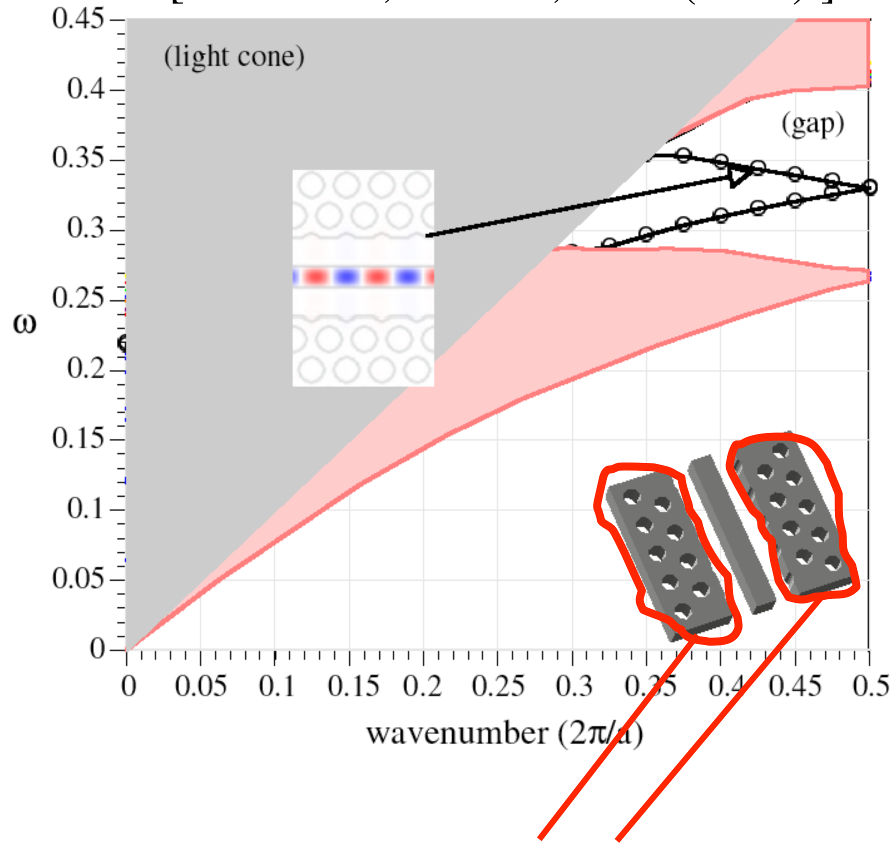
[S. G. Johnson *et al*., *Applied Phys*. *B* **81**, 283 (2005).]

Strip Waveguides in Photonic-Crystal Slabs (3d)

[Johnson *et al.*, *PRB* **62**, 8212 (2000)]



[Lau *et al.*, *APL* **81**, 3915 (2002)]

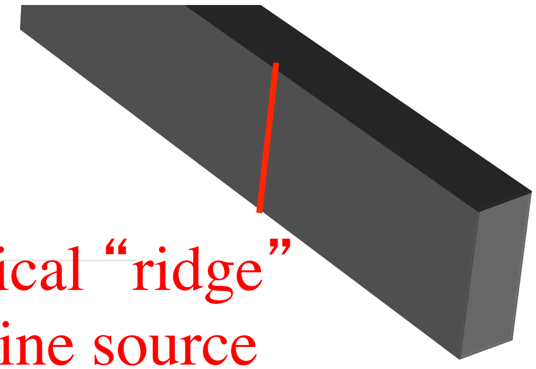


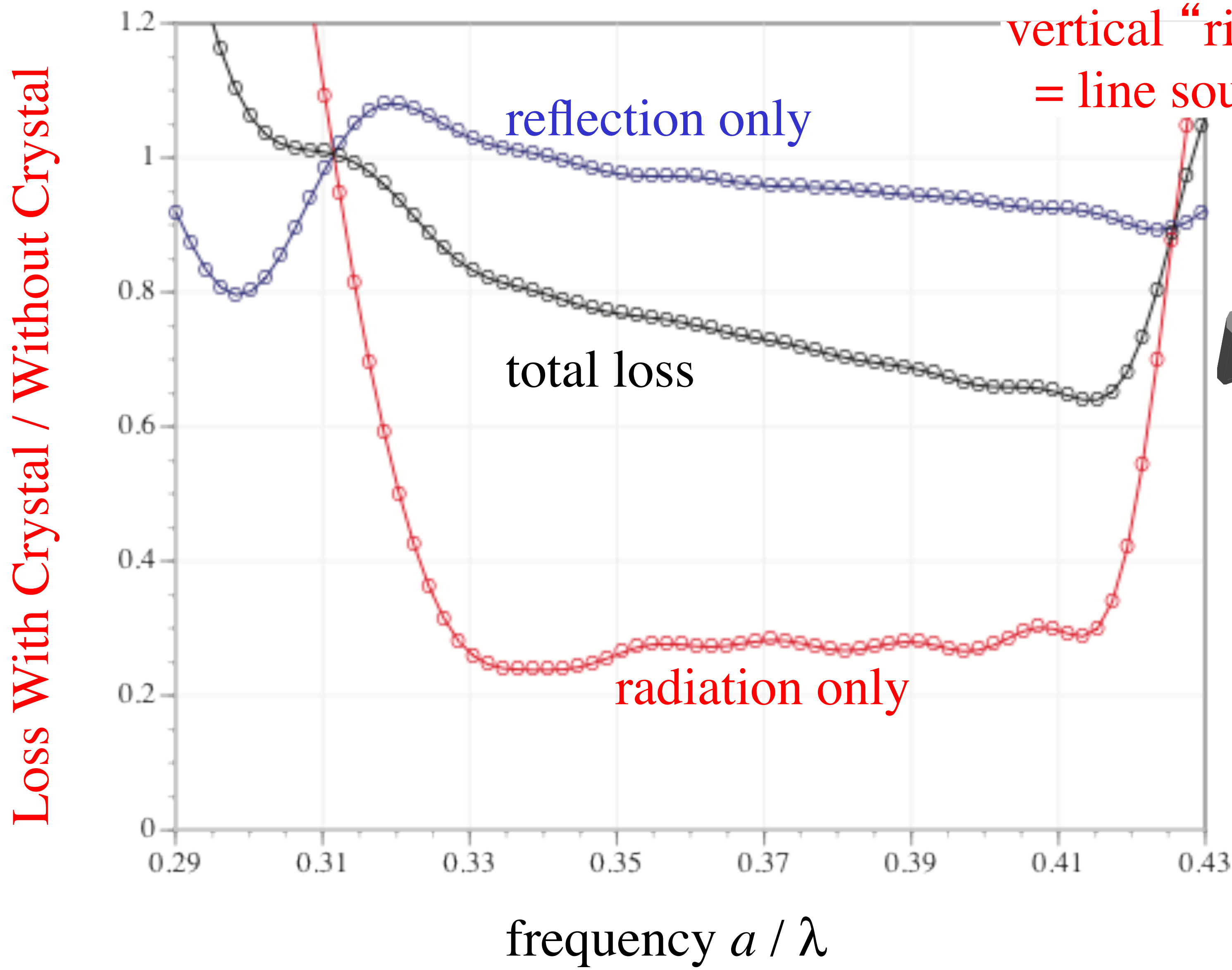
How does *incomplete 3d gap* affect roughness loss?

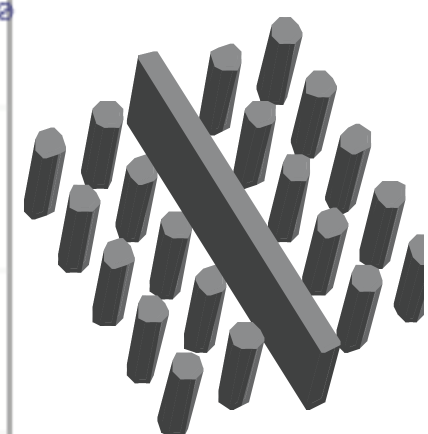
[S. G. Johnson *et al.*, *Applied Phys. B* **81**, 283 (2005).]

Rods: Surface-corrugation

[S. G. Johnson *et al*., *Applied Phys. B* **81**, 283 (2005).]

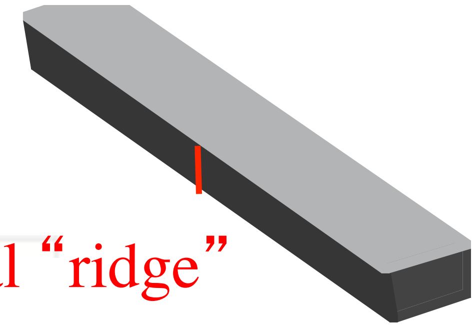




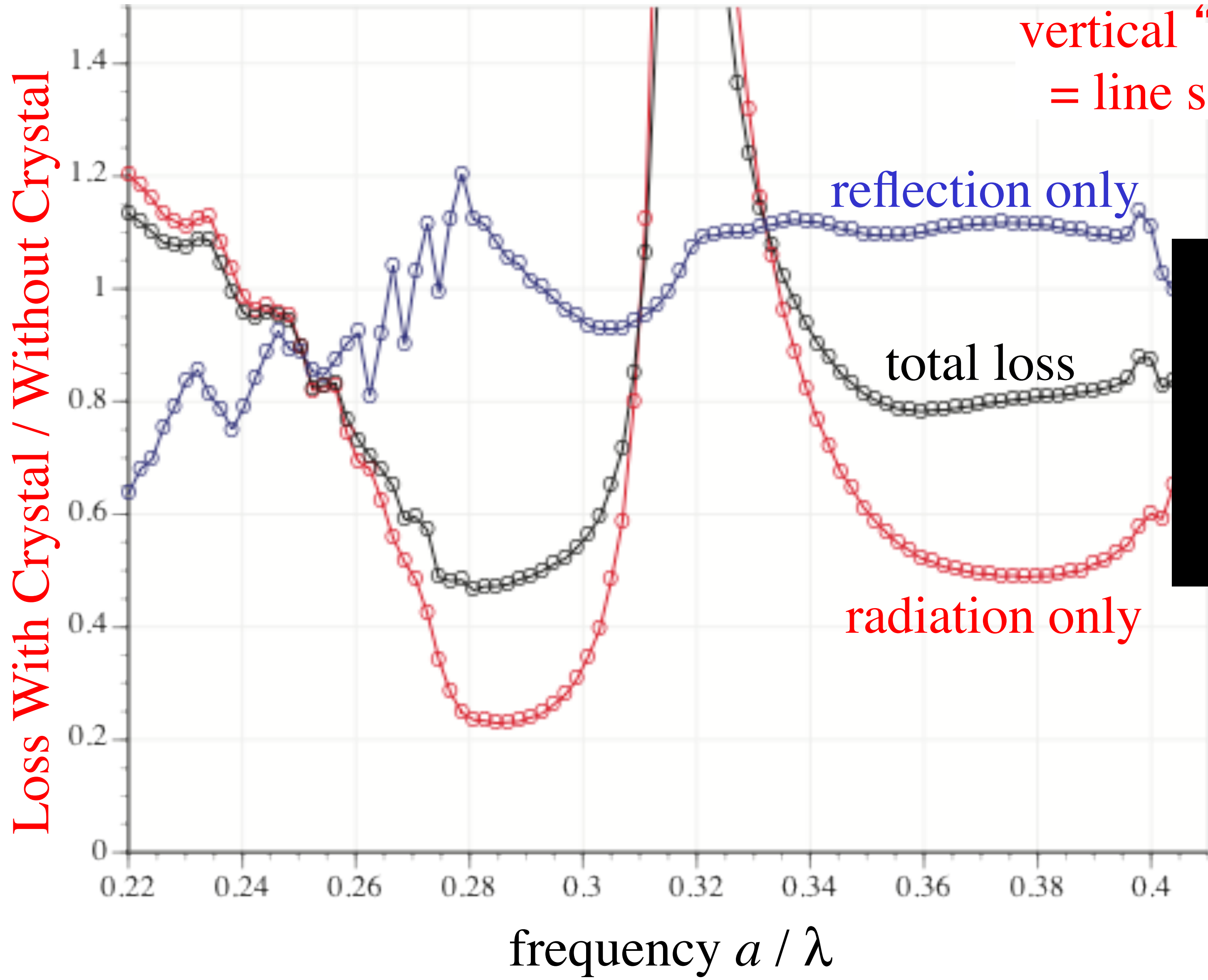


Holes: Surface-corrugation

[S. G. Johnson *et al*., *Applied Phys*. *B* **81**, 283 (2005).]

