Photonic Crystals: Periodic Surprises in Electromagnetism

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Complete Band Gaps:
You can leave home without them.
How else can we confine light?
Total Internal Reflection

$n_i > n_o$

rays at shallow angles $> \theta_c$
are totally reflected

Snell’s Law:

$$n_i \sin \theta_i = n_o \sin \theta_o$$

$$\sin \theta_c = \frac{n_o}{n_i}$$

$< 1$, so $\theta_c$ is real

$i.e.$ TIR can only guide within higher index
unlike a band gap
Total Internal Reflection?

$n_i > n_o$

rays at shallow angles $> \theta_c$
are totally reflected

So, for example,
a discontiguous structure can’t possibly guide by TIR…

the rays can’t stay inside!
Total Internal Reflection?

$n_i > n_o$

rays at shallow angles $> \theta_c$
are totally reflected

So, for example,
a discontiguous structure can’t possibly guide by TIR…

or can it?
Total Internal Reflection Redux

$n_i > n_o$

ray-optics picture is invalid on $\lambda$ scale
(neglects coherence, near field…)

Snell’s Law is really
conservation of $k_\parallel$ and $\omega$:

$$|k_i| \sin \theta_i = |k_o| \sin \theta_o$$

$$|k| = n\omega/c$$

(translational symmetry)

conserved!
Waveguide Dispersion Relations
\(i.e.\) projected band diagrams

\[ n_i > n_o \]

*light cone*
projection of all \( k_\perp \) in \( n_o \)

*light line*:
\[ \omega = ck / n_o \]

\( \omega = ck / n_i \)

higher-index core
*pulls down* state

higher-order modes
at larger \( \omega, \beta \)

weakly guided (field mostly in \( n_o \))

\( n_i > n_o \)
Strange Total Internal Reflection

Index Guiding

light cone

Conserved $k$ and $\omega$
+ higher index to pull down state
= localized/guided mode.
A Hybrid Photonic Crystal:
1d band gap + index guiding

light cone

“band gap”

range of frequencies in which there are no guided modes

slow-light band edge
A Resonant Cavity

index-confined

increased rod radius pulls down “dipole” mode (non-degenerate)

photonic band gap
A Resonant Cavity

The trick is to keep the radiation small… (more on this later)

$k$ not conserved
so coupling to light cone: radiation
Meanwhile, back in reality...

**Air-bridge Resonator:** 1d gap + 2d index guiding

Time for Two Dimensions...

2d is all we really need for many interesting devices

…darn z direction!
How do we make a 2d bandgap?

Most obvious solution?

make 2d pattern really tall
How do we make a 2d bandgap?

If height is finite, we must couple to out-of-plane wavevectors...

$k_z$ not conserved
A 2d band diagram in 3d

Let’s start with the 2d band diagram.

This is what we’d like to have in 3d, too!

Square Lattice of Dielectric Rods
($\varepsilon = 12$, $r=0.2a$)

wavevector

frequency (c/a)

TE bands

TM bands
A 2d band diagram in 3d

Let’s start with the 2d band diagram.

This is what we’d like to have in 3d, too!

No! When we include **out-of-plane** propagation, we get:

- wavevector \( \vec{k} \)
- frequency \( \omega \)
- \( \omega + \delta \omega \)

**projected band diagram fills gap!**

Square Lattice of Dielectric Rods
\( (\varepsilon = 12, r=0.2a) \)

but this empty space looks useful…
Photonic-Crystal Slabs

2d photonic bandgap + vertical index guiding

[ S. G. Johnson and J. D. Joannopoulos, *Photonic Crystals: The Road from Theory to Practice* ]
Square Lattice of Dielectric Rods
($\varepsilon = 12$, $r=0.2a$, $h=2a$)

The Light Cone:
All possible states propagating in the air

The Guided Modes:
Cannot couple to the light cone…
$\rightarrow$ confined to the slab

Thickness is critical.
Should be about $\lambda/2$
(to have a gap & be single-mode)
Symmetry in a Slab

2d: TM and TE modes

slab: odd (TM-like) and even (TE-like) modes

Like in 2d, there may only be a band gap in one symmetry/polarization
Slab Gaps

Square Lattice of Dielectric Rods
($\varepsilon = 12, r=0.2a, h=2a$)

Triangular Lattice of Air Holes
($\varepsilon = 12, r=0.3a, h=0.5a$)

light cone

frequency (c/a)

odd (TM-like) bands
even (TE-like) bands

TM-like gap

TE-like gap
Substrates, for the Gravity-Impaired

(rods or holes)

substrate breaks symmetry:
some even/odd mixing “kills” gap

BUT

with strong confinement
(high index contrast)

mixing can be weak

superstrate restores symmetry

“extruded” substrate
= stronger confinement

(less mixing even without superstrate)
Extruded Rod Substrate

S. Assefa, L. A. Kolodziejski

high index

2 μm
Air-membrane Slabs

who needs a substrate?

