The design and modeling of microstructured optical fiber

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Outline

• What are these fibers (and why should I care)?

• The guiding mechanisms: index-guiding and band gaps

• Finding the guided modes

• Small corrections (with big impacts)
Outline

• What are these fibers (and why should I care)?

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Optical Fibers Today
(not to scale)

more complex profiles
to tune dispersion

“high” index
doped-silica core
$n \sim 1.46$

“LP_{01}”
confined mode
field diameter $\sim 8\mu m$

silica cladding
$n \sim 1.45$

protective polymer
sheath

losses $\sim 0.2$ dB/km
at $\lambda=1.55\mu m$
(amplifiers every 50–100km)

but this is
$\sim$ as good as it gets…

[ R. Ramaswami & K. N. Sivarajan, Optical Networks: A Practical Perspective ]
The Glass Ceiling: *Limits of Silica*

**Loss:** amplifiers every 50–100km

...limited by Rayleigh scattering (*molecular entropy*)
...cannot use “exotic” wavelengths like 10.6µm

**Nonlinearities:** after ~100km, cause dispersion, crosstalk, power limits

(limited by mode area ~ single-mode, bending loss)
also cannot be made (very) large for compact nonlinear devices

**Radical modifications to dispersion, polarization effects?**
...tunability is limited by low index contrast

---

Long Distances

High Bit-Rates

Dense Wavelength Multiplexing (DWDM)

Compact Devices
Breaking the Glass Ceiling:
Hollow-core Bandgap Fibers

1000x better loss/nonlinear limits (from density)

Photonic Crystal

Bragg fiber
[Yeh et al., 1978]
+ omnidirectional = OmniGuides

1d crystal

PCF
[Knight et al., 1998]

2d crystal

(You can also put stuff in here …)
Breaking the Glass Ceiling:
Hollow-core Bandgap Fibers

Bragg fiber
[ Yeh et al., 1978 ]
+ omnidirectional
= OmniGuides

PCF
[ Knight et al., 1998 ]

white/grey = chalco/polymer

silica


[ figs courtesy Y. Fink et al., MIT ]
Breaking the Glass Ceiling:
Hollow-core Bandgap Fibers

Guiding @ 10.6$\mu$m
(high-power CO$_2$ lasers)
loss < 1 dB/m
(material loss $\sim$ 10$^4$ dB/m)
[ Temelkuran et al.,

Guiding @ 1.55$\mu$m
loss $\sim$ 13dB/km
[ Smith, et al.,

OFC 2004: 1.7dB/km
BlazePhotonics

[ R. F. Cregan et al.,
Science 285, 1537 (1999) ]

[ figs courtesy
Y. Fink et al., MIT ]

white/grey = chalco/polymer

silica

5$\mu$m
Breaking the Glass Ceiling II: Solid-core Holey Fibers

Solid-core holey cladding forms effective low-index material

Can have much higher contrast than doped silica…

strong confinement = enhanced nonlinearities, birefringence, …

Breaking the Glass Ceiling II:
Solid-core Holey Fibers

endlessly single-mode
[ T. A. Birks et al.,

polarization-maintaining
[ K. Suzuki,

nonlinear fibers
[ Wadsworth et al.,

low-contrast linear fiber
(large area)
[ J. C. Knight et al.,
Outline

• What are these fibers (and why should I care)?