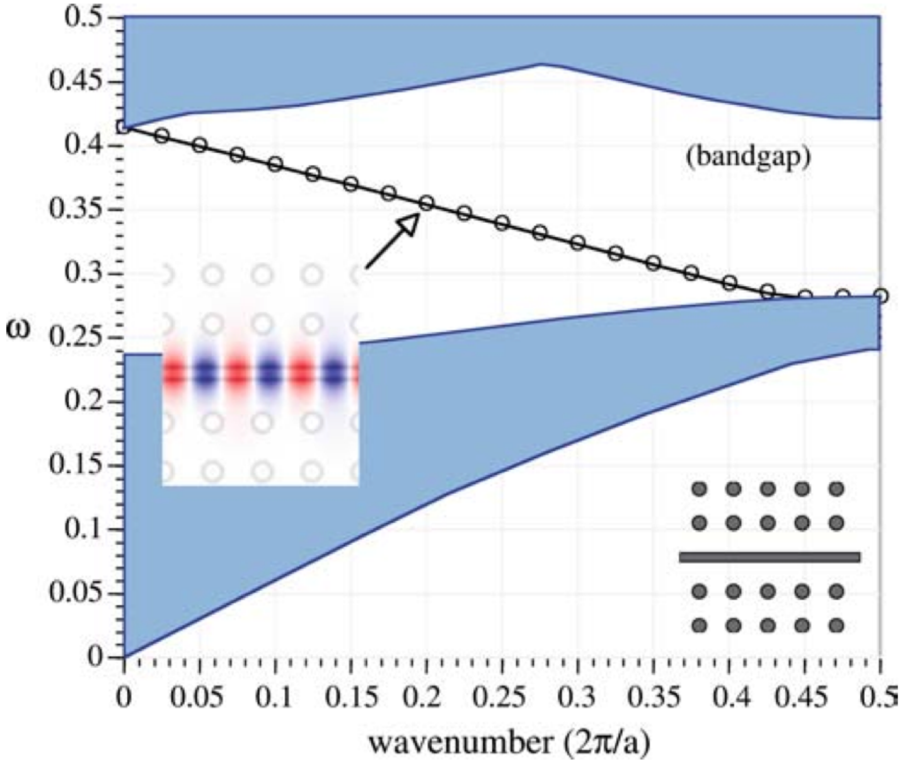


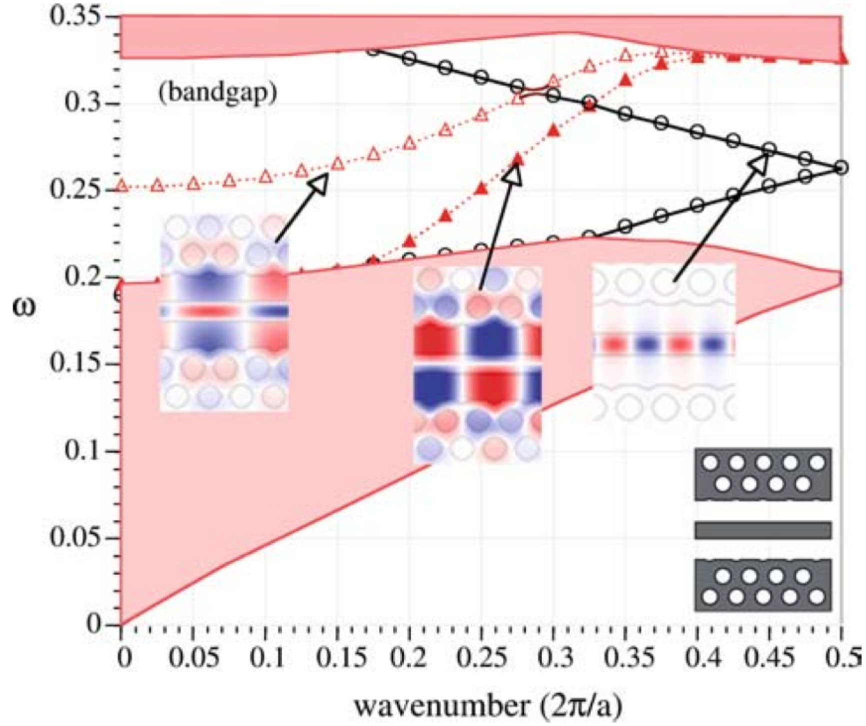
vertical "ridge"
= line source



Rods vs. Holes? *Answer is in 2d.*

[S. G. Johnson *et al.*, *Applied Phys. B* **81**, 283 (2005).]





The hole waveguide is not single mode

— crystal introduces new modes (in 2d)

and new leaky modes (in 3d)

Controlled Deviations: Tapers

[Johnson *et al*., *PRE* **66**, 066608 (2002)]

- An adiabatic theorem for periodic systems:

slow transitions = 100% transmission

— with simple conditions = design criteria

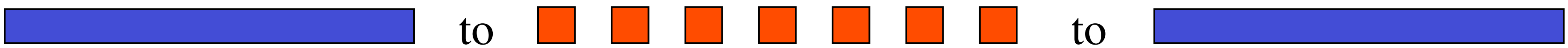
In doing so, we got something more:

a new coupled-mode theory for periodic systems

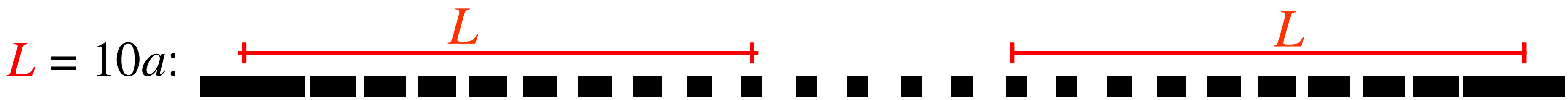
= efficient modeling +

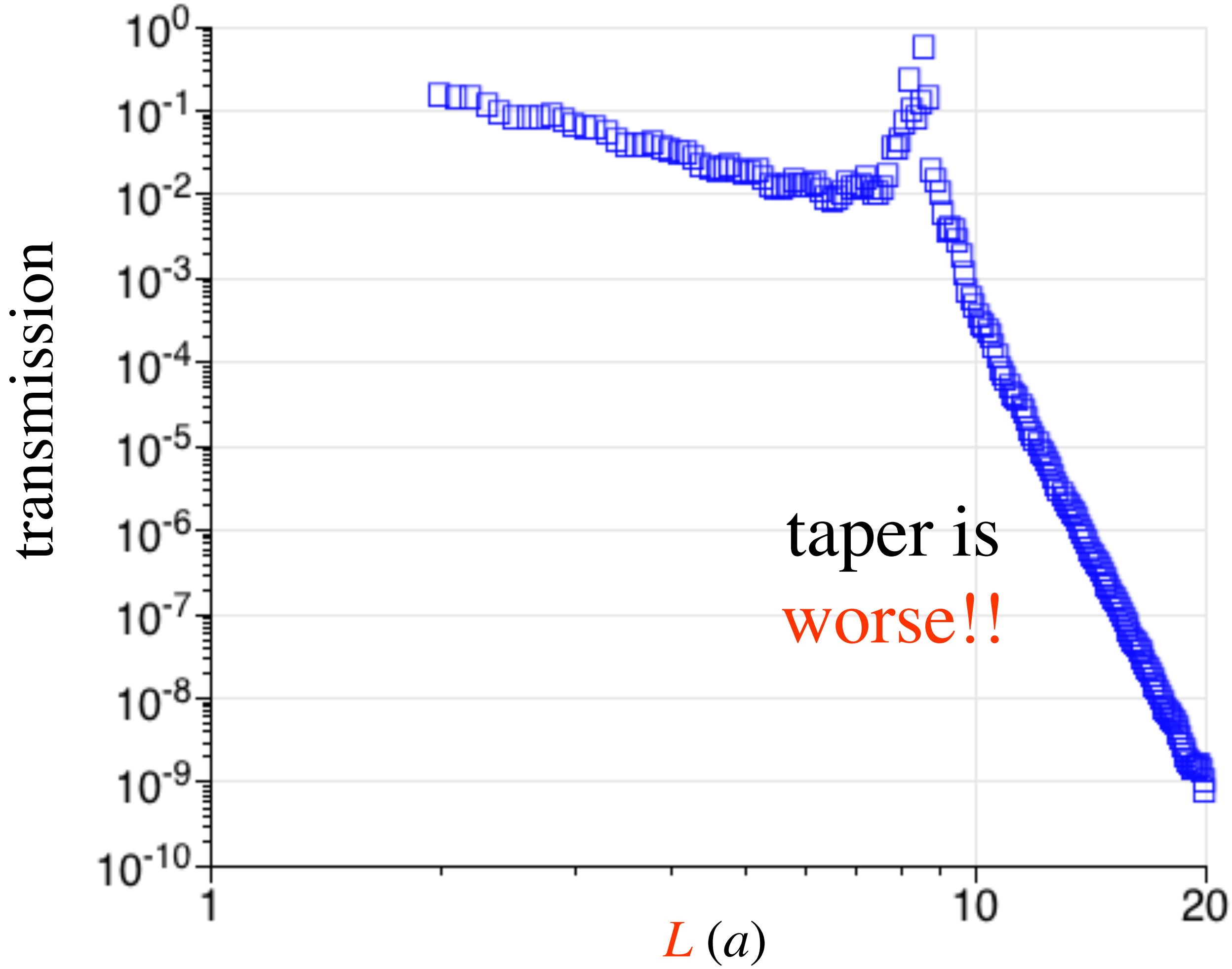
results for other problems

A simple problem?



A simple problem?





What happened
to the adiabatic theorem?

[Johnson *et al*., *PRE* **66**, 066608 (2002)]

There *is* an adiabatic theorem!
…but with two conditions

At all intermediate taper points, the operating mode:

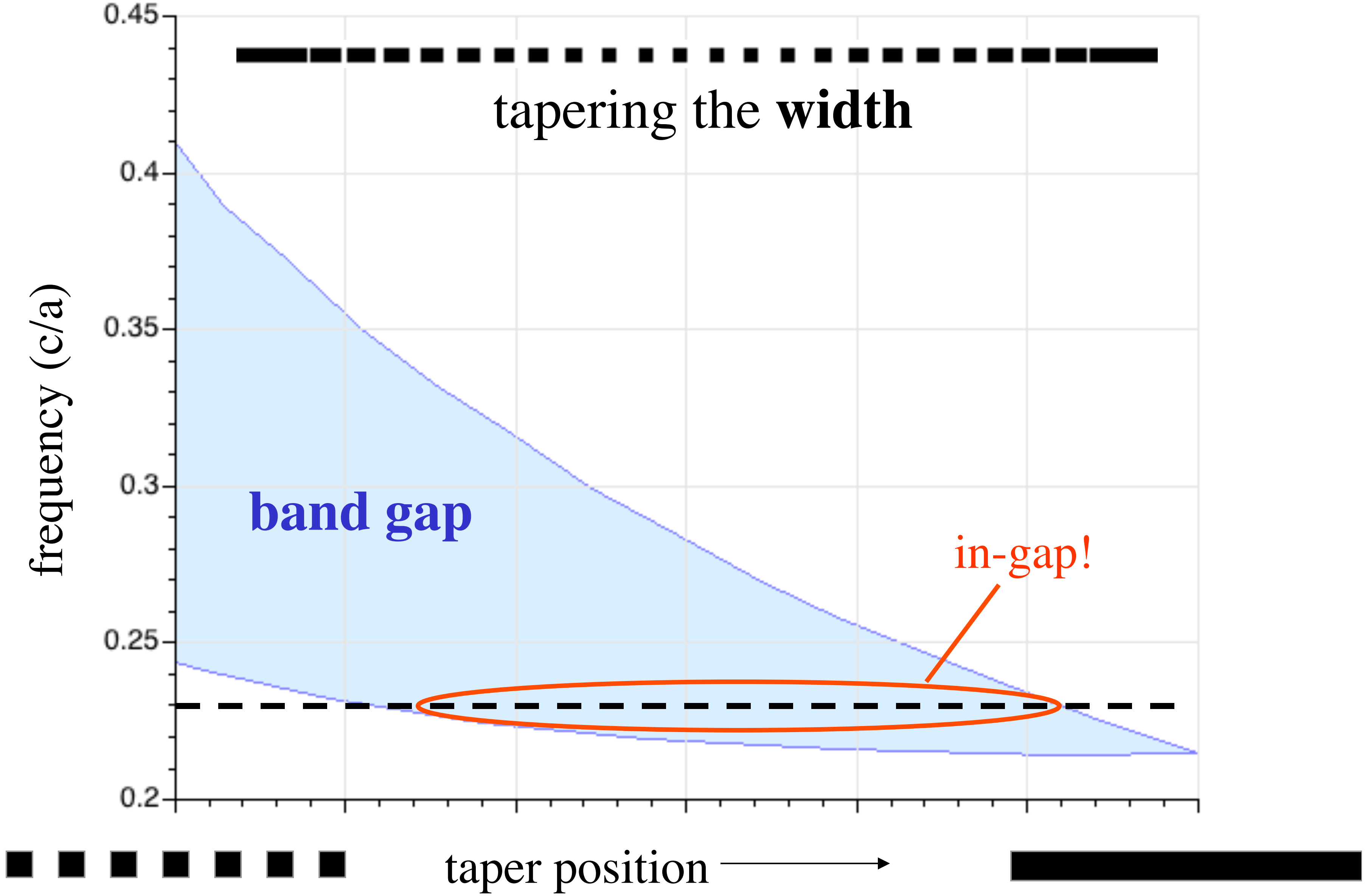
Must be propagating (not in the band gap).

Must be guided (not part of a continuum).