Optimal Slab Thickness

$\sim \lambda/2$, but $\lambda/2$ in what material?

effective medium theory: effective $\varepsilon$ depends on polarization
Photonic-Crystal Building Blocks

point defects (cavities)

line defects (waveguides)
A Reduced-Index Waveguide

We cannot completely remove the rods—no vertical confinement!

Still have conserved wavevector—under the light cone, no radiation

Reduce the radius of a row of rods to “trap” a waveguide mode in the gap.
Reduced-Index Waveguide Modes
Experimental Waveguide & Bend


caution: can easily be multi-mode

bending efficiency

waveguide mode

band gap

wavelength (µm)

1µm GaAs AlO SiO₂

1µm

1200 1300 1400 1500 1600 1700 1800
Inevitable Radiation Losses
whenever translational symmetry is broken

e.g. at cavities, waveguide bends, disorder…

\[ k \text{ is no longer conserved!} \]

\[ \omega \text{ (conserved)} \]

\[ \text{coupling to light cone} = \text{radiation losses} \]
All Is Not Lost

A simple model device (filters, bends, …):

\[
\frac{1}{Q} = \frac{1}{Q_r} + \frac{1}{Q_w}
\]

\[Q = \text{lifetime/period} = \text{frequency/bandwidth}\]

We want: \(Q_r \gg Q_w\)

\[1 - \text{transmission} \sim \frac{2Q}{Q_r}\]

\textit{worst case:} high-Q (narrow-band) cavities
Semi-analytical losses

A low-loss strategy:

\[
\ddot{E}(\ddot{x}) = \int G_\omega (\ddot{x}, \ddot{x}') \cdot \ddot{E}(\ddot{x}') \cdot \Delta \varepsilon(\ddot{x}')
\]

Make field inside defect small
= delocalize mode

Make defect weak
= delocalize mode

far-field (radiation)
defect
Green’s function (defect-free system)
near-field (cavity mode)
defect
Monopole Cavity in a Slab

Lower the $\varepsilon$ of a single rod: push up a monopole (singlet) state.

Use small $\Delta\varepsilon$: delocalized in-plane, & high-Q (we hope)
Delocalized Monopole Q

\[ \Delta \text{frequency above band edge (c/a)} \]

\( e = 6 - 11 \)
Super-defects

Weaker defect with more unit cells.

More delocalized
at the same point in the gap
(i.e. at same bulk decay rate)
Super-Defect vs. Single-Defect Q

\[ Q_r = \frac{100}{1000} \]
Super-Defect vs. Single-Defect Q
Super-Defect State
(cross-section)

$\Delta \varepsilon = -3$, $Q_{\text{rad}} = 13,000$

still $\sim$ localized: \textit{In-plane} $Q_{||}$ is $> 50,000$ for only 4 bulk periods
Hole Slab

ε = 11.56
period a, radius 0.3a
thickness 0.5a

Reduce radius of 7 holes to 0.2a

Q = 2500 near mid-gap (Δfreq = 0.03)

Very robust to roughness (note pixellization, a = 10 pixels).
How do we compute $Q$?
(via 3d FDTD [finite-difference time-domain] simulation)

1. excite cavity with dipole source (broad bandwidth, e.g. Gaussian pulse)

   … monitor field at some point.

   …extract frequencies, decay rates via signal processing (FFT is suboptimal)


**Pro**: no *a priori* knowledge, get all ω’s and Q’s at once

**Con**: no separate $Q_w/Q_r$, $Q > 500,000$ hard, mixed-up field pattern if multiple resonances
How do we compute $Q$?
(via 3d FDTD [finite-difference time-domain] simulation)

2. excite cavity with narrow-band dipole source
(e.g. temporally broad Gaussian pulse)
— source is at $\omega_0$ resonance, which must already be known (via 1)

...measure outgoing power $P$ and energy $U$

$$Q = \omega_0 \frac{U}{P}$$

Pro: separate $Q_w/Q_r$, arbitrary $Q$, also get field pattern

Con: requires separate run 1 to get $\omega_0$,
long-time source for closely-spaced resonances
Can we increase Q without delocalizing?
Semi-analytical losses

Another low-loss strategy:

\[ \vec{E}(\vec{x}) = \int \vec{G}_\omega(\vec{x}, \vec{x}') \cdot \vec{E}(\vec{x}') \cdot \Delta \varepsilon(\vec{x}') \]

- **Green’s function** (defect-free system)
- far-field (radiation)
- **defect**
- near-field (cavity mode)
- **defect**

exploit cancellations from sign oscillations
Need a more compact representation

Cannot cancel infinitely many $E(x)$ integrals

Radiation pattern from localized source...