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• Small corrections (with big impacts)
Universal Truths: Conservation Laws

(1) Linear, time-invariant system: (nonlinearities are small correction)

\[ \text{frequency } \omega \text{ is conserved} \]

(2) \( z \)-invariant system: (bends etc. are small correction)

\[ \text{wavenumber } \beta \text{ is conserved} \]

electric (\( E \)) and magnetic (\( H \)) fields can be chosen:

\[ E(x,y) \, e^{i(\beta z - \omega t)}, \quad H(x,y) \, e^{i(\beta z - \omega t)} \]
Sequence of Computation

1. Plot all solutions of infinite cladding as $\omega$ vs. $\beta$

   empty spaces (gaps): guiding possibilities

2. Core introduces new states in empty spaces
   — plot $\omega(\beta)$ dispersion relation

3. Compute other stuff…
Conventional Fiber: Uniform Cladding

uniform cladding, index $n$

$$\omega = \frac{c}{n} \sqrt{\beta^2 + |k_t|^2}$$

$$\geq \frac{c\beta}{n}$$

(light cone)

(light line: $\omega = c \frac{\beta}{n}$)
Conventional Fiber: **Uniform Cladding**

Uniform cladding, index $n$

Core with higher index $n'$ pulls down index-guided mode(s)

$$\omega = \frac{c}{n} \sqrt{\beta^2 + |k_t|^2}$$

$$\geq \frac{c\beta}{n}$$

$\omega = \frac{c \beta}{n'}$

**light cone**

Higher-order

Fundamental
**PCF: Periodic Cladding**

Bloch’s Theorem for periodic systems: fields can be written:

\[ E(x,y) e^{i(\beta z + k_t x_t - \omega t)} \]
\[ H(x,y) e^{i(\beta z + k_t x_t - \omega t)} \]

periodic functions on primitive cell

transverse (xy) Bloch wavevector \( k_t \)

satisfies eigenproblem (Hermitian if lossless)

\[
\nabla_{k_t,\beta} \times \frac{1}{\varepsilon} \nabla_{k_t,\beta} \times H = \frac{\omega^2}{c^2} H
\]

constraint: \( \nabla_{k_t,\beta} \cdot H = 0 \)

where:

\[
\nabla_{k_t,\beta} = \nabla + i k_t + i \beta \mathbf{\hat{z}}
\]
PCF: Cladding Eigensolution

Finite cell $\Rightarrow$ discrete eigenvalues $\omega_n$

Want to solve for $\omega_n(k_t, \beta)$, & plot vs. $\beta$ for “all” $n$, $k_t$

\[
\nabla_{k_t,\beta} \times \frac{1}{\varepsilon} \nabla_{k_t,\beta} \times H_n = \frac{\omega_n^2}{c^2} H_n
\]

constraint: $\nabla_{k_t,\beta} \cdot H = 0$

where: $\nabla_{k_t,\beta} = \nabla + ik_t + i\beta \hat{z}$

$H(x,y) \ e^{i(\beta z + k_t x_t - \omega t)}$

1. Limit range of $k_t$: irreducible Brillouin zone
2. Limit degrees of freedom: expand $H$ in finite basis
3. Efficiently solve eigenproblem: iterative methods
PCF: Cladding Eigensolution

1. Limit range of $k_t$: irreducible Brillouin zone

   — Bloch’s theorem: solutions are periodic in $k_t$

   \[
   \text{first Brillouin zone} = \text{minimum } |k_t| \text{ “primitive cell”}
   \]

   irreducible Brillouin zone: reduced by symmetry

2. Limit degrees of freedom: expand $H$ in finite basis

3. Efficiently solve eigenproblem: iterative methods
PCF: Cladding Eigensolution

1. Limit range of $k_t$: irreducible Brillouin zone

2. Limit degrees of freedom: expand $\mathbf{H}$ in finite basis
   — must satisfy constraint: $\nabla_{k_t, \beta} \cdot \mathbf{H} = 0$