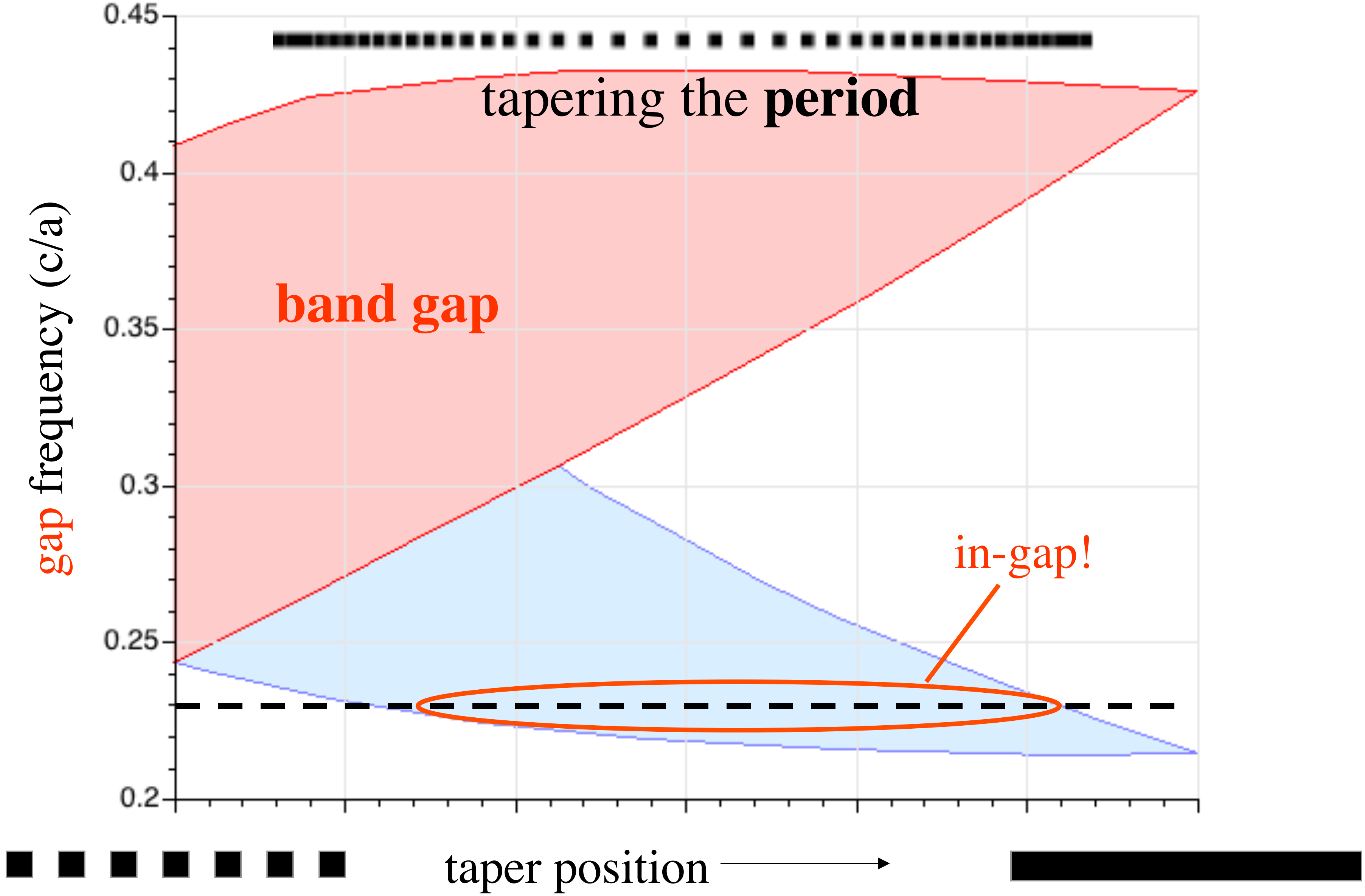
Intuitive!

Easy to violate accidentally in photonic crystals.

A Problematic Taper



Corrected Taper: Shifting the Gap



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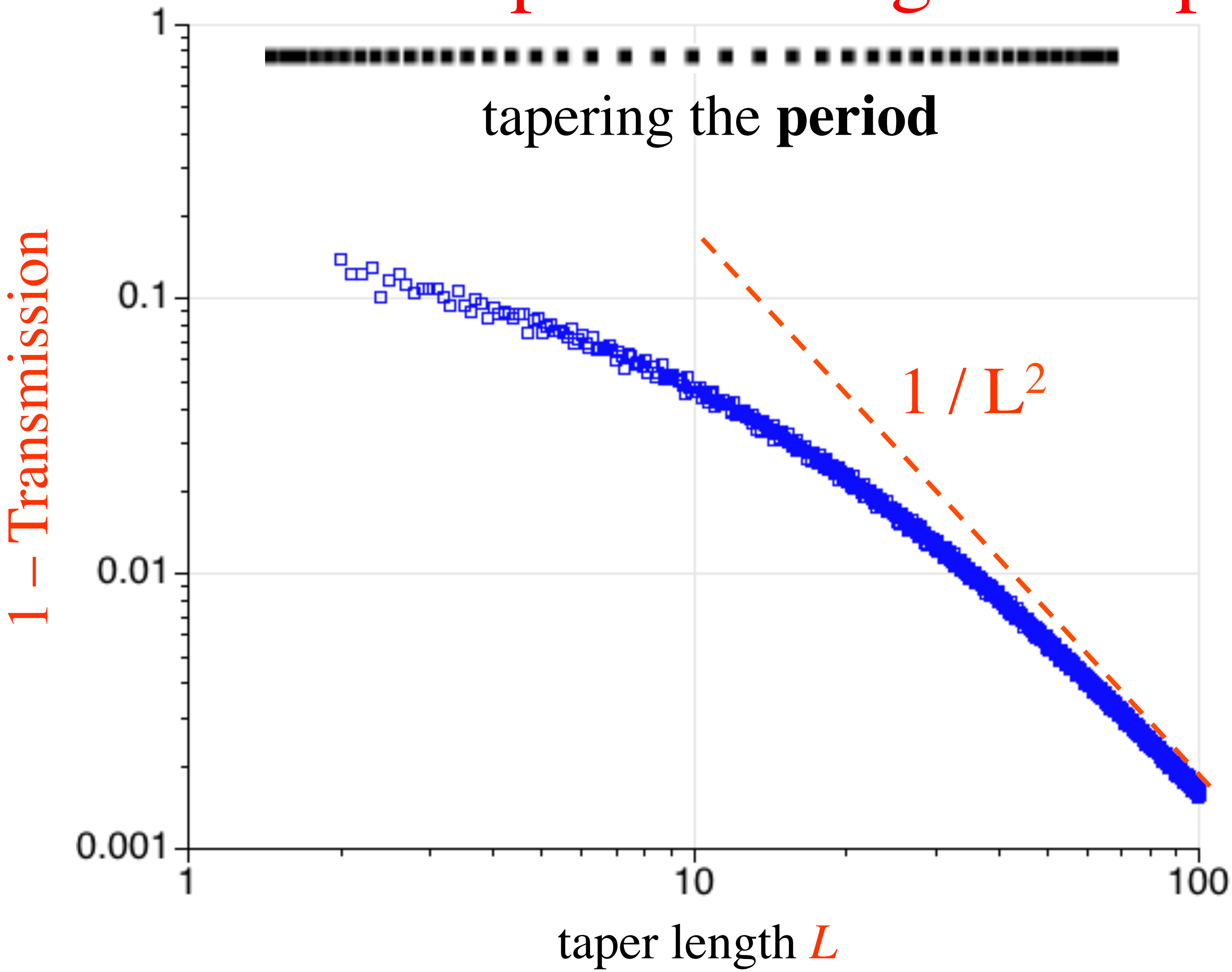

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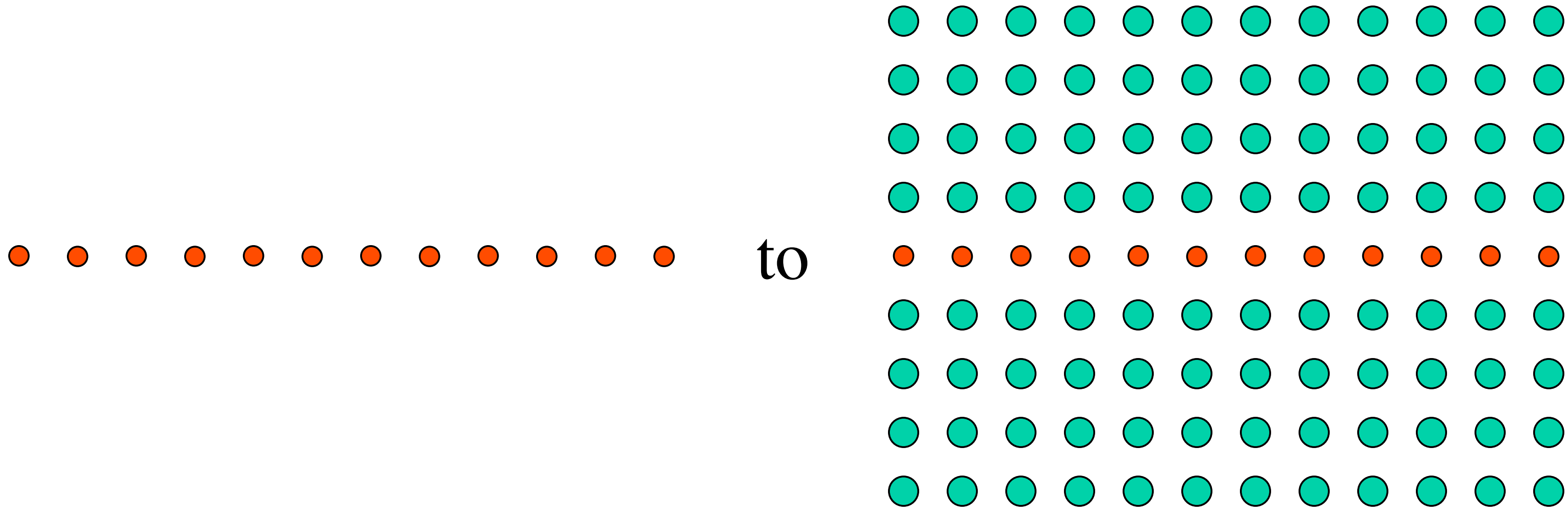
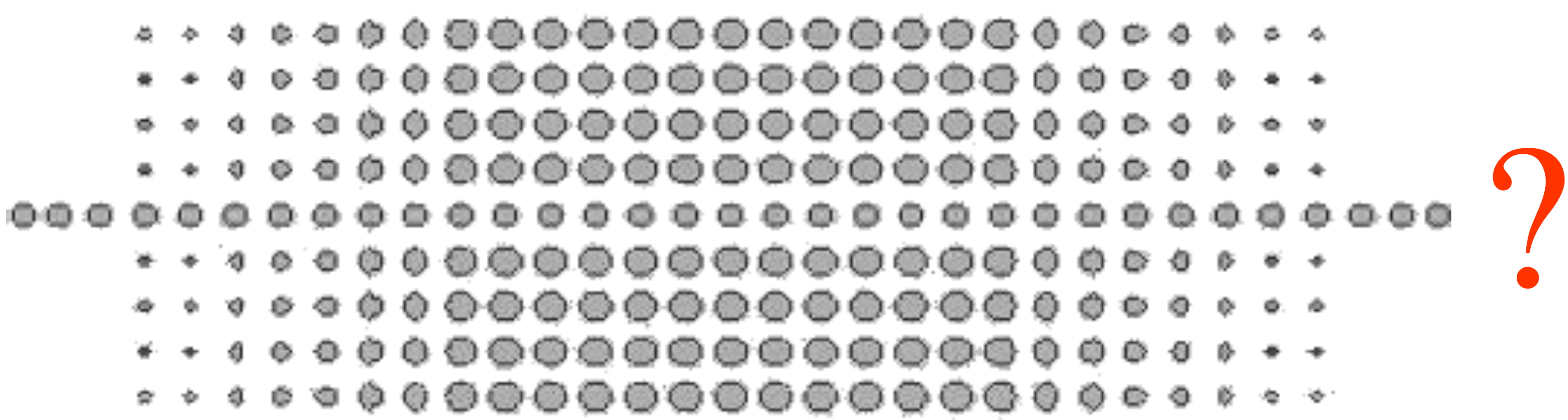
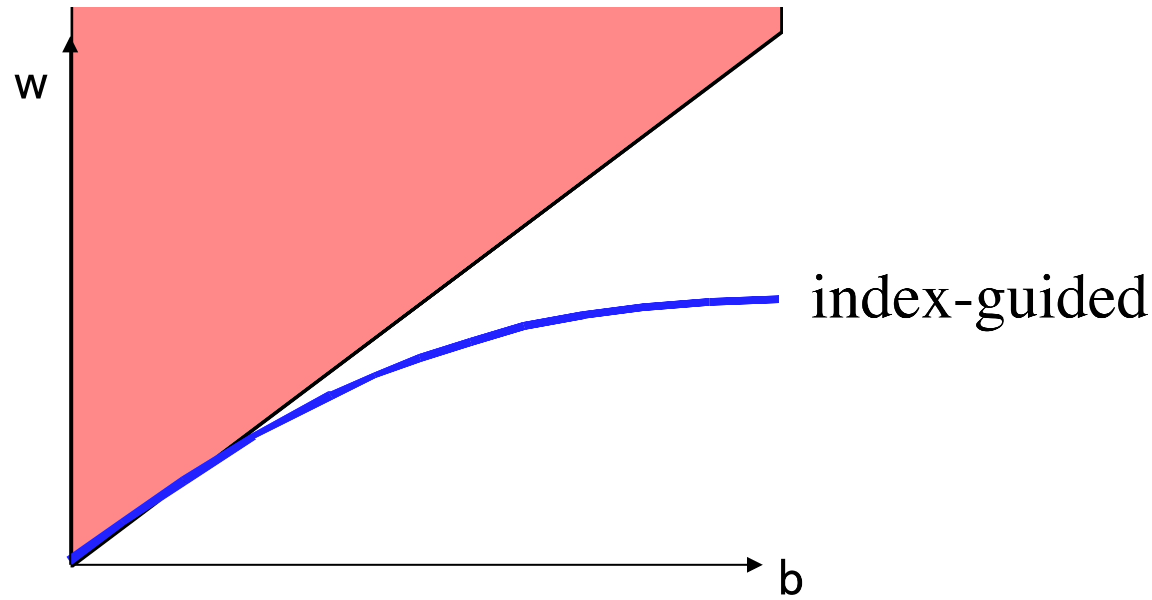
Index-guided to Bandgap-guided