— use multipole expansion & cancel largest moment
Multipole Expansion

[ Jackson, *Classical Electrodynamics* ]

\[
\text{radiated field} = \text{dipole} + \text{quadrupole} + \text{hexapole} + \ldots
\]

Each term’s strength = single integral over near field
…one term is cancellable by tuning one defect parameter
Multipole Expansion

[ Jackson, *Classical Electrodynamics* ]

radiated field =

\[
\text{dipole} + \text{quadrupole} + \text{hexapole} + \ldots
\]

peak Q (cancellation) = transition to higher-order radiation
Multipoles in a 2d example

index-confined

increased rod radius pulls down “dipole” mode (non-degenerate)

as we change the radius, $\omega$ sweeps across the gap
2d multipole cancellation

- $r = 0.35a$: $Q = 1,773$
- $r = 0.375a$: $Q = 28,700$
- $r = 0.40a$: $Q = 6,624$

$Q$ vs. frequency (c/a)
cancel a dipole by opposite dipoles…

cancellation comes from opposite-sign fields in adjacent rods

… changing radius changed balance of dipoles
3d multipole cancellation?

quadrupole mode

(E_z cross section)

enlarge center & adjacent rods

vary side-rod ε slightly
for continuous tuning
(balance central moment with opposite-sign side rods)

quadrupole mode

gap bottom

frequency (c/a)

gap top

Q

0.319243
0.390536
3d multipole cancellation

Q = 408
Q = 1925
Q = 426

near field $E_z$

far field $|E|^2$

nodal planes (source of high Q)
An Experimental (Laser) Cavity


Elongation $p$ is a tuning parameter for the cavity…

…in simulations, Q peaks sharply to $\sim 10000$ for $p = 0.1a$

(likely to be a multipole-cancellation effect)

* actually, there are two cavity modes; $p$ breaks degeneracy
An Experimental (Laser) Cavity


Elongation $p$ is a tuning parameter for the cavity…

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An Experimental (Laser) Cavity

[M. Loncar et al., Appl. Phys. Lett. 81, 2680 (2002)]

Quantum-well lasing threshold of 214µW (optically pumped @830nm, 1% duty cycle)
How can we get *arbitrary* $Q$ with *finite* modal volume?

Only one way:

a **full 3d band gap**

Now there are two ways.

The Basic Idea, in 2d

-start with:
-junction of two waveguides

\[ e_1 < e_2 < e_1' \]
\[ e_2' < e_1' \]

No radiation at junction
if the modes are perfectly matched
Perfect Mode Matching

requires:

same differential equations and boundary conditions

\[ \varepsilon_1 \]
\[ \varepsilon_2 < \varepsilon_1 \]

\[ \varepsilon_1' \]
\[ \varepsilon_2' < \varepsilon_1' \]

Match differential equations…

\[ \varepsilon_2 - \varepsilon_1 = \varepsilon_2' - \varepsilon_1' \]

…closely related to separability

Perfect Mode Matching

requires:
same differential equations and boundary conditions

<table>
<thead>
<tr>
<th>$\varepsilon_1$</th>
<th>$\varepsilon_1'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_2 &lt; \varepsilon_1$</td>
<td>$\varepsilon_2' &lt; \varepsilon_1'$</td>
</tr>
</tbody>
</table>

Match boundary conditions:
field must be TE
($E$ out of plane, in 2d)

(note switch in TE/TM convention)
TE modes in 3d

for cylindrical waveguides, “azimuthally polarized”

$TE_{0n}$ modes
A Perfect Cavity in 3d
(~ VCSEL + perfect lateral confinement)

Perfect index confinement (no scattering) + 1d band gap = 3d confinement
A Perfectly Confined Mode

$\varepsilon_1, \varepsilon_2 = 9, 6$

$\varepsilon_1', \varepsilon_2' = 4, 1$

E energy density, vertical slice
Q limited only by finite size

\[ V = (0.4\lambda)^3 \]

\[ N = 5 \]

\[ N = 10 \]
Q-tips

Three independent mechanisms for high Q:

**Delocalization:** trade off modal size for Q
  \[ Q_r \text{ grows monotonically towards band edge} \]

**Multipole Cancellation:** force higher-order far-field pattern
  \[ Q_r \text{ peaks inside gap} \]
  New nodal planes appear in far-field pattern at peak

**Mode Matching:** allows arbitrary Q, finite V
  Requires special symmetry & materials
Forget these devices…

I just want a mirror.

ok
Projected Bands of a 1d Crystal
(a.k.a. a Bragg mirror)

- Modes in crystal
- Incident light
- $k_\parallel$ conserved
- $\omega = ck_\parallel$
- Light line of air
- 1d band gap
- Modes in crystal
- (normal incidence)
Omnidirectional Reflection


in these ω ranges, there is no overlap between modes of air & crystal

all incident light (any angle, polarization) is reflected from flat surface

needs: sufficient index contrast & $n_{hi} > n_{lo} > 1$
Omnidirectional Mirrors in Practice


contours of omnidirectional gap size

Index ratio, $n_2 / n_1$

Smaller index, $n_1$

$\Delta \lambda / \lambda_{mid}$

Reflectance (%)

Te / polystyrene

normal

$45^\circ$ s

$45^\circ$ p

$80^\circ$ s

$80^\circ$ p

Wavelength (microns)