Planewave (FFT) basis

$\mathbf{H}(x_t) = \sum_G H_G e^{iG \cdot x_t}$

constraint: $H_G \cdot (G + k + \beta \hat{z}) = 0$

uniform “grid,” periodic boundaries, simple code, $O(N \log N)$

Finite-element basis

constraint, boundary conditions:
Nédélec elements


nonuniform mesh, more arbitrary boundaries, complex code & mesh, $O(N)$

3. Efficiently solve eigenproblem: iterative methods
PCF: Cladding Eigensolution

1. Limit range of $k_t$: irreducible Brillouin zone

2. Limit degrees of freedom: expand $H$ in finite basis ($N$)

\[
|H\rangle = H(x_t) = \sum_{m=1}^{N} h_m b_m(x_t)
\]

solve: $\hat{A}|H\rangle = \omega^2 |H\rangle$

finite matrix problem: $Ah = \omega^2 Bh$

3. Efficiently solve eigenproblem: iterative methods

\[
\langle f | g \rangle = \int f^* \cdot g \\
A_{ml} = \langle b_m | \hat{A} | b_l \rangle \\
B_{ml} = \langle b_m | b_l \rangle
\]
PCF: Cladding Eigensolution

1. Limit range of $\mathbf{k}_i$: irreducible Brillouin zone
2. Limit degrees of freedom: expand $\mathbf{H}$ in finite basis
3. Efficiently solve eigenproblem: iterative methods

$$Ah = \omega^2 Bh$$

Slow way: compute $A$ & $B$, ask LAPACK for eigenvalues
  — requires $O(N^2)$ storage, $O(N^3)$ time

Faster way:
  — start with initial guess eigenvector $h_0$
  — iteratively improve
  — $O(Np)$ storage, $\sim O(Np^2)$ time for $p$ eigenvectors
    (p smallest eigenvalues)
PCF: Cladding Eigensolution

1. Limit range of $k$: irreducible Brillouin zone
2. Limit degrees of freedom: expand $H$ in finite basis
3. Efficiently solve eigenproblem: iterative methods

$$Ah = \omega^2 Bh$$

Many iterative methods:

— Arnoldi, Lanczos, Davidson, Jacobi-Davidson, …, Rayleigh-quotient minimization
PCF: Cladding Eigensolution

1. Limit range of $k_t$: irreducible Brillouin zone
2. Limit degrees of freedom: expand $H$ in finite basis
3. Efficiently solve eigenproblem: iterative methods

$$Ah = \omega^2 Bh$$

Many iterative methods:
- Arnoldi, Lanczos, Davidson, Jacobi-Davidson, …,
- Rayleigh-quotient minimization

for Hermitian matrices, smallest eigenvalue $\omega_0$ minimizes:

$$\omega_0^2 = \min_h \frac{h' Ah}{h' Bh}$$

“variational theorem” minimize by conjugate-gradient, (or multigrid, etc.)
PCF: Holey Silica Cladding

\[ r = 0.1a \]

\[ \omega = \beta c \]

light cone

dimensionless units: Maxwell’s equations are scale-invariant
PCF: Holey Silica Cladding

\[ r = 0.17717a \]

\[ \omega = \beta c \]

\[ \beta \left( \frac{2\pi}{a} \right) \]

\[ \omega \left( \frac{2\pi c}{a} \right) \]

light cone

\[ n = 1.46 \]
PCF: Holey Silica Cladding

\[ r = 0.22973a \]

\[ \beta \left( \frac{2\pi}{a} \right) \]

light cone

\[ \omega = \beta c \]
PCF: Holey Silica Cladding

\[ r = 0.30912a \]

\[ \beta = \frac{2\pi}{a} \]

\[ \omega = \beta c \]

Light cone
PCF: Holey Silica Cladding

\[ r = 0.34197a \]

\[ \beta (2\pi/a) \]

\[ \omega (2\pi c/a) \]

light cone

\[ \omega = \beta c \]

\[ n = 1.46 \]
PCF: Holey Silica Cladding

\[ r = 0.37193a \]

\[ \omega = \beta c \]