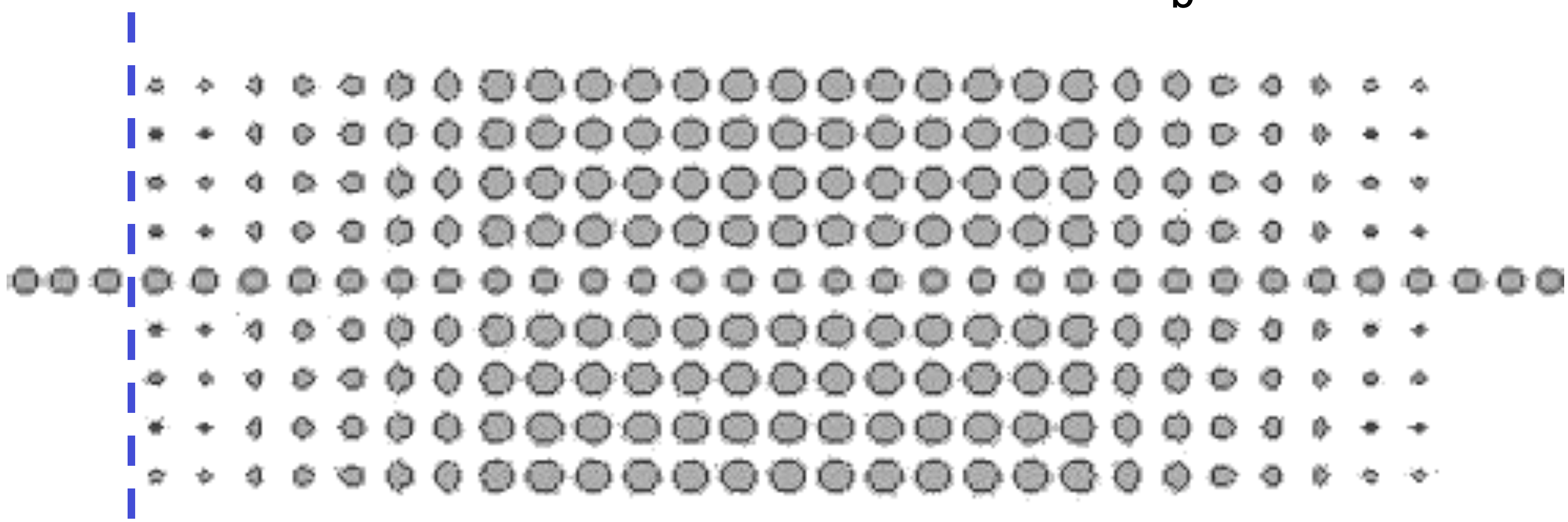




Index-guided to Bandgap-guided

cartoon:





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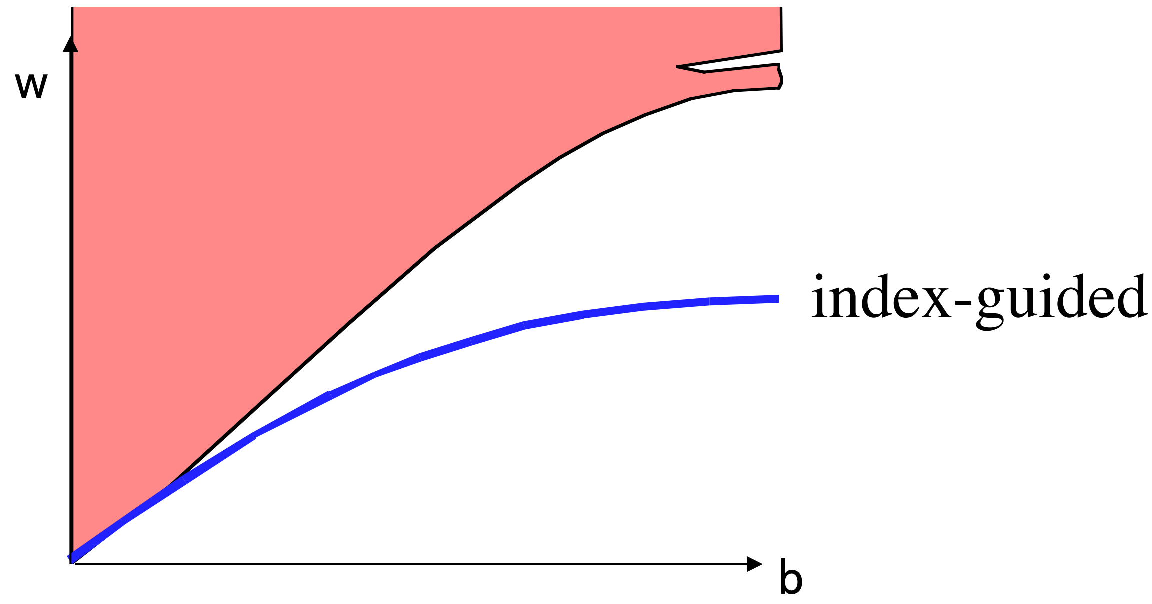

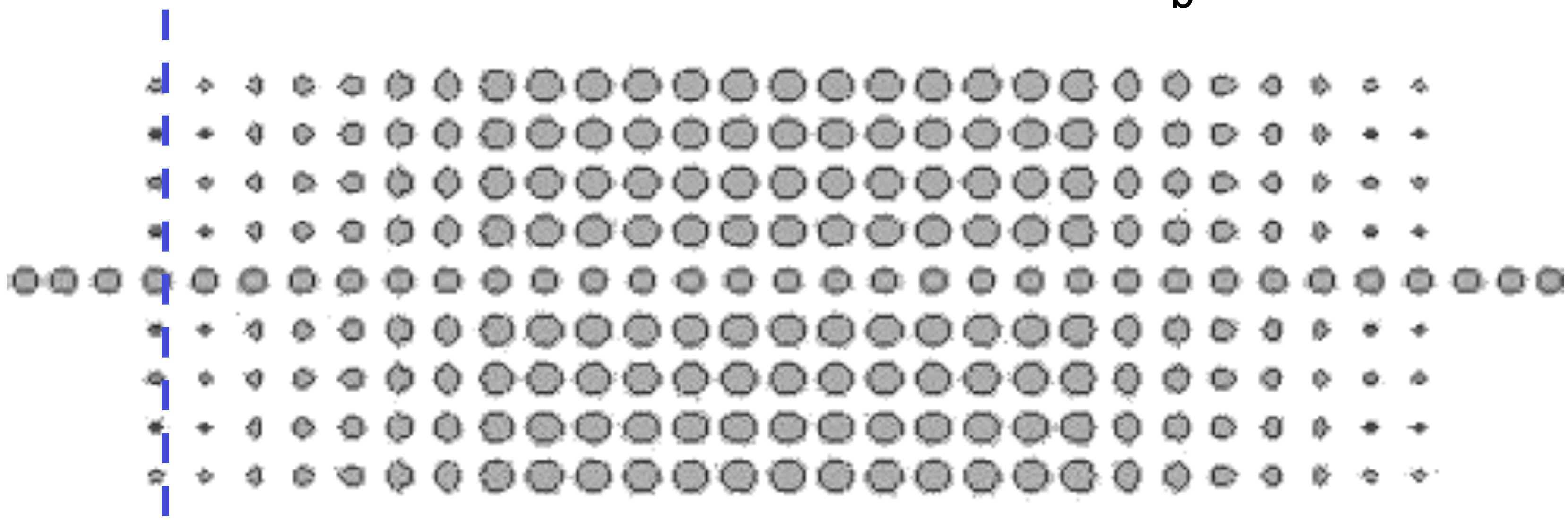

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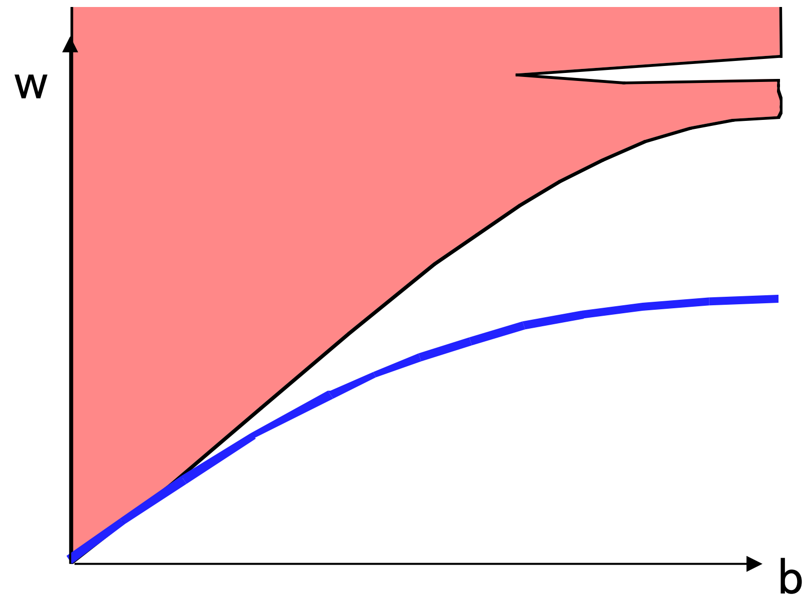

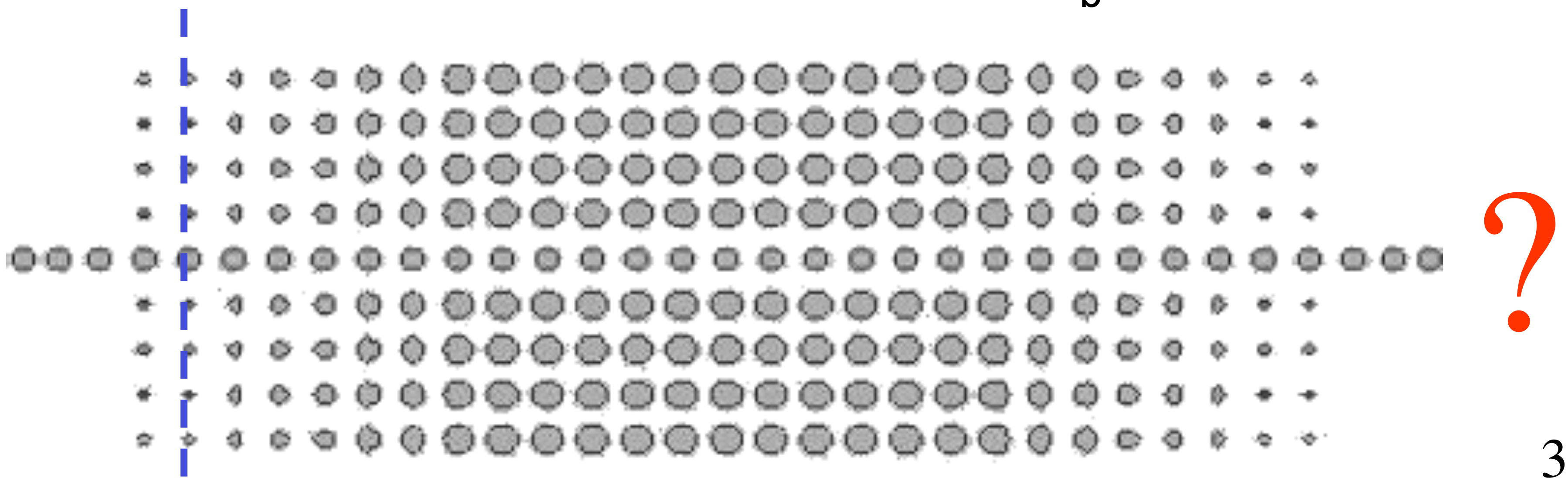

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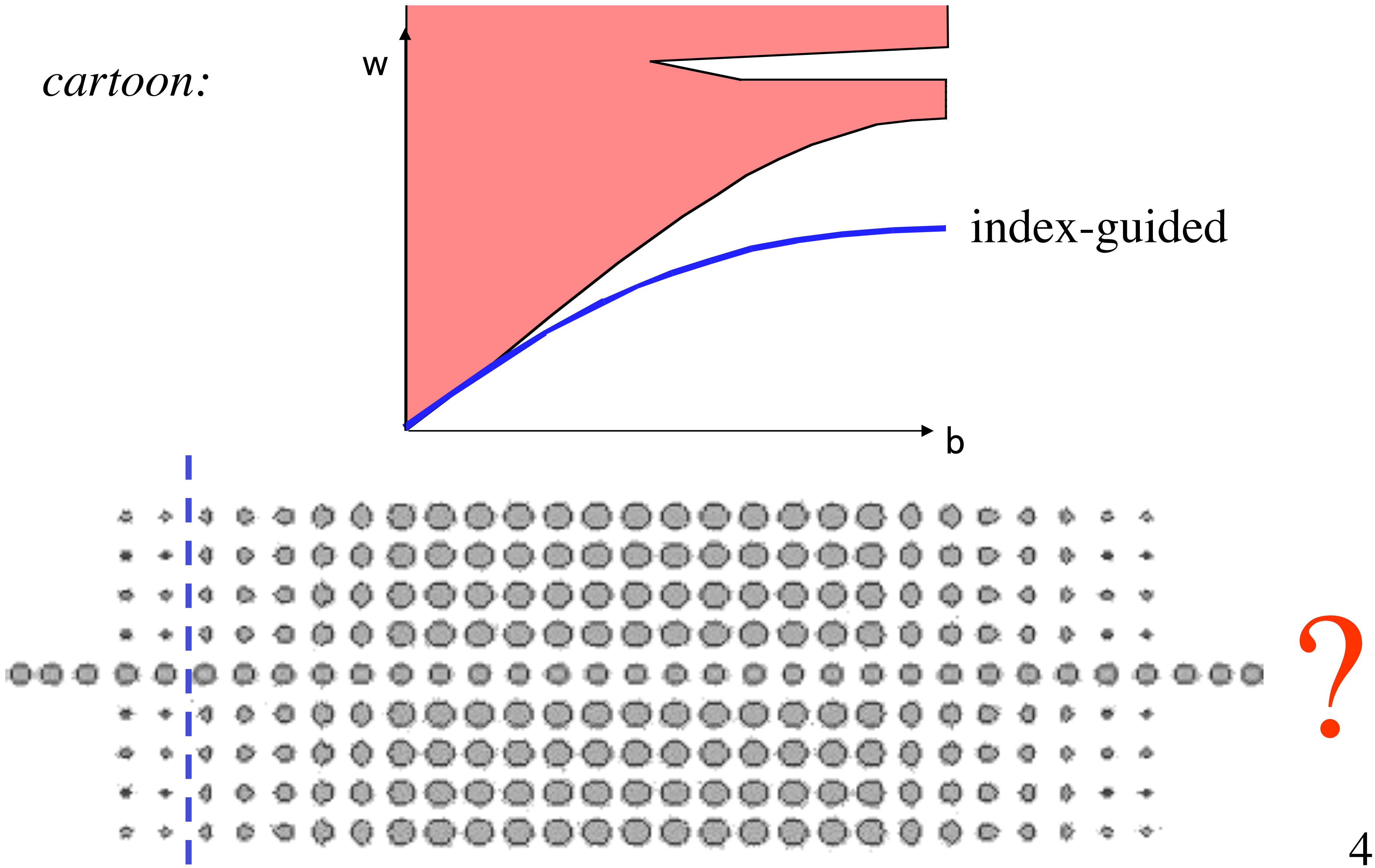

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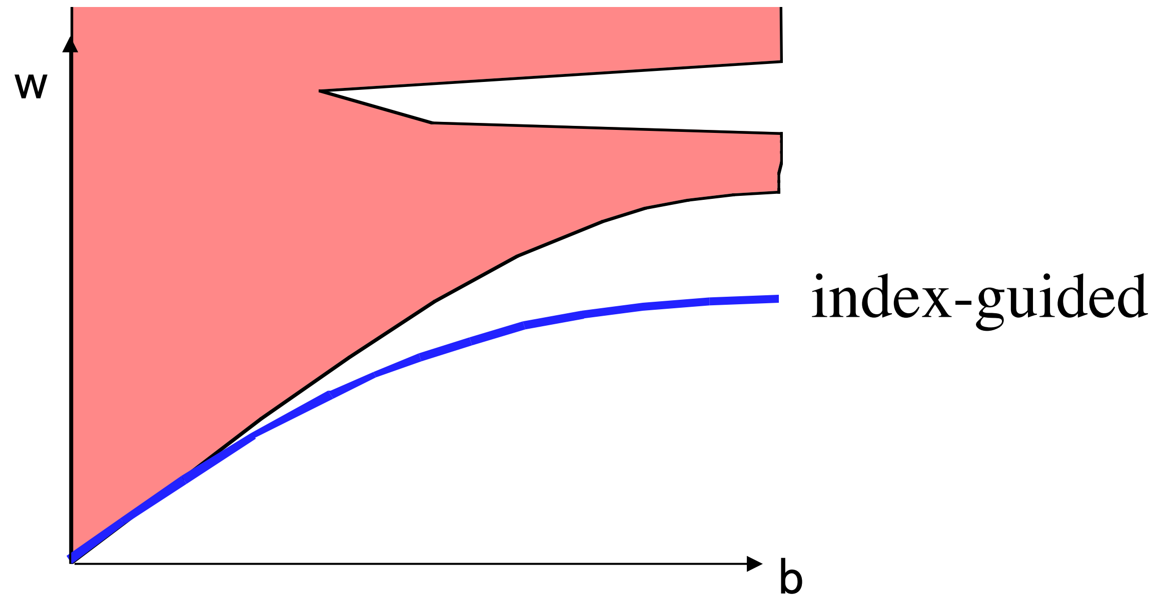

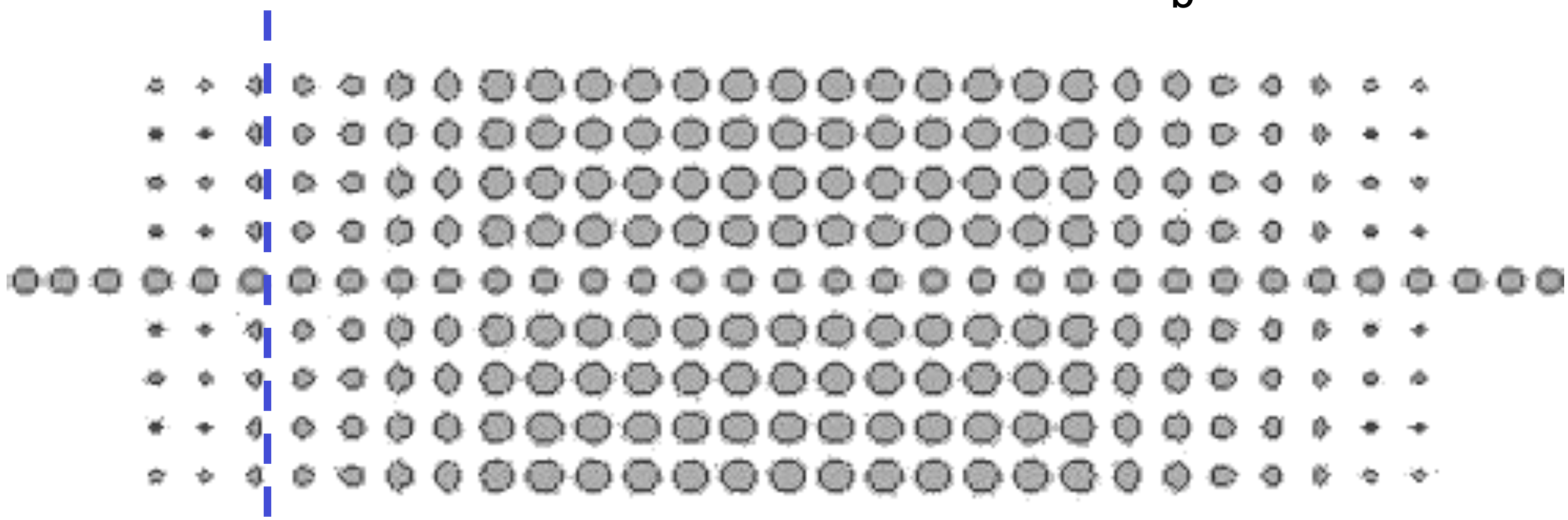

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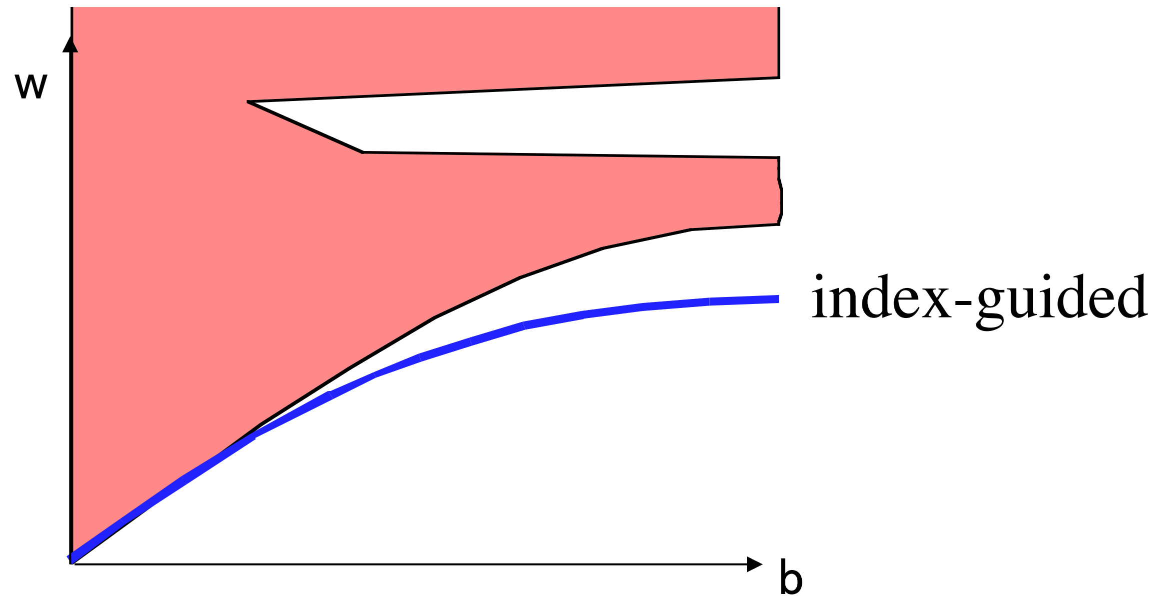