Light cone

Diagram showing the relation between \( \omega \) and \( \beta \) for a Holey Silica Cladding PCF.
PCF: Holey Silica Cladding

\[ r = 0.4a \]

\[ \omega = \beta c \]
PCF: Holey Silica Cladding

\[ r = 0.42557a \]

\[ \beta (2\pi/a) \]

\[ \omega = \beta c \]

light cone
PCF: Holey Silica Cladding

\[ r = 0.45a \]

light cone

index-guided modes
go here

gap-guided modes
go here
PCF: Holey Silica Cladding

\( r = 0.45a \)

above air line: guiding in air core is possible

below air line: surface states of air core
Bragg Fiber Cladding

at large radius, becomes \(~\) planar

\[ n_{\text{hi}} = 4.6 \]
\[ n_{\text{lo}} = 1.6 \]

Bragg fiber gaps (1d eigenproblem)

radial \( k_r \)
(Bloch wavevector)

\( \beta \)

0 by conservation of angular momentum

\( \beta = 0: \) normal incidence
Omnidirectional Cladding

Bragg fiber gaps (1d eigenproblem)

ω

wavenumber \( \beta \)

β = 0: normal incidence

omnidirectional (planar) reflection

e.g. light from fluorescent sources is trapped

for \( n_{hi} / n_{lo} \) big enough and \( n_{lo} > 1 \)

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Sequence of Computation

1. Plot all solutions of infinite cladding as $\omega$ vs. $\beta$

   ![Diagram showing $\omega$ vs. $\beta$ plot with a shaded area labeled "light cone" indicating empty spaces (gaps): guiding possibilities.]

2. Core introduces new states in empty spaces — plot $\omega(\beta)$ dispersion relation

3. Compute other stuff…
Computing Guided (Core) Modes

\[ \nabla_\beta \times \frac{1}{\varepsilon} \nabla_\beta \times \mathbf{H}_n = \frac{\omega_n^2}{c^2} \mathbf{H}_n \]

constraint: \[ \nabla_\beta \cdot \mathbf{H} = 0 \]

where: \[ \nabla_\beta = \nabla + i\beta \hat{\mathbf{z}} \]

magnetic field = \[ \mathbf{H}(x,y) \ e^{i(\beta z - \omega t)} \]

Same differential equation as before, …except no \( k_t \)

— can solve the same way

New considerations:

1. Boundary conditions
2. Leakage (finite-size) radiation loss
3. Interior eigenvalues
Computing Guided (Core) Modes

1. Boundary conditions

Only care about guided modes:
   - exponentially decaying outside core

Effect of boundary cond. decays exponentially
   - mostly, boundaries are irrelevant!
   periodic (planewave), conducting, absorbing all okay

2. Leakage (finite-size) radiation loss

3. Interior eigenvalues
Guided Mode in a Solid Core

small computation: only lowest-\( \omega \) band!

(~ one minute, planewave)

holey PCF light cone

fundamental mode
(two polarizations)

endlessly single mode: \( \Delta n_{\text{eff}} \) decreases with \( \lambda \)

\[
1.46 - \frac{\beta c}{\omega} = 1.46 - n_{\text{eff}}
\]

\( r = 0.3a \)
Fixed-frequency Modes?

Here, we are computing $\omega(\beta')$, but we often want $\beta(\omega')$ — $\lambda$ is specified

No problem!

Just find root of $\omega(\beta') - \omega'$, using Newton’s method:

$$\beta' \leftarrow \beta' - \frac{\omega - \omega'}{d\omega/d\beta}$$

(Factor of 3–4 in time.)

group velocity $= \text{power} / (\text{energy density})$

(a.k.a. Hellman-Feynman theorem, a.k.a. first-order perturbation theory, a.k.a. “$k$-dot-$p$” theory)
Computing Guided (Core) Modes

1. Boundary conditions

Only care about guided modes:
   — exponentially decaying outside core

Effect of boundary cond. decays exponentially
   — mostly, boundaries are irrelevant!
   periodic (planewave), conducting, absorbing all okay