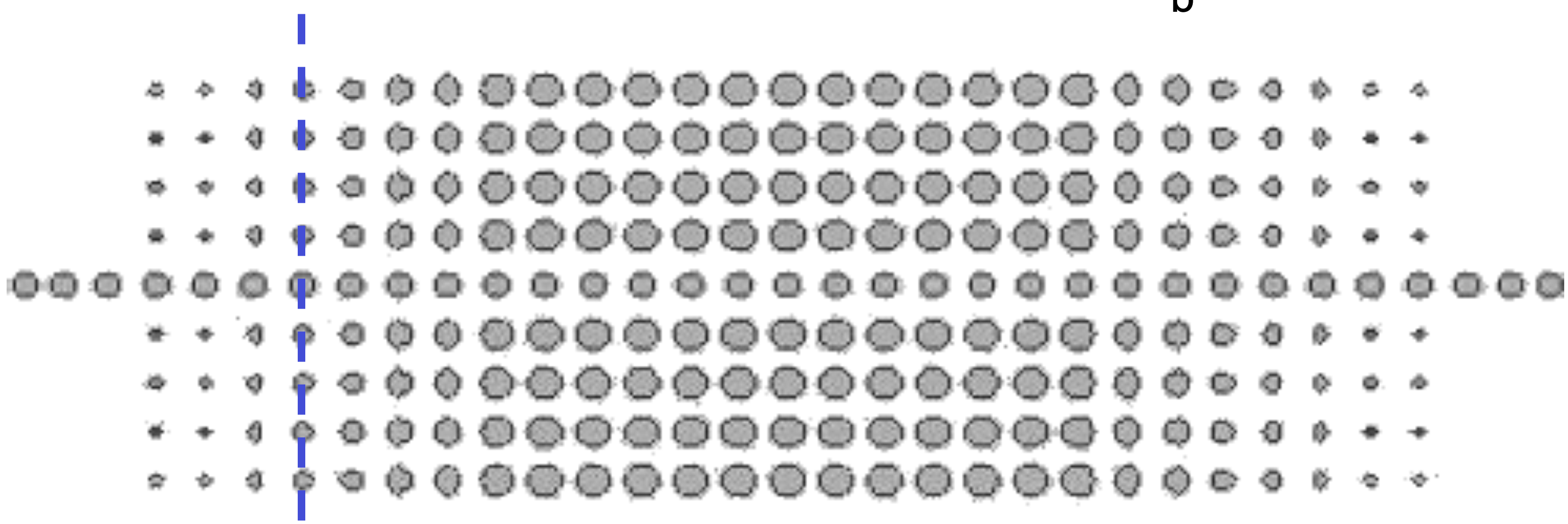

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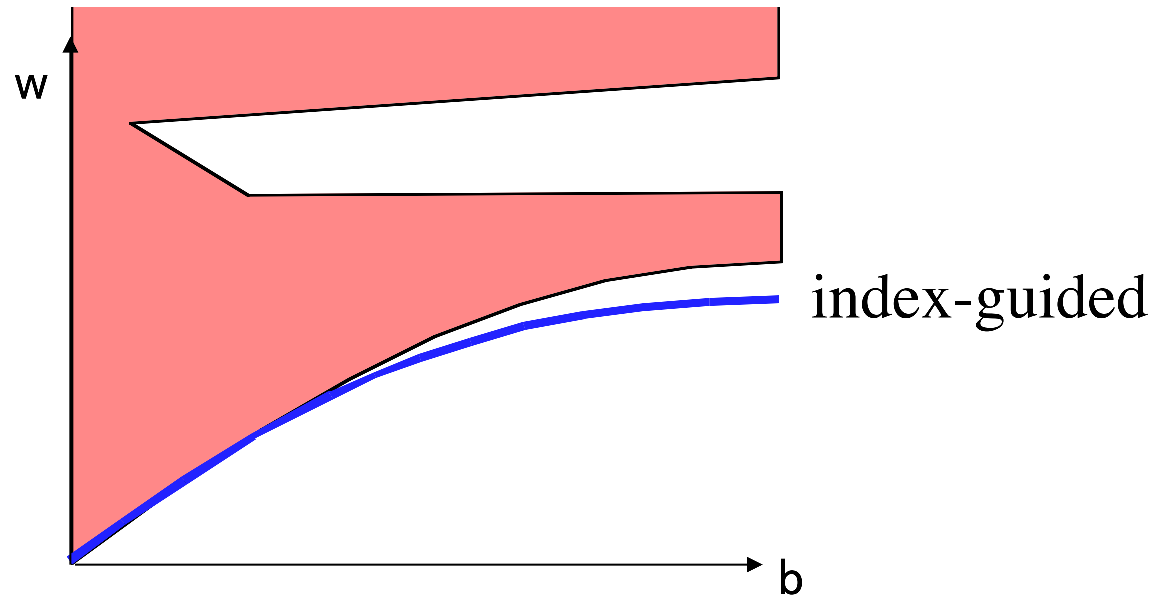

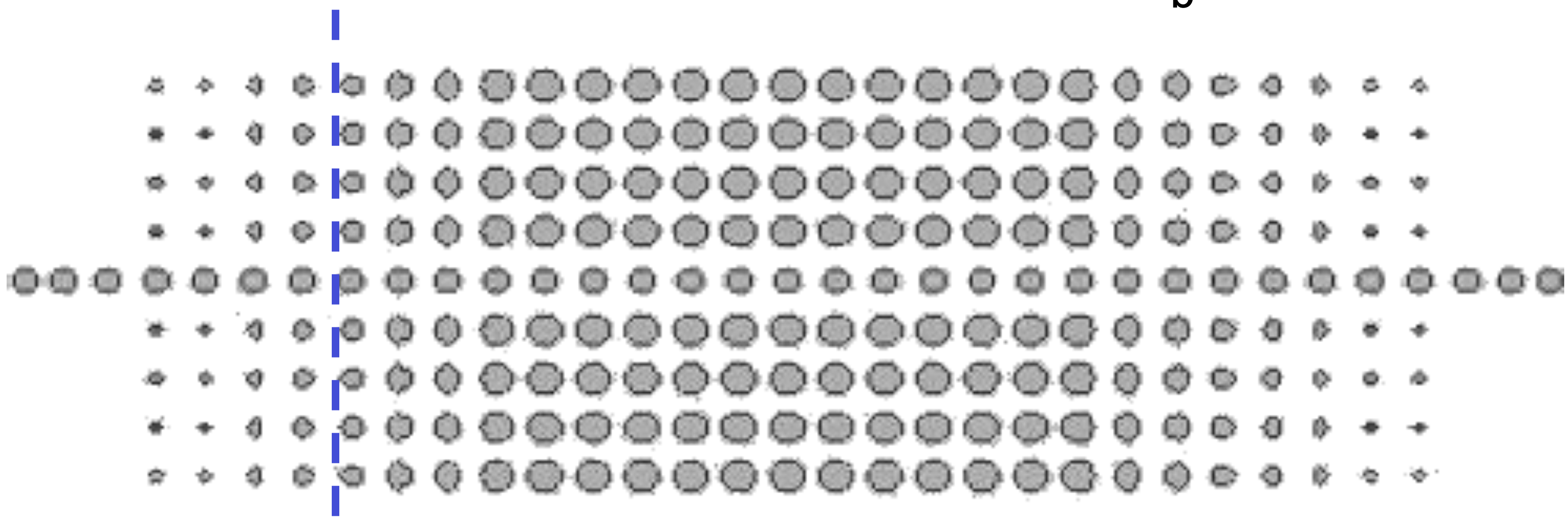

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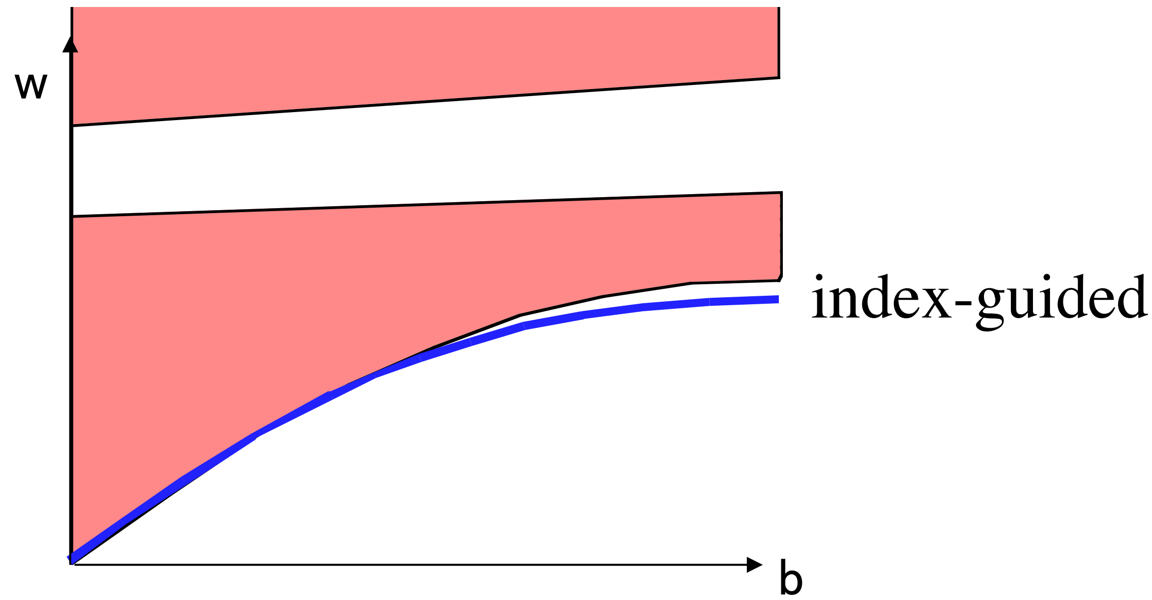

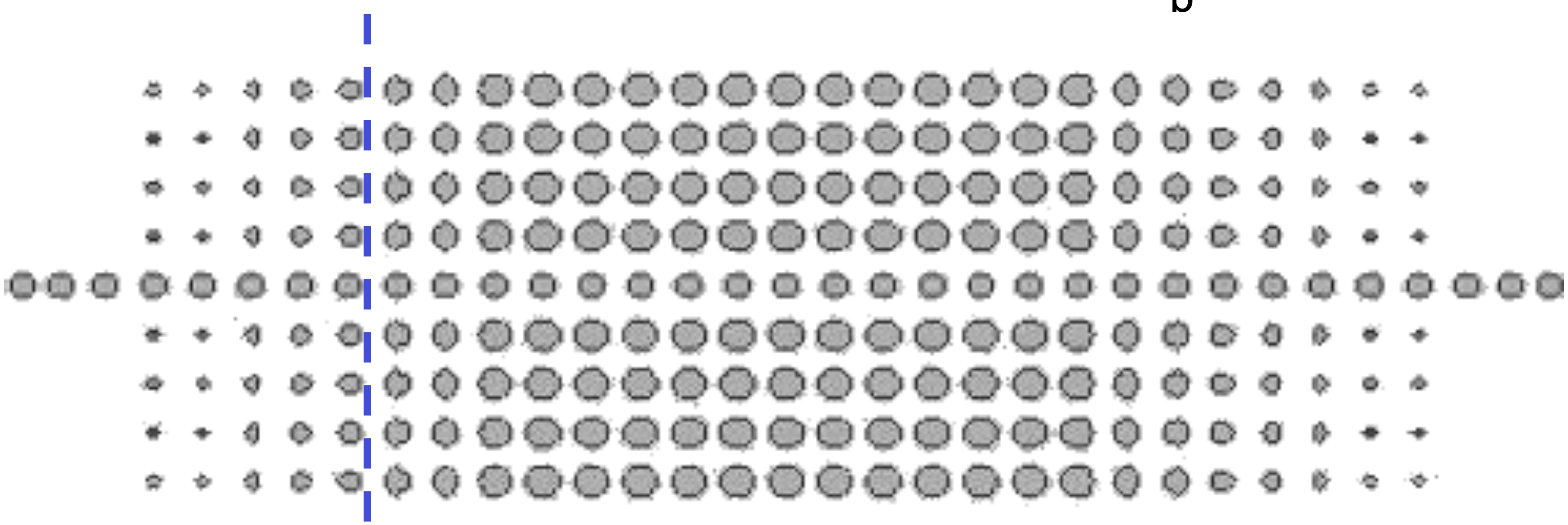

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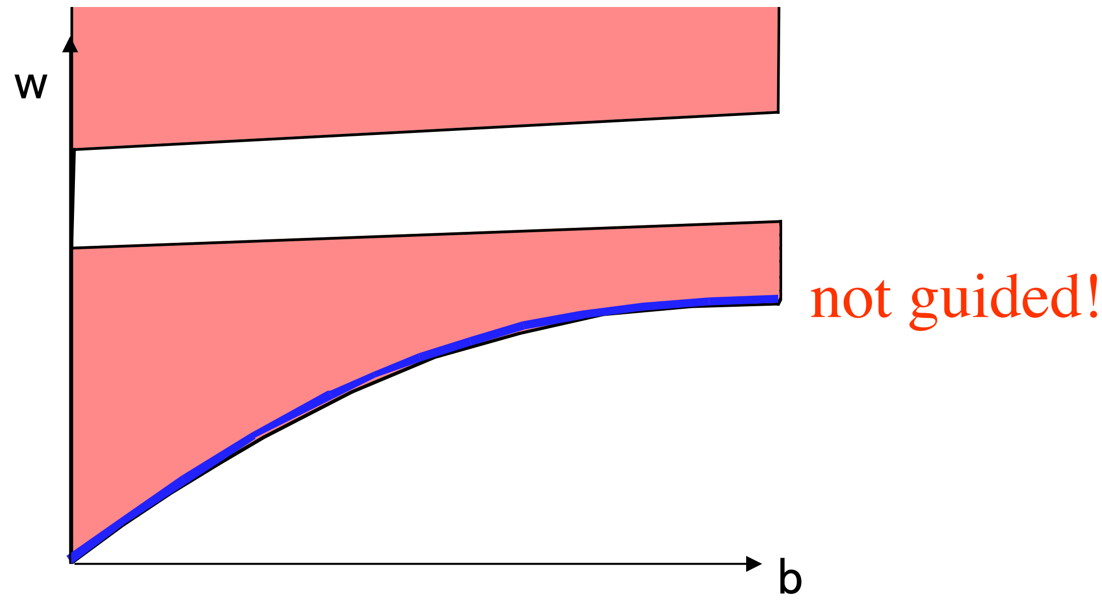

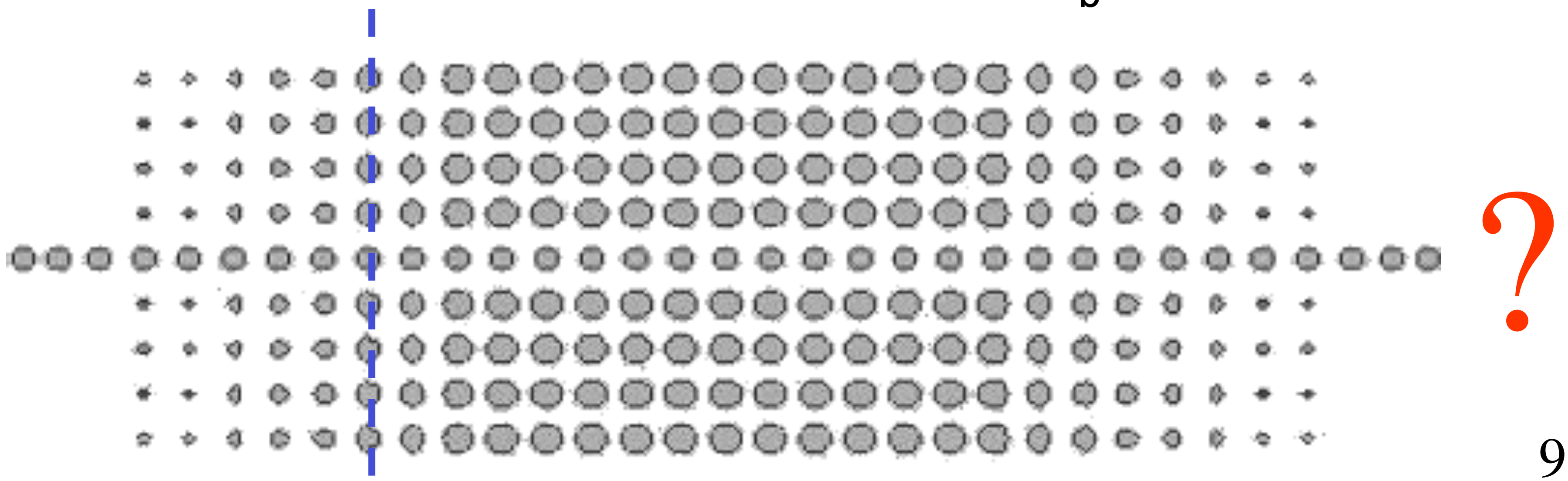

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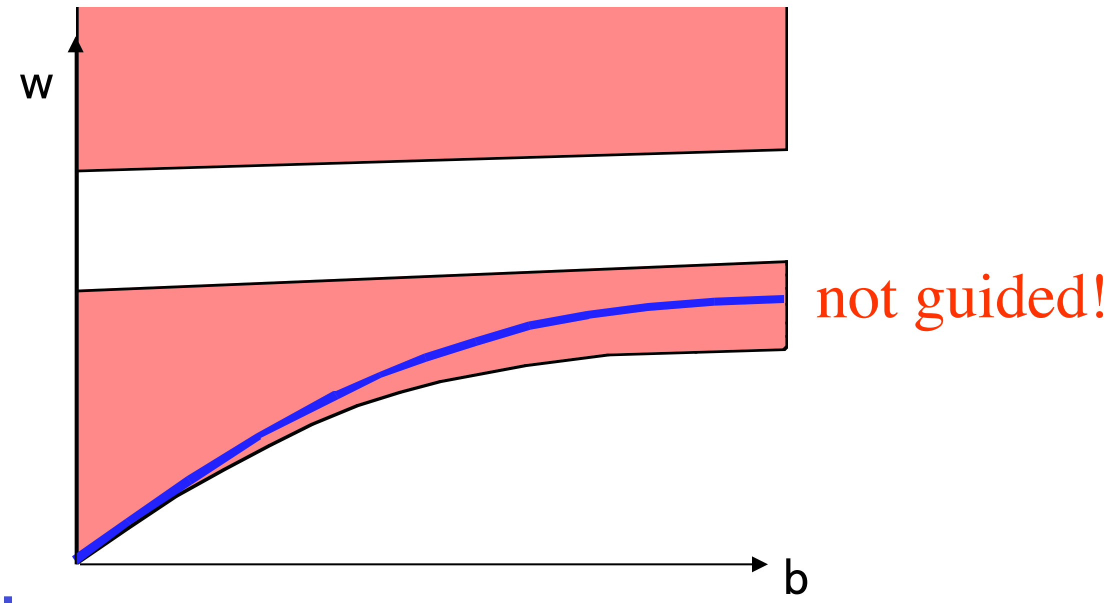

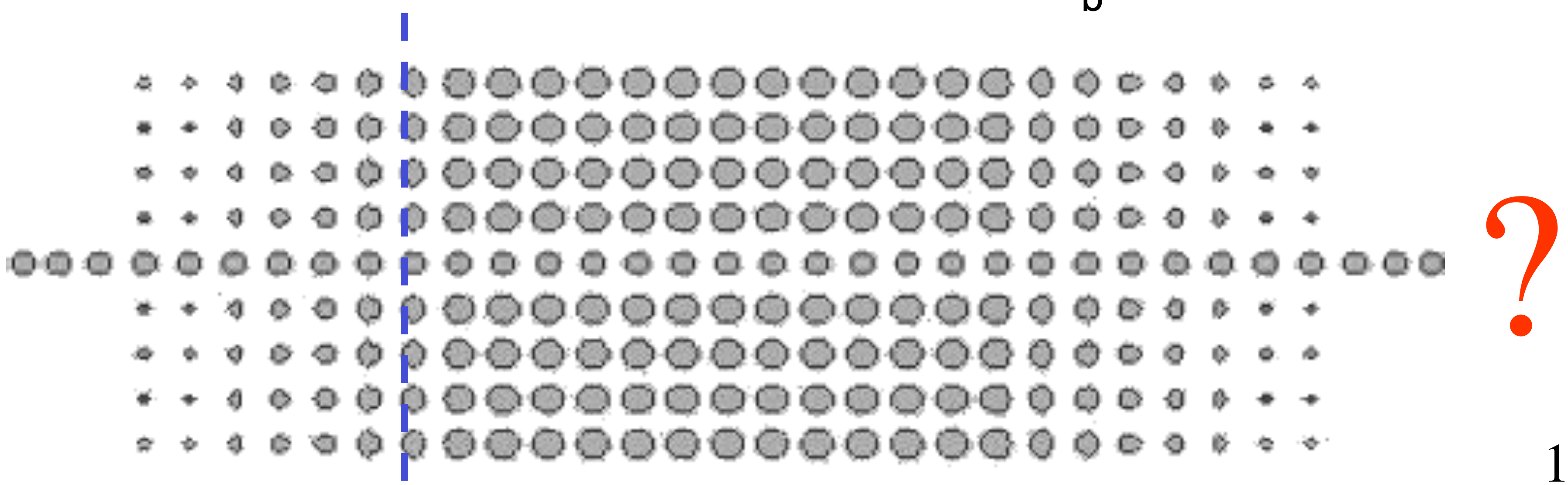

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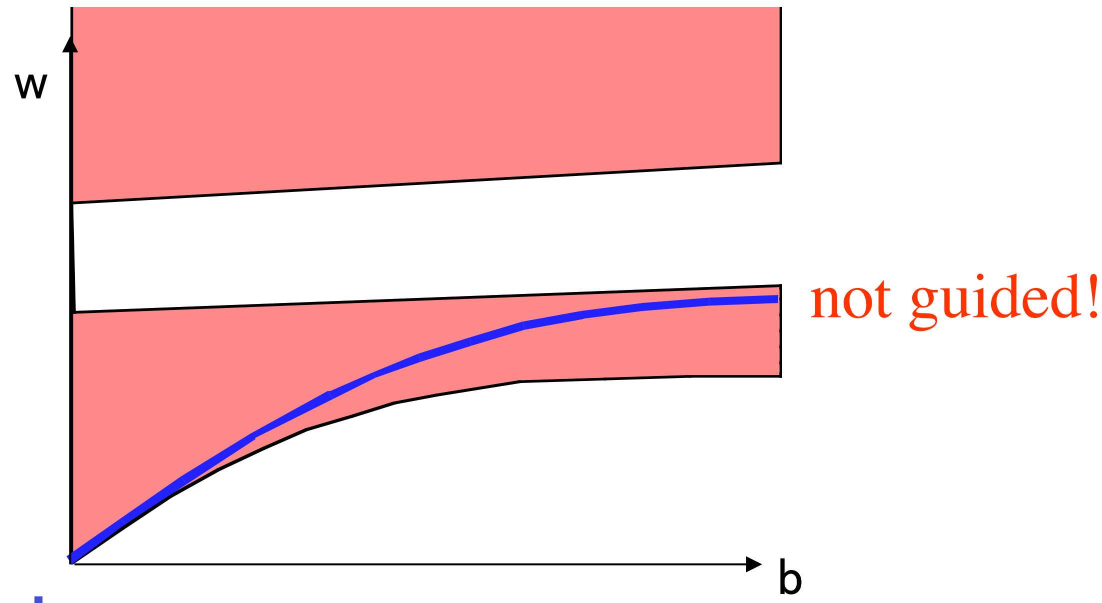

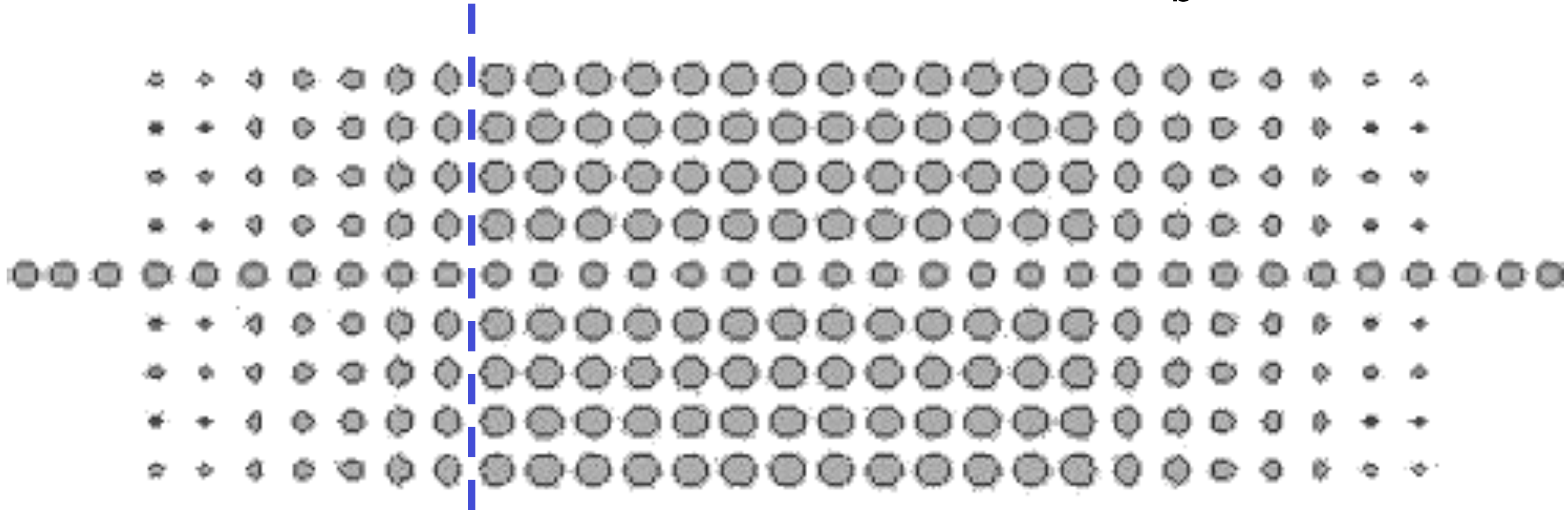

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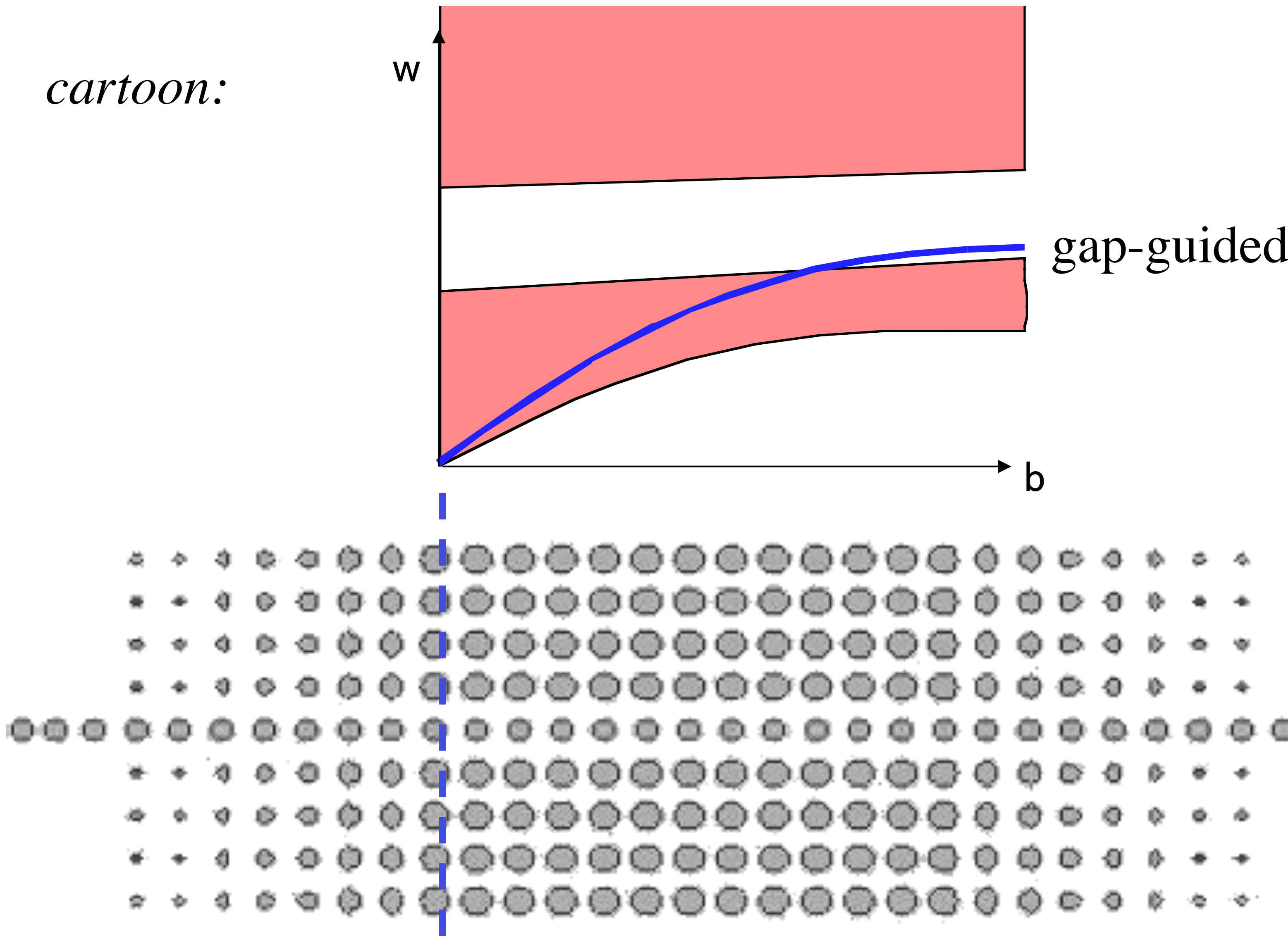