…except when we want
(small) finite-size losses…

2. Leakage (finite-size) radiation loss

3. Interior eigenvalues
Computing Guided (Core) Modes

1. Boundary conditions
2. Leakage (finite-size) radiation loss
   Use PML absorbing boundary layer
   perfectly matched layer
   …with iterative method that works for
   non-Hermitian (dissipative) systems:
   Jacobi-Davidson, …

Or imaginary-distance BPM:
   in imaginary $z$, largest $\beta$ (fundamental) mode grows exponentially

3. Interior eigenvalues
Computing Guided (Core) Modes

1. Boundary conditions
2. Leakage (finite-size) radiation loss
   
   imaginary-distance BPM
   

3. Interior eigenvalues

2 rings

3 rings
Computing Guided (Core) Modes

1. Boundary conditions
2. Leakage (finite-size) radiation loss
3. Interior eigenvalues

Gap-guided modes lie above continuum (~ N states for N-hole cell)

...but most methods compute smallest ω (or largest β)

Computing Guided (Core) Modes

1. Boundary conditions
2. Leakage (finite-size) radiation loss
3. Interior (of the spectrum) eigenvalues

- Gap-guided modes lie above continuum (~ $N$ states for $N$-hole cell)
- but most methods compute smallest $\omega$ (or largest $\beta$)

- Compute $N$ lowest states first: deflation (orthogonalize to get higher states) [see previous slide]
- Use interior eigensolver method—closest eigenvalues to $\omega_0$ (mid-gap)
  - Jacobi-Davidson,
  - Arnoldi with shift-and-invert,
  - smallest eigenvalues of $(A-\omega_0^2)^2$
  - convergence often slower
- Other methods: FDTD, etc…
Interior Eigenvalues by FDTD
finite-difference time-domain

Simulate Maxwell’s equations on a discrete grid,
+ PML boundaries + $e^{i\beta z}$ z-dependence

- Excite with broad-spectrum dipole (↑) source

complex $\omega_n$

signal processing


Response is many sharp peaks,
one peak per mode

decay rate in time gives loss: $\text{Im}[\beta] = - \frac{\text{Im}[\omega]}{d\omega/d\beta}$
Interior Eigenvalues by FDTD

finite-difference time-domain

Simulate Maxwell’s equations on a discrete grid, + PML boundaries + $e^{i\beta z}$ z-dependence

• Excite with broad-spectrum dipole (↑) source

Response is many sharp peaks, one peak per mode

mode field profile

narrow-spectrum source
An Easier Problem: Bragg-fiber Modes

In each concentric region, solutions are Bessel functions:

\[ c J_m(kr) + d Y_m(kr) \times e^{im \phi} \]

\[ k = \sqrt{\left( \frac{\omega}{c} \right)^2 \varepsilon - \beta^2} \]

“angular momentum”

At circular interfaces match boundary conditions with 4 × 4 transfer matrix

…search for complex \( \beta \) that satisfies: finite at \( r=0 \), outgoing at \( r=\infty \)

[ Johnson, Opt. Express 9, 748 (2001) ]
Hollow Metal Waveguides, Reborn

metal waveguide modes

OmniGuide fiber modes

1970’s microwave tubes @ Bell Labs

frequency $\omega$

wavenumber $\beta$

modes are directly analogous to those in hollow metal waveguide
An Old Friend: the \textbf{TE}_{01} mode

**lowest-loss mode,**
just as in metal

(near) \textit{node at interface}
\hspace{1cm} = strong confinement
\hspace{1cm} = low losses

\textit{non-degenerate mode}
\hspace{1cm} — cannot be split
\hspace{1cm} = no birefringence or PMD
Bushels of Bessels
— A General Multipole Method