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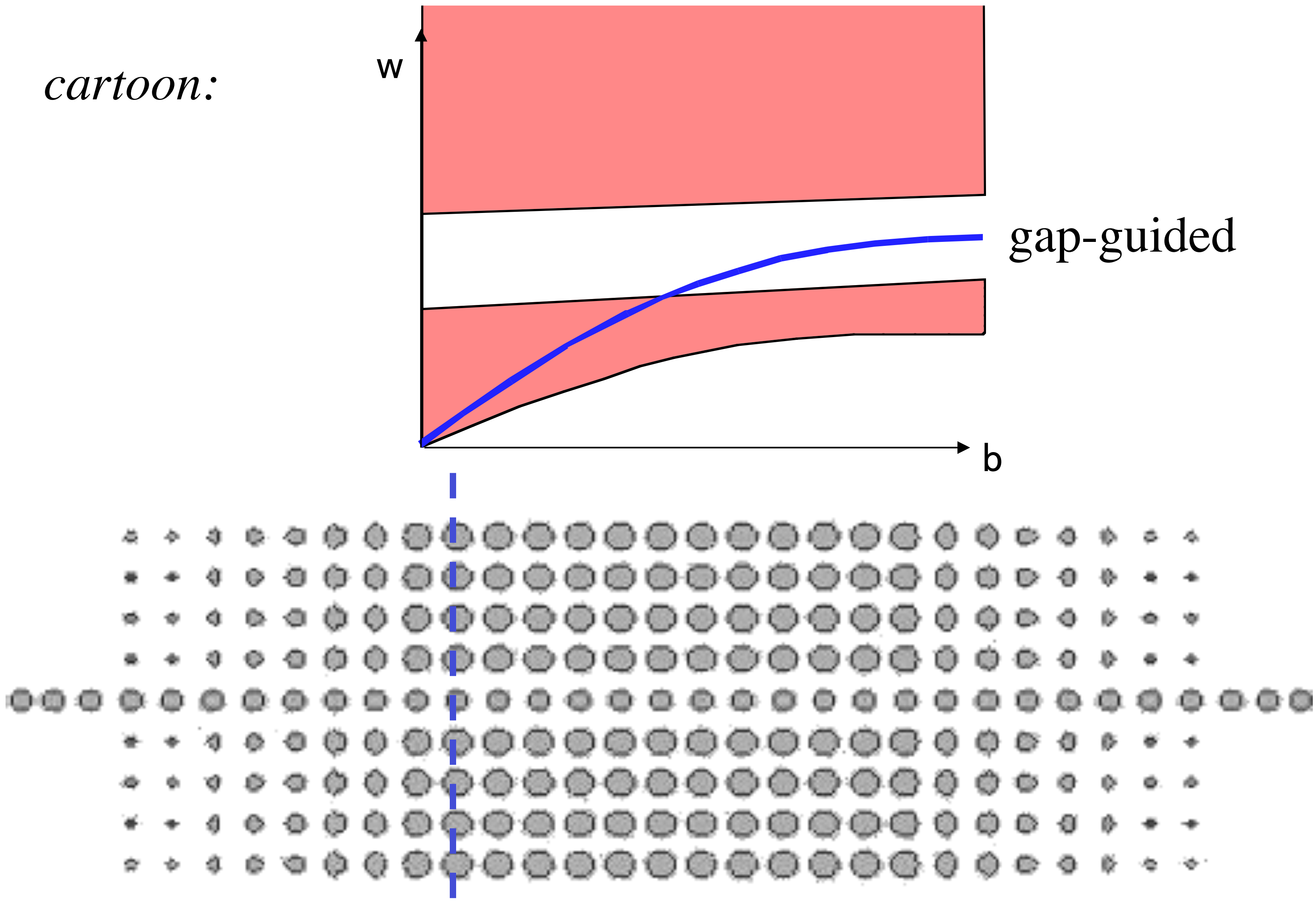

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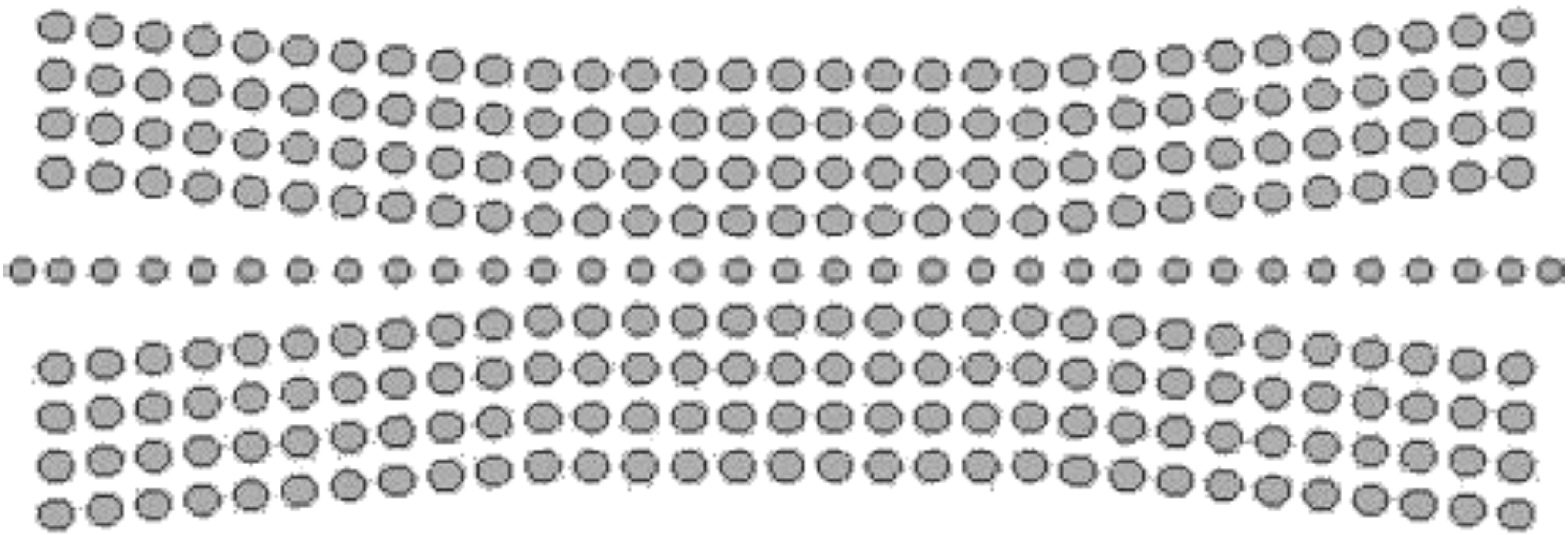
A Working Transition

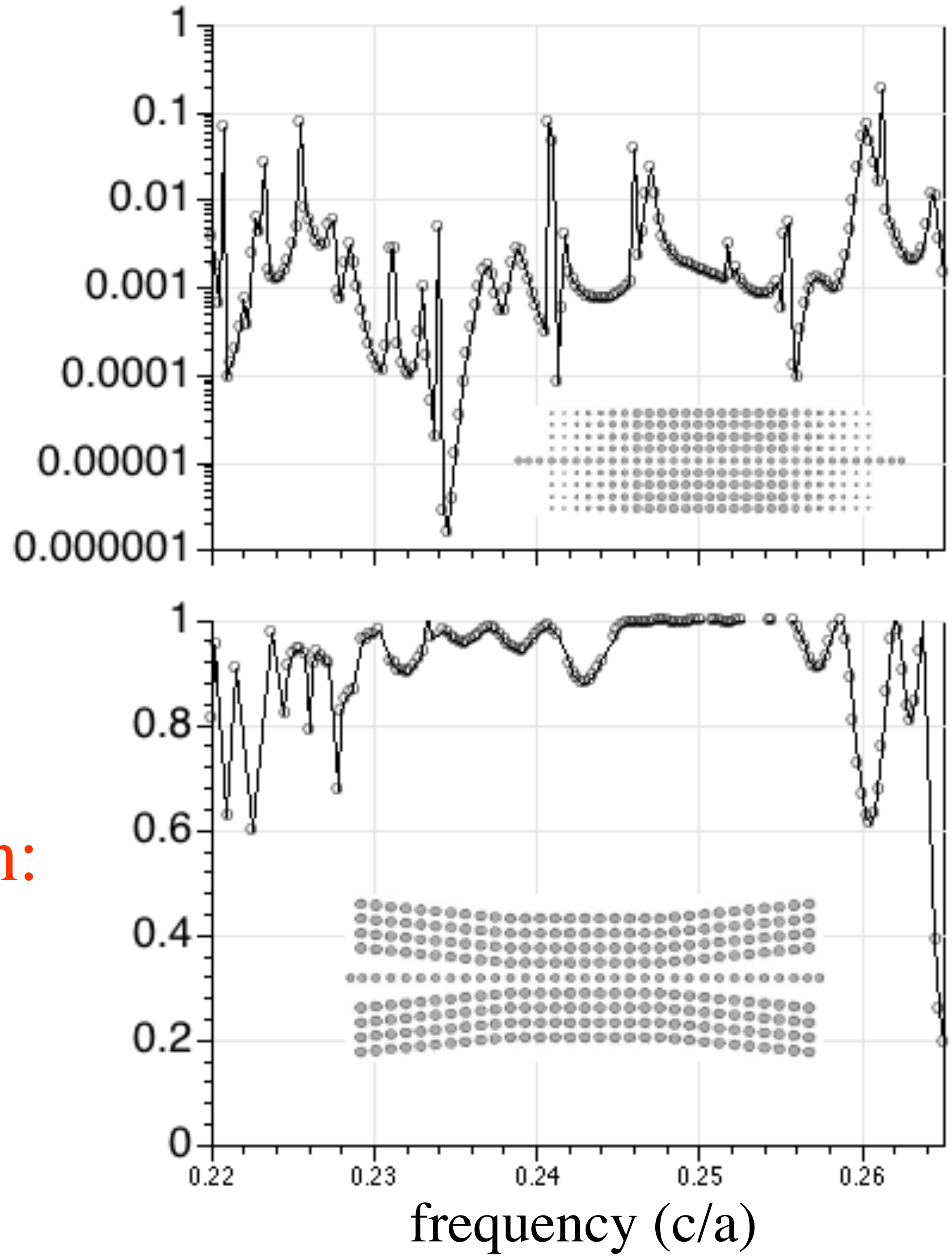


continuum always lies below guided band

... just far away

Bad Transmission:

Good Transmission:

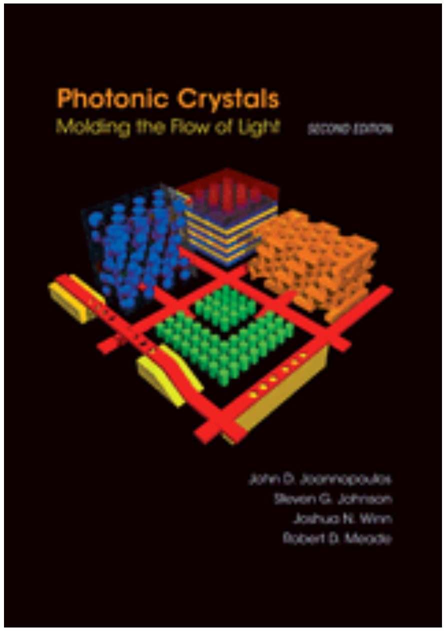


The story of photonic crystals:

~~Finding New Materials / Processes~~

→ Designing New Structures

Free Materials Online



Photonic Crystals book: jdj.mit.edu/book

Tutorial slides: jdj.mit.edu/photons/tutorial

Free electromagnetic simulation software
(FDTD, mode solver, etc.)
jdj.mit.edu/wiki