Each cylinder has its own Bessel expansion:

$$\text{field} \sim \sum_{m}^{M} c_{m} J_{m} + d_{m} Y_{m}$$

$(m\text{ is not conserved})$

With $N$ cylinders,
get $2NM \times 2NM$ matrix of boundary conditions

Solution gives full complex $\beta$,
but takes $O(N^3)$ time
— more than 4–5 periods is difficult

future: “Fast Multipole Method” should reduce to $O(N \log N)$?
Outline

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• Finding the guided modes

• Small corrections (with big impacts)
All Imperfections are Small
(or the fiber wouldn’t work)

- **Material absorption:** small imaginary $\Delta \varepsilon$
- **Nonlinearity:** small $\Delta \varepsilon \sim |E|^2$
- **Acircularity** (birefringence): small $\varepsilon$ boundary shift
- **Bends:** small $\Delta \varepsilon \sim \Delta x / R_{\text{bend}}$
- **Roughness:** small $\Delta \varepsilon$ or boundary shift

Weak effects, long distances: hard to compute directly
— use perturbation theory
Perturbation Theory and Related Methods
(Coupled-Mode Theory, Volume-Current Method, etc.)

*Given solution for ideal system*

compute approximate effect of small changes

...solves hard problems starting with easy problems

& provides (semi) analytical insight
Perturbation Theory
for Hermitian eigenproblems

given eigenvectors/values: \( \hat{O}|u\rangle = u|u\rangle \)

…find change \( \Delta u \) \& \( \Delta |u\rangle \) for small \( \Delta \hat{O} \)

**Solution:**
expand as power series in \( \Delta \hat{O} \)

\[
\Delta u = 0 + \Delta u^{(1)} + \Delta u^{(2)} + \ldots
\]

\[
\Delta u^{(1)} = \frac{\langle u|\Delta \hat{O}|u\rangle}{\langle u|u\rangle}
\]

& \( \Delta |u\rangle = 0 + \Delta |u\rangle^{(1)} + \ldots \)

(first order is usually enough)
Perturbation Theory

for electromagnetism

\[ \Delta \omega^{(1)} = \frac{c^2}{2 \omega} \frac{\langle H | \Delta \hat{A} | H \rangle}{\langle H | H \rangle} \]

\[ = - \frac{\omega}{2} \int \Delta \varepsilon |E|^2 \]

\[ = \frac{d\omega}{d\beta} \]

…e.g. absorption gives imaginary \( \Delta \omega \)

= decay!
A Quantitative Example

Gas can have low loss & nonlinearity

...but what about the cladding?

...some field penetrates!

& may need to use very "bad" material to get high index contrast
Suppressing Cladding Losses

Material absorption: small imaginary $\Delta \varepsilon$

Mode Losses \( \div \) Bulk Cladding Losses

Large differential loss

$\text{TE}_{01}$ strongly suppresses cladding absorption

(like ohmic loss, for metal)
High-Power Transmission
at 10.6µm (no previous dielectric waveguide)

Polymer losses @10.6µm ~ 50,000dB/m…

…waveguide losses ~ 1dB/m


[ figs courtesy Y. Fink et al., MIT ]
Quantifying Nonlinearity

Kerr nonlinearity: small $\Delta \varepsilon \sim |E|^2$

$\Delta \beta \sim \text{power } P \sim 1 / \text{lengthscale for nonlinear effects}$

$\gamma = \Delta \beta / P$

= nonlinear-strength parameter determining
self-phase modulation (SPM), four-wave mixing (FWM), …

(unlike “effective area,”
tells where the field is,
not just how big)
Suppressing Cladding Nonlinearity

Mode Nonlinearity* ÷ Cladding Nonlinearity

Will be dominated by nonlinearity of air

~10,000 times weaker than in silica fiber (including factor of 10 in area)

* “nonlinearity” = $\Delta \beta^{(1)} / P = \gamma$

\[ \lambda (\mu m) \]

\[ 10^{-9} \]

\[ 10^{-8} \]

\[ 10^{-7} \]

\[ 10^{-6} \]
Acircularity & Perturbation Theory
(or any shifting-boundary problem)

\[ \Delta \varepsilon = \varepsilon_1 - \varepsilon_2 \]
\[ \Delta \varepsilon = \varepsilon_2 - \varepsilon_1 \]

… just plug \( \Delta \varepsilon \)'s into perturbation formulas?

FAILS for high index contrast!

beware field discontinuity…
fortunately, a simple correction exists

[ S. G. Johnson et al., *PRE* 65, 066611 (2002) ]
Acircularity & Perturbation Theory
(or any shifting-boundary problem)

\[ \Delta \varepsilon = \varepsilon_1 - \varepsilon_2 \]

\[ \Delta \varepsilon = \varepsilon_2 - \varepsilon_1 \]

\[ \Delta \omega^{(1)} = -\frac{\omega_{\text{surf.}}}{2} \int \Delta h \left[ \Delta \varepsilon |E_\parallel|^2 - \Delta \frac{1}{\varepsilon} |D_\perp|^2 \right] \int \varepsilon |E|^2 \]

[ S. G. Johnson et al., PRE 65, 066611 (2002) ]
Loss from Roughness/Disorder

imperfection acts like a volume current

\[ \vec{J} \sim \Delta \varepsilon \vec{E}_0 \]

volume-current method

or Green’s functions with first Born approximation
Loss from Roughness/Disorder

imperfection acts like a volume current

\[ \vec{J} \sim \Delta \varepsilon \vec{E}_0 \]

For surface roughness, including field discontinuities:

\[ \vec{J} \sim \Delta \varepsilon \vec{E}_\parallel - \varepsilon \Delta \varepsilon^{-1} \vec{D}_\perp \]
Loss from Roughness/Disorder

uncorrelated disorder adds *incoherently*

So, compute power $P$ radiated by *one* localized source $J$, and *loss rate* $\sim P \ast$ (mean disorder strength)
Effect of an *Incomplete* Gap
on uncorrelated surface roughness loss

Conventional waveguide
(matching modal area)

…with Si/SiO₂ Bragg mirrors (1D gap)
50% lower losses (in dB)
same reflection

some radiation blocked
Considerations for Roughness Loss

• Band gap can suppress some radiation
  — typically by at most $\sim 1/2$, depending on crystal

• Loss $\sim \Delta \varepsilon^2 \sim 1000$ times larger than for silica

• Loss $\sim$ fraction of $|\mathbf{E}|^2$ in solid material
  — factor of $\sim 1/5$ for 7-hole PCF
  — $\sim 10^{-5}$ for large-core Bragg-fiber design

• Hardest part is to get reliable statistics for disorder.
Using perturbations to design big effects
Perturbation Theory and Dispersion

when two distinct modes cross & interact, unusual dispersion is produced

\[ \omega \]

\[ \beta \]

no interaction/coupling

mode 1

mode 2
Perturbation Theory and Dispersion

when two distinct modes cross & interact, unusual dispersion is produced

coupling: anti-crossing
Two Localized Modes
= Very Strong Dispersion

weak coupling
= rapid slope change
= high dispersion
(> 500,000 ps/nm-km
+ dispersion-slope matching)

(Different-Symmetry) Slow-light Modes

= Anomalous Dispersion

\[ \omega \]

\[ \beta = 0 \]

slow-light band edges at \( \beta = 0 \)

\[ \beta = 0 \] point has additional symmetry:
- modes can be purely TE/TM polarized
- force different symmetry modes together

(Different-Symmetry) Slow-light Modes
= Anomalous Dispersion

(Different-Symmetry) Slow-light Modes = Anomalous Dispersion

Uses gap at $\beta=0$:

- perfect metal [1960]
- or Bragg fiber
- or high-index PCF ($n > 2.5$)

Further Reading

**Reviews:**


**This Presentation, Free Software, Other Material:**

http://ab-initio.mit.edu/photons/tutorial